

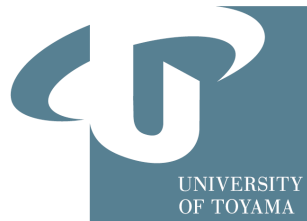
博士学位請求論文

**Higgs physics in the supersymmetric  
grand unified theory  
with the Hosotani mechanism**

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March 2014



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To my parents

## ACKNOWLEDGMENTS

I would like to express my sincere gratitude to my supervisor, Prof. Shinya Kanemura for providing me much instruction in particle physics and its philosophy. I am very grateful to Prof. Mitsuru Kakizaki and Prof. Toshifumi Yamashita for fruitful collaborations. I thank Prof. Shigeki Matsumoto, Prof. Yasuhiro Okada, Prof. Koji Tsumura and Dr. Takehiro Nabeshima for useful collaborations. I am also grateful to Prof. Takeshi Kurimoto for many instructive advices. I would like to thank Prof. Yutaka Hosotani, Prof. Kaori Kobayashi, Prof. Yoshiki Moriwaki and Prof. Takashi Tayama for careful reading of this thesis. Special thanks are also given to the secretary of our laboratory, Ms. Asami Takagi for her kind cooperations and helps. I would also like to thank the other members of Theoretical Physics Group, University of Toyama, Dr. Hiroshi Yokoya, Dr. Kei Yagyu, Dr. Hiroaki Sugiyama, Dr. Makoto Nakamura, Ms. Mariko Kikuchi, Mr. Naoki Machida, Mr. Toshinori Matsui, Mr. Ryo Hosaka, Mr. Hiroyuki Okuhara, Mr. Akiteru Santa and Mr. Norihito Moki for their hospitality.

## Abstract

In 2012, a new boson was discovered with the mass 126 GeV at the Large Hadron Collider (LHC). After that, the spin of this boson turned out to be zero. It was also confirmed that this boson couples to many of the standard model (SM) particles. Consequently, the new particle was identified to be a Higgs boson, which plays a role of triggering spontaneous breaking of the electroweak gauge symmetry and generating masses of elementary particles. Furthermore, its coupling strength to the other particles seems to be consistent with the prediction in the SM within the error of the current data. Therefore, the SM was found to be a very successful model even with including the Higgs sector.

However, none of the researchers believes that the SM is a fundamental theory of particle physics. In the SM, only Electromagnetic Force and Weak Force are unified to Electroweak Force at the energy scale above 100 GeV, which is written as a quantum gauge field theory with the gauge symmetry  $SU(2)_L \times U(1)_Y$ . On the other hand, Strong Force is described as an independent gauge theory with  $SU(3)_C$ , the quantum chromodynamics. Consequently, the gauge structure of the SM is given by  $SU(3)_C \times SU(2)_L \times U(1)_Y$ . From the viewpoint of the history of unification of law in physics, the SM is nothing but a low energy effective theory of a more fundamental theory such as the grand unified theory (GUT), in which the gauge structure in the SM is derived from a more simple structure such as  $SU(5)$ ,  $SO(10)$ , and so on.

Another reason why the SM is not a fundamental theory arises from its Higgs sector. As the Higgs field is the order parameter of electroweak gauge symmetry breaking, it must be a scalar field. It has been known that the introduction of such a scalar field into a gauge field theory is problematic, yielding quadratic ultraviolet divergences in the radiative correction to the mass of the Higgs boson. Renormalization of these quadratic divergences bring a serious problem of huge fine-tuning, so-called the hierarchy problem. Clearly, a new physics model beyond the SM is necessary at the TeV scale, in which the quadratic divergences are canceled and the hierarchy problem disappears.

Introduction of supersymmetry (SUSY) at the TeV scale is a charming idea. SUSY is the symmetry between bosons and fermions. The quadratic divergences from a bosonic loop contribution and from a fermionic loop contribution are canceled with each other in SUSY theories. Supersymmetric extensions of the SM have been extensively investigated for previous three decades. The minimal supersymmetric standard model (MSSM) predicts a set of new particles at the TeV scale, SUSY partner particles of the SM ones. In addition, the Higgs sector is predicted to be composed of two Higgs doublets in the MSSM, in which the mass of the SM-like Higgs boson is preferred to be less than about 120 GeV without large fine-tuning. The recent LHC results that the mass of the Higgs boson is 126 GeV and that no other new particles has not been found yet seem to indicate the tension between the MSSM and the hierarchy problem. Some extended models from the MSSM are also investigated to relax this tension. In the next-to-minimal supersymmetric standard model (NMSSM), the mass of the Higgs boson can be greater than that in the MSSM, and the experiment value of the Higgs boson mass may be explained easier.

SUSY-GUTs are attractive, because the hierarchy problem disappears and the gauge couplings are unified with better accuracy as compared to the GUT without SUSY. The unification scale is typically  $10^{16}$  GeV. However, SUSY-GUTs also have a new fine-tuning problem. In these models, there is the mass splitting between the color triplet and the  $SU(2)_L$  doublet Higgs

fields, which arises from common multiplet. This is so-called the doublet-triplet splitting problems. On the other hand, in general, it is very difficult to test models of GUTs at the collider experiments. The GUT scale is typically  $\mathcal{O}(10^{16})$  GeV that is inferred from gauge coupling unification scale. Due to the decoupling theorem, it is difficult to test GUTs at the high energy collider experiments. Currently, tests of GUTs rely on checking relations among parameters of new particles by flavor experiments indirectly.

In this thesis, we discuss a new type of GUTs, the supersymmetric grand unified theory with the Hosotani mechanism (SGGHU). This model was proposed to solve the doublet-triplet problem. This model predicts the existence of adjoint chiral superfields whose quantum numbers are equal to the gauge bosons in the SM, and masses are at the supersymmetric breaking scale, namely at the TeV scale. The Higgs sector is extended with additional  $SU(2)_L$  triplet and singlet chiral multiplets. Therefore, properties of particles in the Higgs sector are different from the SM, the MSSM and other models. In this thesis, phenomenological analyses of the SGGHU, especially the mass spectrum of the particles in the model and the coupling constants of the Higgs sector are investigated. We calculate deviations in coupling constants of the standard model-like Higgs boson and the mass spectrum of the additional Higgs bosons. We find that our model is distinguishable from the others by precision measurements of these couplings and masses of the additional Higgs bosons. We show the testability of our model at the collider experiments such as the luminosity up-graded Large Hadron Collider and the International Linear Collider.



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# Chapter 1

## Introduction

The standard model (SM) for particle physics has been successful in describing high-energy phenomena at collider experiments. Particles in the SM cannot have masses under the gauge symmetry  $SU(3)_C \times SU(2)_L \times U(1)_Y$ . In order to give masses to the SM particles, the electroweak symmetry ( $SU(2)_L \times U(1)_Y$ ) is broken into the electromagnetic symmetry ( $U(1)_{EM}$ ) by introducing an  $SU(2)_L$  doublet which obtains the vacuum expectation value (VEV) by the spontaneous symmetry breaking. Consequently, quarks, charged leptons and weak gauge bosons obtain their mass. In July 2012 at the Large Hadron Collider (LHC) a new boson with the mass of 126 GeV was found [1]. Since spin-2 was excluded with 99% C.L. by ATLAS [2] and CMS [3], it was confirmed that the spin of this boson is 0. Its coupling to many of the SM particles was also confirmed. From these facts, a new particle was identified to be a Higgs boson. In addition, its coupling strength to the SM particles is almost correspond to the prediction in the SM within the error of the current data. The SM is likely to be correct at the moment.

However, there are good reasons to consider new physics beyond the SM. We already know phenomenological problems such as the existence of dark matter [4], baryon asymmetry of the universe and neutrino oscillation, all of which cannot be explained in the SM. The SM also has a theoretical problem, so-called the hierarchy problem. Since quadratic ultraviolet divergences appear in the radiative correction to the mass of the Higgs boson, a huge cancelation between its bare mass and loop contribution is required for the renormalization of the Higgs boson mass. It is a serious fine-tuning. We need a new physics model which can solve this problem.

The Higgs sector in the SM has the minimal form with only one  $SU(2)_L$  doublet scalar field, which is just a hypothesis. Actually, some of new physics models have extended Higgs sectors. For instance, the Higgs sector in the minimal supersymmetric standard model (MSSM) is composed of two doublet scalar fields. The Higgs sector in some models which can explain the mass of the neutrino contains an additional  $SU(2)_L$  triplet scalar field. If the form of the Higgs sector is experimentally determined, we can decide a new paradigm for physics beyond the SM. Even though no other new particles is found directly at the collider experiments, we can obtain properties of the Higgs sector by an indirect way. Measuring the Higgs boson couplings with high accuracy, we could obtain deviations in Higgs couplings from the SM predictions. Using these deviations and finding the pattern of deviations at future collider experiments, the Higgs sector could be determined by fingerprinting with the data. With this guide, we focus on physics of the Higgs sector.

It has been known that the supersymmetry (SUSY) can solve the hierarchy problem. SUSY is the symmetry between bosons and fermions. Since the loop effect of scalar bosons is opposite

from that of fermions, the quadratic divergences from a bosonic loop contribution and from a fermionic loop are canceled in SUSY models. In the MSSM, the Higgs boson is lighter than the  $Z$ -boson at the tree-level. For this reason, the 126 GeV Higgs boson needs large loop corrections with heavy superparticles. On the other hand, since an extended Higgs sector with a singlet scalar field from the MSSM Higgs sector, so-called the next-to-MSSM (NMSSM [5]), has an additional quartic coupling via the F-term, the Higgs boson can become relatively heavy without requiring heavy superparticles as compared with the one of the MSSM [6, 7].

From the viewpoint of the history of unification of law in physics, it is natural to consider that the SM is a low energy effective theory of a more fundamental theory such as grand unified theory (GUT [8]). Since the GUTs unify the gauge groups in the SM and quantize the electric charge, the GUTs are very attractive models.

Since the SUSY solves the hierarchy problem, SUSY-GUTs [9] are well-motivated models of physics beyond-the-SM. However, in the SUSY-GUTs, the GUT breaking scale is typically  $\mathcal{O}(10^{16})$  GeV inferred from the gauge coupling unification. Due to the decoupling theorem [10], it is difficult to test the SUSY-GUTs at collider experiments, because effects of such a extremely heavy particles on the low energy effective theory are negligible. At the same time, there is another difficulty. In the  $SU(5)$  SUSY-GUT, the  $SU(2)_L$  doublet fields necessarily accompany color triplets Higgs fields. The color triplet Higgs fields are supposed to be as heavy as the GUT scale for the proton lifetime, but  $SU(2)_L$  doublet Higgs fields should be around  $\mathcal{O}(10^2)$  GeV for the electroweak symmetry breaking. That is, the  $SU(5)$  SUSY-GUT also has a fine-tuning problem of the mass splitting between the color triplet and  $SU(2)_L$  doublet Higgs fields, which are generated from common multiplet, so-called doublet-triplet (DT) splitting problem [11–16].

In this thesis, we consider the supersymmetric grand unified theory with the Hosotani mechanism, so-called the supersymmetric grand Gauge-Higgs unification (SGGHU). In this model, the unified symmetry is broken by the Hosotani mechanism [17]. The extra-dimensional component of the gauge field causes the symmetry breaking. It is known that the SGGHU realizes naturally the DT splitting [18]. Furthermore this model predicts the existence of chiral adjoint superfields, color octet,  $SU(2)_L$  triplet and singlet at the SUSY breaking scale, whose gauge quantum numbers are the same as the SM gauge bosons. We investigate the Higgs sector of this model which is extended from that of the MSSM by the  $SU(2)_L$  triplet and the singlet chiral multiplets. Due to couplings between the MSSM Higgs doublet and new Higgs triplet and singlet, the SM-like Higgs boson mass can become large as compared with the prediction of the MSSM. We derive values of parameters in the low energy effective theory using the Renormalization Group Equations (RGEs), and evaluate the masses and couplings of the SGGHU Higgs sector particles. We show the testability of our model at future collider experiments such as the International Linear Collider (ILC).

This thesis is organized as follows. In Chapter. 2, we review the SM. After the spontaneous symmetry breaking, quarks, charged leptons, weak gauge bosons, and the Higgs boson obtain their mass. The parameter in the Higgs sector is restricted by theoretical bounds. In Chapter. 3, we review the THDM. There are two  $SU(2)_L$  doublet scalar fields in this Higgs sector. We analyze theoretical bounds and experimental constraints for parameters in the Higgs sector. In Chapter. 4, we discuss the SGGHU. In particular, we focus on the Higgs sector which is extended by the triplet and singlet. We show the testability of our model at collider experiments. Conclusions are given in Chapter 5. In Appendix, we specify the Higgs sector in the NMSSM and the SGGHU. Mass matrices of Higgs bosons, neutralinos and charginos, and coupling

constants among Higgs bosons and other particles are summarized.



# Chapter 2

## Standard Model

We here review the Higgs sector in the SM. In the SM, the Higgs sector includes only an  $SU(2)_L$  doublet scalar field which is the minimal form. After the spontaneous gauge symmetry breaking, quarks, charged leptons, weak gauge bosons and Higgs boson obtain their masses. Then we consider the bounds for the Higgs boson mass.

### 2.1 Masses of SM particles

#### 2.1.1 Higgs boson mass and gauge boson masses

The Higgs sector of the SM is the minimal form with the one Higgs doublet as

$$\mathcal{L}^{\text{Higgs}} = |D_\mu \Phi|^2 - V(\Phi), \quad V(\Phi) = -\mu^2 |\Phi|^2 + \lambda |\Phi|^4, \quad (2.1)$$

$$\Phi = \begin{pmatrix} w^+ \\ \frac{1}{\sqrt{2}}(v + h + iz) \end{pmatrix}, \quad (2.2)$$

where  $w^+$  and  $z$  are NG bosons and these are eaten by the longitudinal mode of gauge bosons after the spontaneous symmetry breaking.  $v$  is the VEV of the doublet Higgs.  $D_\mu$  is covariant derivative

$$D_\mu \Phi = \left( \partial_\mu - ig_2 W_\mu^a \frac{\tau^a}{2} - ig_Y B_\mu \frac{1}{2} \right) \Phi. \quad (2.3)$$

From the vacuum condition as the following

$$\left. \frac{\partial V}{\partial h} \right|_0 = 0, \quad (2.4)$$

we obtain  $\mu^2 = \lambda v^2$ . The mass of the Higgs boson is

$$m_h^2 = 2\lambda v^2. \quad (2.5)$$

We obtain the masses of gauge bosons from the kinematic term after the spontaneous

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$Q_L^i$	<b>3</b>	<b>2</b>	+1/6
$u_R^i$	<b>3</b>	<b>1</b>	+2/3
$d_R^i$	<b>3</b>	<b>1</b>	-1/3
$L_L^i$	<b>1</b>	<b>2</b>	-1/2
$e_R^i$	<b>1</b>	<b>1</b>	-1
$\Phi$	<b>1</b>	<b>2</b>	+1/2

Table 2.1: Quantum number of fermions

symmetry breaking. The  $Z$  boson, the  $W$  boson and the photon are the following:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \quad (2.6)$$

$$Z_\mu = \frac{1}{\sqrt{2}}(g_2 W_\mu^3 - g_Y B_\mu), \quad (2.7)$$

$$A_\mu = \frac{1}{\sqrt{2}}(g_Y W_\mu^3 + g_2 B_\mu). \quad (2.8)$$

The mass of  $W$  boson and  $Z$  boson are  $m_W = \frac{g_2}{2}v$  and  $m_Z = \frac{\sqrt{g_2^2 + g_Y^2}}{2}v$ , respectively. However the photon remain massless. Using the mixing angle  $\theta_w$ ,  $Z$  and  $A$  is written by

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos \theta_w & -\sin \theta_w \\ \sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}, \quad (2.9)$$

where

$$\cos \theta_w = \frac{g_2}{\sqrt{g_2^2 + g_Y^2}}, \quad \sin \theta_w = \frac{g_Y}{\sqrt{g_2^2 + g_Y^2}}. \quad (2.10)$$

### 2.1.2 Fermion masses

The masses of fermions also are obtained by the spontaneous symmetry breaking. In the SM, quantum number of fermions is Tab.(2.1). The Yukawa sector is written as

$$\mathcal{L}^Y = -[\overline{Q}_L^i Y_d^{ij} \Phi d_R^j + \overline{Q}_L^i Y_u^{ij} \Phi^C u_R^j + \overline{L}_L^i Y_l^{ij} \Phi e_R^j + \text{h.c.}], \quad (2.11)$$

where  $\Phi^C = i\tau_2 \Phi^*$ ,  $Y_{u,d,l}$  is a  $3 \times 3$  complex matrix. These Yukawa terms are diagonalized by a bi-unitary transformation as

$$Q_L^i \rightarrow U^{ij} Q^j, \quad L_L^i \rightarrow U_l^{ij} Q^j, \quad u_R^i \rightarrow V_u^{ij} u_R^j, \quad d_R^i \rightarrow V_d^{ij} d_R^j, \quad e_R^i \rightarrow V_l^{ij} e_R^j. \quad (2.12)$$

$$\mathcal{L} \rightarrow -[\overline{Q}_L^i Y_d^{\text{diag}} \Phi d_R^j + \overline{Q}_L^i Y_u^{\text{diag}} \Phi^C u_R^j + \overline{L}_L^i Y_l^{\text{diag}} \Phi e_R^j + \text{h.c.}], \quad (2.13)$$

These diagonalized Yukawa matrices are given by

$$Y_u^{\text{diag}} = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}, \quad Y_d^{\text{diag}} = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad Y_l^{\text{diag}} = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}. \quad (2.14)$$

Therefore, the mass of fermion is the following

$$m_f = \frac{y_f}{\sqrt{2}}v. \quad (2.15)$$

## 2.2 Bound from theory

Bound of the Higgs mass can be obtained from theoretical consideration. We here mention the unitarity bound and the triviality bound.

### 2.2.1 Tree-level Unitarity bound

From the unitarity of S-matrix, we can restrict the parameter of the Higgs sector [19], namely, the coupling  $\lambda$  (or the mass of the Higgs boson  $m_h = 2\lambda v$ ) in the SM. Due to the optical theorem, The total cross section  $\sigma_{\text{tot}}$  can be written by the imaginary part of the scattering amplitude with the scattering angle  $\theta = 0$  as

$$\sigma_{\text{tot}} = \frac{1}{s} \text{Im}[\mathcal{M}(\theta = 0)], \quad (2.16)$$

where  $s$  is the squared center of mass energy. Since the main contribution of the  $\sigma_{\text{tot}}$  comes from 2 body  $\rightarrow$  2 body process,  $\sigma_{\text{tot}}$  is also written by

$$\sigma_{\text{tot}} \geq \frac{1}{s} \int d \cos \theta \frac{|\mathcal{M}|}{32\pi}. \quad (2.17)$$

On the other hand, the amplitude  $\mathcal{M}$  can be expanded in terms of the  $J$ th partial wave amplitude  $a_J$  as

$$\mathcal{M} = 16\pi^2 \sum_{J=0}^{\infty} (2J+1) P_J(\cos \theta) a_J. \quad (2.18)$$

By combining the Eqs. (2.16), (2.17) and (2.18), we obtain

$$\text{Re}(a_J)^2 + \left\{ \text{Im}(a_J) - \frac{1}{2} \right\}^2 \leq \left( \frac{1}{2} \right)^2 \quad (2.19)$$

This equation suggests that  $a_J$  has to be on the circle with the radius 1/2 and the center coordinate (0,1/2) in the complex plane. Therefore,  $|\text{Re}(a_J)| \leq 1/2$  at the tree level. We apply this constraint to the elastic scattering process of longitudinal component of the  $W$  boson,  $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ . In the high energy limit, 0th partial wave amplitude can be calculated as

$$a_0 \sim -\frac{G_F m_h^2}{4\sqrt{2}\pi}. \quad (2.20)$$

From Eq. (2.19), we obtain the upper bound of the Higgs boson mass

$$m_h < \frac{2\pi\sqrt{2}}{G_F} \sim (873 \text{ GeV})^2. \quad (2.21)$$

By considering the partial wave unitarity on the four-channel system,  $W_L^+W_L^-$ ,  $1/\sqrt{2}Z_LZ_L$ ,  $1/\sqrt{2}HH$  and  $HZ_L$ , we can obtain the stronger constraint. In the high energy limit, the scattering process for longitudinal component of the weak gauge bosons can be replaced by the NG boson modes which is known as the equivalence theorem [20]. In the basis of  $\{w^+w^-, zz, hh, hz\}$ , this  $4 \times 4$  s-wave amplitude matrix  $t$  is given as

$$t = -\frac{G_F m_h^2}{4\pi\sqrt{2}} \begin{pmatrix} 1 & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & 0 \\ \frac{1}{\sqrt{8}} & \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{\sqrt{8}} & \frac{1}{4} & \frac{3}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}. \quad (2.22)$$

Since this matrix  $t$  has eigenvalues  $\{3/2, 1/2, 1/2, 1/2\} \times G_F m_h^2 / 4\pi\sqrt{2}$ , we obtain

$$m_h^2 < (712 \text{ GeV})^2. \quad (2.23)$$

### 2.2.2 Triviality bound

Quartic coupling constant is important in the Higgs sector. Because if quartic coupling constant is less than zero, then there is possibility that the vacuum is unstable. Using RGEs, we estimate the quartic coupling constant at the arbitrary scale. In other words, we can know the breaking scale of this model. In the SM, the running of the quartic coupling constant is written as [21]

$$\frac{d\lambda}{d\log Q} \simeq \frac{1}{16\pi^2} [24\lambda^2 + 12y_t^2\lambda - 6y_t^4], \quad (2.24)$$

where  $\lambda$  is quartic coupling constant and  $Q$  is an arbitrary scale. As shown in above the equation, running of  $\lambda$  is sensitive for  $y_t$ . Therefore the top mass  $m_t$  is an important parameter. In Ref. [22], when  $m_t = 171.0$  GeV, the SM is unbroken at the GUTs scale. On the other hand, when  $m_t = 175.3$  GeV, the vacuum is meta-stable above  $\mathcal{O}(10^8)$  GeV.



# Chapter 3

## Two Higgs Doublet Model

We review the Higgs sector in the Two Higgs doublet model (THDM). The THDM has two  $SU(2)_L$  doublet scalar fields. It is one of the simple extended Higgs model. There are many motivations for THDM such as SUSY [23], radiative seesaw models [24–26] and electroweak baryogenesis [27, 28]. In the Higgs sector, there are many parameters as compared with that of the SM. On the other words, additional Higgs bosons other than SM-like Higgs boson appear. Next, we consider bounds of parameters in Higgs sector from the experimental and theoretical constraints. Finally, we show the allowed region of parameters.

### 3.1 Higgs sector

THDM contains two  $SU(2)_L$  doublet scalar fields. The most general Higgs potential is written as

$$\begin{aligned}
 V(\Phi_1, \Phi_2) = & m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - m_3^2 (\Phi_1^\dagger \Phi_2 + \text{h.c.}) \\
 & + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\
 & + \left[ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \lambda_6 (\Phi_1^\dagger \Phi_1) (\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \text{h.c.} \right], \quad (3.1)
 \end{aligned}$$

where  $\Phi_1$  and  $\Phi_2$  are  $SU(2)$  doublet fields with  $Y = 1/2$ ,  $m_{1,2}$  and  $\lambda_{1-4}$  are real, and  $m_3$  and  $\lambda_{5-7}$  are complex. Yukawa sector is given by

$$\mathcal{L}_{\text{THDM}}^Y = -[\overline{Q}_L^i Y_d^{ij} (\Phi_1 + \Phi_2) d_R^j + \overline{Q}_L^i Y_u^{ij} (\Phi_1^C + \Phi_2^C) u_R^j + \overline{L}_L^i Y_l^{ij} (\Phi_1 + \Phi_2) e_R^j + \text{h.c.}]. \quad (3.2)$$

As we can see from Eq.(3.2), Yukawa sector cannot be diagonalized simultaneously. In other words, general THDM contain flavor changing neutral current(FCNC) at tree-level. In order to avoid the tree-level FCNC, we consider the THDM with discrete  $Z_2$  symmetry under which two Higgs fields behave as  $\Phi_1 \rightarrow \Phi_1$  and  $\Phi_2 \rightarrow -\Phi_2$  but we allow soft breaking term [29]. There are four kinds of Yukawa interactions under the discrete  $Z_2$  symmetry which are listed in Tab. 3.1 [30, 31]. Then the Higgs potential is given by

$$\begin{aligned}
 \mathcal{V}(\Phi_1, \Phi_2) = & m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - m_3^2 (\Phi_1^\dagger \Phi_2 + \text{h.c.}) \\
 & + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\
 & + \left[ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \text{h.c.} \right]. \quad (3.3)
 \end{aligned}$$

	$\Phi_1$	$\Phi_2$	$u_R$	$d_R$	$l_R$	$Q_L, L_L$
Type I	+	-	-	-	-	+
Type II	+	-	-	+	+	+
Type X	+	-	-	-	+	+
Type Y	+	-	-	+	-	+

Table 3.1: The charge assignments of the discrete  $Z_2$  symmetry

$\lambda_{6,7}$  terms are forbidden by the discrete  $Z_2$  symmetry. This Higgs sector has real six parameters and two complex parameters. We take two complex parameters  $\lambda_5$  and  $m_3$  to be real assuming that CP is conserved in the Higgs sector. The Higgs doublet fields are parametrized as

$$\Phi_i = \begin{pmatrix} w^+ \\ \frac{1}{\sqrt{2}}(v_i + h_i + iz_i) \end{pmatrix}. \quad (3.4)$$

where the VEVs  $v_1$  and  $v_2$  satisfy  $\sqrt{v_1^2 + v_2^2} = v \simeq 246$  GeV and  $\tan \beta = v_2/v_1$ . The mass eigenstates are obtained by rotating the component fields as

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R(\alpha) \begin{pmatrix} H \\ h \end{pmatrix}, \quad \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = R(\beta) \begin{pmatrix} z \\ A \end{pmatrix}, \quad \begin{pmatrix} w_1^\pm \\ w_2^\pm \end{pmatrix} = R(\beta) \begin{pmatrix} w^\pm \\ H^\pm \end{pmatrix}, \quad (3.5)$$

where  $w^\pm$  and  $z$  are Nambu-Goldstone bosons,  $h$ ,  $H$ ,  $A$  and  $H^\pm$  are respectively two CP-even, one CP-odd and charged Higgs bosons, and

$$R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}. \quad (3.6)$$

From the vacuum conditions:

$$\begin{aligned} m_1^2 &= m_3^2 \tan \beta - \frac{\lambda_1}{2} v_2^2 - \frac{1}{2} \bar{\lambda} v_2^2, \\ m_2^2 &= m_3^2 \cot \beta - \frac{\lambda_2}{2} v_1^2 - \frac{1}{2} \bar{\lambda} v_1^2, \end{aligned} \quad (3.7)$$

where  $\bar{\lambda} = \lambda_3 + \lambda_4 + \lambda_5$ . Using Eq.(3.5) and Eq.(3.7), we obtain physical scalar masses as

$$\begin{aligned} m_{H^\pm}^2 &= M^2 - \frac{1}{2}(\lambda_4 + \lambda_5)v^2, \\ m_A^2 &= M^2 - \lambda_5 v^2, \\ m_{H,h}^2 &= \frac{1}{2} \left[ M_{11}^2 + M_{22}^2 \pm \sqrt{(M_{11}^2 + M_{22}^2)^2 + 4M_{12}^2} \right]. \end{aligned} \quad (3.8)$$

Matrix elements  $M_{ij}$  are

$$\begin{aligned} M_{11}^2 &= \lambda_1 v^2 \cos^4 \beta + \lambda_2 v^2 \sin^4 \beta + 2\bar{\lambda} v^2 \sin^2 \beta \cos^2 \beta, \\ M_{22}^2 &= M^2 + (\lambda_1 + \lambda_2 - 2\bar{\lambda}) \sin^2 \beta \cos^2 \beta, \\ M_{12}^2 &= \frac{1}{2}(-\lambda_1 \cos^2 \beta + \lambda_2 \sin^2 \beta + \bar{\lambda} \cos 2\beta) v^2 \sin \beta \cos \beta, \end{aligned} \quad (3.9)$$

where soft breaking parameter  $M^2 = m_3^2/(\sin\beta\cos\beta)$  [32]. The eight parameters  $\lambda_{1-5}$  and  $m_{1-3}^2$  are replaced by mixing angles  $\alpha$  and  $\beta$ , physical masses  $m_A$ ,  $m_h$ ,  $m_H$  and  $m_{H^\pm}$ , the VEV  $v$  and soft breaking parameter  $M^2$ . In particular, the quartic coupling constants are expressed in terms of physical Higgs boson masses, mixing angles and the soft breaking parameter as

$$\lambda_1 = \frac{1}{v^2 \cos^2 \beta} (-M^2 \sin^2 \beta + m_H^2 \cos^2 \alpha + m_h^2 \sin^2 \alpha), \quad (3.10)$$

$$\lambda_2 = \frac{1}{v^2 \sin^2 \beta} (-M^2 \cos^2 \beta + m_H^2 \sin^2 \alpha + m_h^2 \cos^2 \alpha), \quad (3.11)$$

$$\lambda_3 = \frac{1}{v^2} [-M^2 + (m_H^2 - m_h^2) \frac{\sin 2\alpha}{\sin 2\beta} + 2m_{H^\pm}^2], \quad (3.12)$$

$$\lambda_4 = \frac{1}{v^2} (M^2 + m_A^2 - 2m_{H^\pm}^2), \quad (3.13)$$

$$\lambda_5 = \frac{1}{v^2} (M^2 - m_A^2). \quad (3.14)$$

The coupling constants of the CP-even Higgs boson with the weak gauge boson  $hWW$  and  $HWW$  are proportional to  $\sin(\beta - \alpha)$  and  $\cos(\beta - \alpha)$ , respectively. When  $\sin(\beta - \alpha) = 1$ , only  $h$  couples the weak gauge bosons. Then  $h$  behaves as the SM Higgs boson. Therefore, in this case  $h$  is called the SM-like Higgs boson. It is interesting that THDM has different couplings of the Higgs boson for each types of Yukawa interactions. In this thesis, we do not mention difference for types of Yukawa interactions, although we refer to constraints which are independent of that.

## 3.2 Bounds of parameters

We here consider the constraints of the parameters of the Higgs sector from experimental and theoretical bounds, namely, the oblique corrections, the tree-level unitarity and the vacuum stability which restrict the parameters without depending on the type of Yukawa interactions in THDM.

### 3.2.1 Oblique corrections

New physics effects on the electroweak oblique parameters are parameterized by the  $S$ ,  $T$  and  $U$  parameters [33]. In the THDM, the contributions to the electroweak parameters from the scalar boson loops are given by

$$S_\Phi = -\frac{1}{4\pi} [F'_\Delta(m_{H^\pm}, m_{H^\pm}) - \sin^2(\beta - \alpha) F'_\Delta(m_H, m_A) - \cos^2(\beta - \alpha) F'_\Delta(m_h, m_A)], \quad (3.15)$$

$$T_\Phi = -\frac{\sqrt{2}G_F}{16\pi^2\alpha_{EM}} \left\{ -F_\Delta(m_A, m_{H^\pm}) \right. \\ \left. + \sin^2(\beta - \alpha) [F_\Delta(m_H, m_A) - F_\Delta(m_H, m_{H^\pm})] \right. \\ \left. + \cos^2(\beta - \alpha) [F_\Delta(m_h, m_A) - F_\Delta(m_h, m_{H^\pm})] \right\}, \quad (3.16)$$

where

$$F_{\Delta}(m_0, m_1) = \frac{m_0^2 + m_1^2}{2} - \frac{m_0^2 m_1^2}{m_0^2 - m_1^2} \ln \frac{m_0^2}{m_1^2}, \quad (3.17)$$

$$F'_{\Delta}(m_0, m_1) = -\frac{1}{3} \left[ \frac{4}{3} - \frac{m_0^2 \ln m_0^2 - m_1^2 \ln m_1^2}{m_0^2 - m_1^2} - \frac{m_0^2 + m_1^2}{(m_0^2 - m_1^2)^2} F_{\Delta}(m_0, m_1) \right]. \quad (3.18)$$

For the case with  $m_0 \sim m_1$ , we have

$$F_{\Delta}(m_0, m_1) \sim \frac{2(m_0 - m_1)^2}{3} - \frac{(m_0 - m_1)^4}{30m_1^3} + \dots, \quad (3.19)$$

$$F'_{\Delta}(m_0, m_1) \sim +\frac{1}{3} \ln m_1^2 + \frac{m_0 - m_1}{6m_1} + \dots. \quad (3.20)$$

When all the additional heavy scalar bosons are degenerate  $m_A = m_H = m_{H\pm}$ , we obtain  $S_{\Phi} = T_{\Phi} = 0$ . In the SM-like limit  $\sin(\beta - \alpha) = 1$  with the further assumption  $m_A = m_H$ , we have

$$S_{\Phi} = -\frac{1}{12\pi} \log \frac{m_{H\pm}^2}{m_A^2}, \quad (3.21)$$

$$T_{\Phi} = +\frac{\sqrt{2}G_F}{12\pi^2\alpha_{EM}}(m_A - m_{H\pm})^2. \quad (3.22)$$

The quadratic dependence on the mass difference between heavy scalar bosons can be early understood by the approximate formula for  $m_A \sim m_{H\pm}$  in Eq. (3.22). Therefore, the deviation of the  $T$  parameter is insensitive to  $M$ .

### 3.2.2 Tree-level unitarity and vacuum stability

In the SM, the mass of the Higgs boson is constrained by the tree level unitarity. It has been studied by considering  $6 \times 6$  scattering matrix of two body scalar states ( $hh, hz, zz, \omega^+\omega^-, h\omega^+, z\omega^+$ ) where each eigenvalues of scattering matrices are restricted to be less than an criteria  $\xi$  as  $a_0 \leq \xi$  [19]. For  $\xi = 1/2$ , the Higgs boson mass is bounded to be less than about 710 GeV. In the THDM, there are 14 neutral [34], 8 singly charged and a doubly charged two body states [35]. In our numerical analysis, absolute values of all eigenvalues for the s-wave amplitude matrix are required to be less than 1/2 as for a criteria to keep perturbativity [36].

For the constraint from vacuum stability, the Higgs potential is required to be positive for a large value of the order parameter. In the SM, this condition is expressed by  $\lambda > 0$  at the tree level. In the THDM, the condition for vacuum stability is given by [37–39]

$$\sqrt{\lambda_1\lambda_2} + \lambda_3 + \min[0, \lambda_4 - |\lambda_5|] > 0. \quad (3.23)$$

## 3.3 Combination of Constraints

Combining above constraints, we can show that the results in the case with  $\sin(\beta - \alpha) = 1$ ,  $m_H^2 = m_A^2 = M^2$  and  $m_h = 117, 140, 240, 500$  GeV (on the  $m_A - \Delta m$  plane) in Fig. 3.1 as in Ref. [40], where  $\Delta m = m_A - m_{H\pm}$ . The masses of neutral scalars and the soft-breaking mass

parameter are taken to be degenerate  $m_A^2 = m_H^2 = M^2$ . This choice of the parameters would be rather special in the sense that there is no  $\tan\beta$  dependence in this case. In Eqs.(3.10) and (3.11) with  $\sin(\beta - \alpha) = 1$ , terms dependent on  $\tan\beta$  are proportional to  $m_H^2 - M^2$ , and then one of  $\lambda_1$  and  $\lambda_2$  tends to large when  $\tan\beta \neq 1$ . Therefore, the parameter space is more restricted by the unitarity constraints in the case without degeneracy. Also when  $\sin(\beta - \alpha) = 1$  is slightly relaxed, the bound from tree level unitarity becomes sensitive to  $\tan\beta$ . For larger values of  $\tan\beta$ , the bound becomes more restrictive. Consequently, the tree level unitarity bound shown in Fig. 3.1 can be regarded as the most conservative which is independent of the values of  $\tan\beta$ .

In conclusion, we can restrict parameters of the Higgs sector from theoretical bounds and experimental constraints in the THDM. The magnitude of the mass difference between additional heavy scalar bosons can be determined to satisfy the electroweak precision data. Since large coupling constant in the Higgs sector violate the tree-level unitarity, the large mass difference is also restricted. For the  $m_h \sim 126$  GeV, as shown in the upper two panels of Fig. 3.1, the only mass difference is restricted.

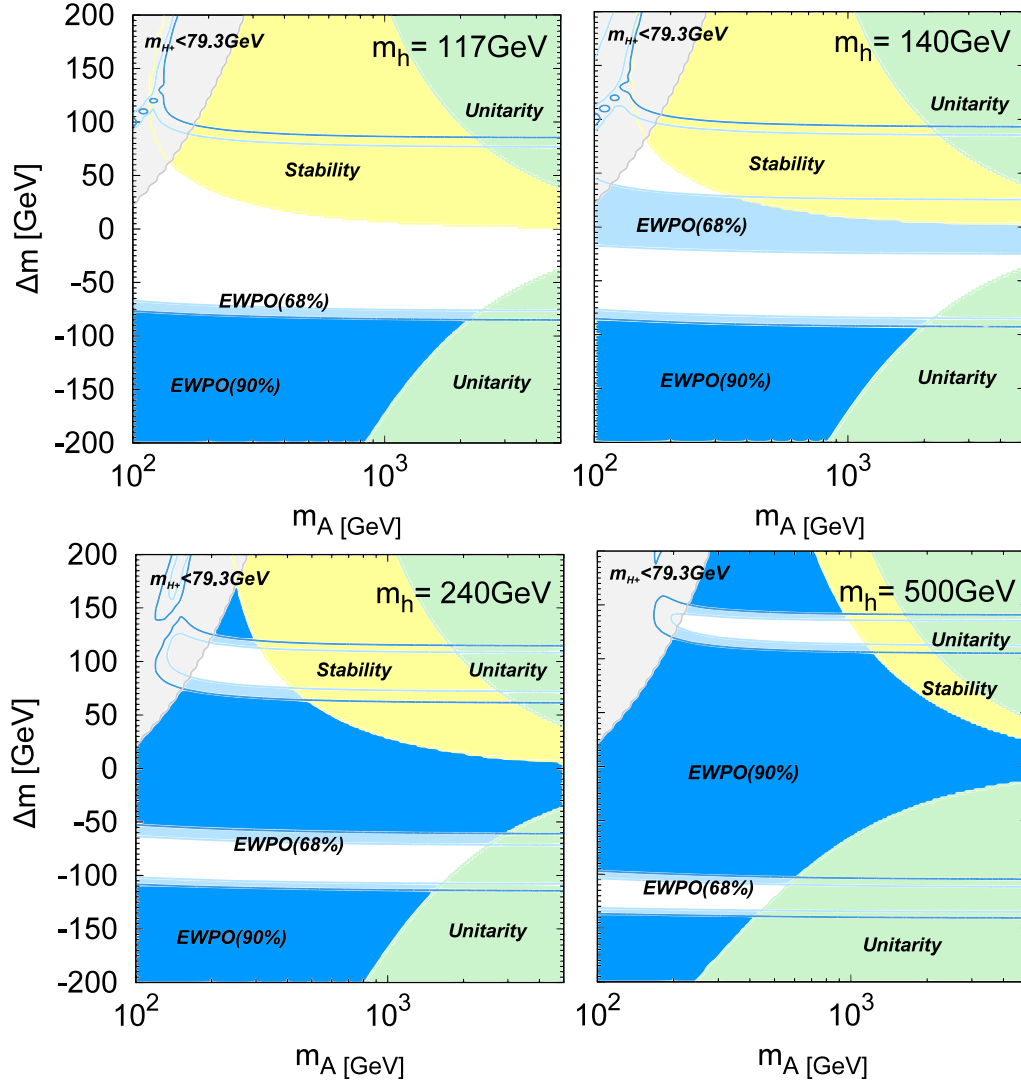


Figure 3.1: Theoretical and experimental constraints in the parameter space of the THDM. Uncolored regions are allowed by all the constraints we here considered, i.e., tree level unitarity/stability and electroweak precision data, and direct search bound of charged Higgs boson,  $m_{H^\pm} < 79.3$  GeV. The mass and mixing parameters are chosen as  $M^2 = m_H^2 = m_A^2$ , with the SM-like limit  $\sin(\beta - \alpha) = 1$ . In this limit, constraints are independent from  $\tan \beta$ .

## Chapter 4

# Supersymmetric Grand Gauge-Higgs Unification

In this chapter, first we review briefly  $SU(5)$  GUT model which is the simplest GUT model, and the DT splitting is the fine-tuning problem of mass difference between the colored triplet Higgs field and the doublet Higgs field in SUSY-GUTs. Then, we discuss the Higgs sector in the supersymmetric grand gauge-Higgs unification. In our model, the DT splitting is absent and the Higgs sector is extended from the MSSM one by adding the triplet and the singlet superfield. Finally, we show deviations in the Higgs couplings with the SM particles from the SM predictions for some benchmark points, and in masses of additional Higgs bosons from the MSSM predictions.

### Grand Unified Theory [8]

The GUT unifies the SM gauge symmetry,  $SU(3)_C \times SU(2)_L \times U(1)_Y$ . Here, we show the  $SU(5)$  GUT which is the minimal GUT model.

The quarks and leptons of the SM are embedded in representation  $\bar{\mathbf{5}}$  and  $\mathbf{10}$  of  $SU(5)$ . The representation  $\bar{\mathbf{5}}$  and  $\mathbf{10}$  of  $SU(5)$  are given by

$$\begin{aligned}\bar{\mathbf{5}} &= \{(\bar{\mathbf{3}}, \mathbf{1})_{1/3}, (\mathbf{1}, \mathbf{2})_{-1/2}\}, \\ \mathbf{10} &= \{(\mathbf{3}, \mathbf{2})_{1/6}, (\bar{\mathbf{3}}, \mathbf{1})_{-2/3}, (\mathbf{1}, \mathbf{1})_1\},\end{aligned}\tag{4.1}$$

where  $(\mathbf{X}, \mathbf{Y})_Z$  indicates the representation  $\mathbf{X}$  of the  $SU(3)_C$ , the representation  $\mathbf{Y}$  of the  $SU(2)_L$  and the hypercharge  $Z$ . The adjoint representation  $\mathbf{24}$  of  $SU(5)$  include the gauge bosons of the SM. The adjoint representation  $\mathbf{24}$  of  $SU(5)$  is written as

$$\mathbf{24} = \{(\mathbf{8}, \mathbf{1})_0, (\mathbf{1}, \mathbf{3})_0, (\mathbf{1}, \mathbf{1})_0, (\mathbf{3}, \mathbf{2})_{-5/6}, (\bar{\mathbf{3}}, \mathbf{2})_{5/6}\}\tag{4.2}$$

where  $(\mathbf{3}, \mathbf{2})_{-5/6}$  and  $(\bar{\mathbf{3}}, \mathbf{2})_{5/6}$  are new gauge bosons, so-called X boson. In the GUT, the baryon number and lepton number are not conserved because quarks and leptons are included in the same multiplet. GUT models predict the proton decay because the X boson causes transitions from the quark to the lepton. The amplitudes for the proton decay are inversely proportional to the mass of the X boson. Since the proton decay has not been detected, the X boson is supposed to be heavy. Its mass is specifically the GUT scale at the least. The Higgs doublet which causes the electroweak symmetry breaking is embedded in the representation  $\mathbf{5}$  of the  $SU(5)$ . This representation  $\mathbf{5}$  is given by

$$\mathbf{5} = \{(\mathbf{3}, \mathbf{1})_{-1/3}, (\mathbf{1}, \mathbf{2})_{1/2}\}.\tag{4.3}$$

### Doublet triplet splitting

Here, I mention the doublet triplet splitting which is a fine-tuning problem in the SUSY-GUT models. As a simple example, we focus on the SUSY SU(5) GUT. This model has two Higgs superfields  $\overline{H}(\mathbf{5}) = (\overline{H}_C, H_d)^T$  and  $H(\mathbf{5}) = (H_C, H_u)^T$ . The superpotential include the following Yukawa terms

$$W \supset (QQH_C), (QL\overline{H}_C), (\overline{U}_R\overline{E}_RH_C), (\overline{U}_R\overline{D}_R\overline{H}_C). \quad (4.4)$$

Using these Yukawa terms, we can construct the dimension five operators that have the form

$$(QQQL), (\overline{U}_R\overline{E}_R\overline{U}_R\overline{D}_R). \quad (4.5)$$

The couplings of these effective interactions are expected to be of order

$$g_{\text{eff}}^{(5)} \sim y^2/M, \quad (4.6)$$

where  $y$  is typical coupling constants, and  $M$  is a typical masses of superfields. Since these interactions Eq. (4.5) are violating the conservation of the baryon and lepton number at the one loop level, this model predicts the proton decay. The rate of the proton decay relates to the effective coupling Eq. (4.6). To avoid the proton decay,  $M$  should be large, so  $H_C$  is of order the GUT scale [11–16].

The unified symmetry is broken, when the representation **24** Higgs field  $A$  obtains the VEV as the following

$$\langle A \rangle = \begin{pmatrix} 2V & 0 \\ 0 & -3V \end{pmatrix}, \quad (4.7)$$

where  $V$  is of order the GUT scale because this  $V$  proves the mass of X boson. The superpotential which include Higgs fields is written as

$$W_{DT} = \overline{H}(\mathbf{5})[m + \lambda_5 A(\mathbf{24})]H(\mathbf{5}). \quad (4.8)$$

After the unified symmetry breaking, the  $SU(3)_C$  triplet Higgs fields and the  $SU(2)_L$  doublet fields obtain their mass as following

$$\begin{aligned} m_C &= m + 2\lambda_5 V, \\ m_2 &= m - 3\lambda_5 V. \end{aligned} \quad (4.9)$$

As is known [41], the doublet Higgs fields are the electroweak scale while  $V$  is the GUT scale. It is difficult to make this mass difference between the colored triplet and doublet Higgs fields in the same multiplet. It is just a fine-tuning.



## 4.1 Model

### 4.1.1 Review of Supersymmetric Grand GHU

In this subsection, we review briefly the grand GHU scenario proposed in Ref. [18]. This scenario is a kind of the grand unification where the  $SU(5)$  unified gauge symmetry is broken by the Hosotani mechanism [17]. The simplest setup that can accommodate the chiral fermions is a five-dimensional (5D)  $SU(5)$  model compactified on an  $S^1/\mathbb{Z}_2$  orbifold with its radius being of the GUT scale. We first discuss the non-SUSY version of the simplest setup discussed in Ref. [42] for illustration purpose, and then supersymmetrize it [18].

The Hosotani mechanism works on higher-dimensional gauge theories for gauge symmetry breaking. To be more concrete, the zero modes of extra-dimensional components of the gauge fields, which behave as scalar fields after the compactification, develop VEVs to break the gauge symmetry. In order to apply this mechanism to the  $SU(5)$  unified gauge symmetry breaking, massless adjoint scalar fields, with respect to the  $SU(5)$  symmetry that remains unbroken against the boundary conditions (BCs), should appear. It is known that such components tend to be projected out in models that realize the chiral fermions due to the orbifold boundary conditions. In Ref. [42], this difficulty is dodged via the so-called diagonal embedding method [43] which is proposed in the context of the string theory. In our field theoretical setup on the  $S^1/\mathbb{Z}_2$  orbifold, we impose two copies of the gauge symmetry with an additional discrete symmetry that exchanges the two gauge symmetries. In other words, the symmetry is  $SU(5) \times SU(5) \times \mathbb{Z}_2$  in our  $SU(5)$  model. Here, we name the gauge fields for the two  $SU(5)$  groups  $A_M^{(1)}$  and  $A_M^{(2)}$ , respectively, where  $M = \mu (= 0-3), 5$  is a 5D Lorentzian index, and define the eigenstates of the  $\mathbb{Z}_2$  action as  $X^{(\pm)} = (X^{(1)} \pm X^{(2)})/\sqrt{2}$ . We set the BCs around the two endpoints of the  $S^1/\mathbb{Z}_2$ ,  $y_0 = 0$  and  $y_\pi = \pi R$ , as

$$A_\mu^{(1)}(y_i - y) = A_\mu^{(2)}(y_i + y), \quad A_5^{(1)}(y_i - y) = -A_5^{(2)}(y_i + y), \quad (4.10)$$

for  $i = 0, \pi$ , where  $y$  denotes the 5th dimensional coordinate. With these BCs, we see that  $A_\mu^{(+)}$  and  $A_5^{(-)}$  obey the Neumann BC at each endpoint to have the zero-modes, and thus that the gauge symmetry remaining unbroken in the 4D effective theory is the diagonal part of the  $SU(5) \times SU(5)$  (or our GUT symmetry is *embedded* into the *diagonal* part) and an adjoint scalar field is actually realized.

An interesting point is that the  $A_5^{(-)}$  is not a simple adjoint scalar field but composes a Wilson loop since it is a part of the gauge field. The Wilson loop is given by

$$W = \mathcal{P} \exp \left( i \int_0^{2\pi R} \frac{g}{\sqrt{2}} A_5^{(-)a} (T_1^a - T_2^a) dy \right) \rightarrow \exp (i \text{diag} (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)), \quad (4.11)$$

where  $\mathcal{P}$  denotes the path-ordered integral,  $g$  is the common gauge coupling constant,  $T_1$  and  $T_2$  are the generators of the two  $SU(5)$  symmetries, and  $a$  is an  $SU(5)$  adjoint index. In the last expression, we show the expression on the fundamental representation for concreteness, and we have used the (remaining)  $SU(5)$  rotation to diagonalize  $A_5^{(-)}$ . This expression shows that the VEV (and actually the system itself) is invariant under the shift  $\theta_i \rightarrow \theta_i + 2\pi$ .

The form of the VEV which is discussed in Ref. [42]. We are interested in is given by  $\theta_1 = \theta_2 = \theta_3 = 2\pi$  and  $\theta_4 = \theta_5 = -3\pi$ , *i.e.*  $\langle W \rangle = \text{diag}(1, 1, 1, -1, -1) \equiv P_W$ . This VEV does

not affect the triplet component of the  $\mathbf{5}$  representation but does affect the doublet to *split* them. This “missing VEV”, which is forbidden for a simple adjoint scalar field by the traceless condition, is allowed since the Wilson loop is valued on a group instead of an algebra and thus is free from the condition. This fact plays an essential role to solve the DT splitting problem.

In this chapter, for simplicity, we do not consider matter fields that are non-singlet under the both gauge groups. We introduce for instance a fermion  $\Psi(\mathbf{R}, \mathbf{1})^{(1)}$  with  $\mathbf{R}$  being a representation of the  $SU(5)$  group and its  $\mathbb{Z}_2$  partner  $\Psi(\mathbf{1}, \mathbf{R})^{(2)}$ . Here, we call the above pair a “bulk  $\mathbf{R}$  multiplet”. Their BCs are given by  $\Psi^{(1)}(y_\pi - y) = -\eta_i^\Psi \gamma_5 \Psi^{(2)}(y_\pi + y)$  where  $\eta_i = \pm 1$  is a parameter associated with each fermion. As one of  $\eta_i$  can be reabsorbed by changing  $\gamma_5$ , *i.e.* by the charge conjugation, we set  $\eta_0 = +1$  and  $\eta_\pi = \eta$  hereafter. Then,  $\Psi_L^{(+)}$  and  $\Psi_R^{(-)}$  have the zero-modes when  $\eta = +1$  while none have when  $\eta = -1$ , when the VEV of  $A_5$  is vanishing.

We note that it is always possible to gauge away the VEV of  $A_5$  ( $\propto \theta$ ). In this basis, called the Scherk-Schwartz basis, the  $SU(5)$  breaking effect appears only on the BCs as

$$\Psi^{(1)}(y_\pi - y) = -\eta_i^\Psi \gamma_5 W_{\mathbf{R}} \Psi^{(2)}(y_\pi + y), \quad (4.12)$$

where  $W_{\mathbf{R}}$  is the Wilson line phase acting on  $\mathbf{R}$ . In concrete, for  $\mathbf{R} = \mathbf{5}$  with  $\eta = -1$  when the above VEV  $\langle W \rangle = P_W$  is realized, the doublet component has the zero-mode while the triplet does not.

The same story discussed above can be applied also in SUSY models if we replace all the fields by the corresponding superfields. Thus, once the desired VEV  $P_W$  is obtained, the DT splitting is easily realized by introducing a bulk  $\mathbf{5}$  hypermultiplet with  $\eta = -1$  for the MSSM Higgs fields. In a similar way, if we introduce bulk  $\mathbf{10}$  hypermultiplets with  $\eta = +1$ , light vector-like pairs  $(U^c, \bar{U}^c)$  and  $(E^c, \bar{E}^c)$  appear. This is utilized to recover the gauge coupling unification later.

Notice that the zero-modes appear always in vector-like pairs from the bulk fields. The chiral fermions can be put simply on each boundary. Interestingly, when the VEV  $\langle W \rangle = P_W$  is realized, bulk fields serve vector-like pairs in  $SU(5)$  incomplete multiplets while the boundary fields which do not couple to  $A_5$  and thus neither to the  $SU(5)$  breaking do chiral fermions in  $SU(5)$  full multiplets.

The remaining task to show that the DT splitting problem is actually solved is to examine when the VEV is realized. Here, we do not request that the vacuum resides on the global minimum but just require only that it is stable so that the lifetime is long enough. For this purpose, we have to check if there is no huge tadpole term for the fluctuation of  $\theta_i$  around the desired vacuum,  $\delta\theta_i$ , and if it is not tachyonic around the desired vacuum. Since there are two largely different scales, the compactification scale and the SUSY breaking scale, the RG analysis should be performed.

Before going on the low energy effective theory, we note that the exchanging  $\mathbb{Z}_2$  symmetry, under which  $\delta\theta_i$  is odd, remains unbroken on the relevant vacuum even though  $\theta_i$  is non-trivial. This is understood by the transformation of the Wilson line which is the order parameter. Under the  $\mathbb{Z}_2$  action  $W$  transforms as  $W \rightarrow W^*$  and the VEV  $\langle W \rangle$  is invariant since it is real. This  $\mathbb{Z}_2$  invariance prohibit the tadpole terms. In the following, we introduce soft  $\mathbb{Z}_2$  breaking as small as the SUSY breaking scale, and thus a small tadpole term will be generated.

### 4.1.2 Low Energy Effective Theory

The supersymmetric grand gauge-Higgs unification predicts the existence of chiral adjoint superfields whose quantum number is the same as the standard model gauge fields. These new chiral adjoint fields are originally embedded in the five-dimensional vector multiplets. In the SUSY limit, these masses of new chiral adjoint superfields vanish because they are included in the five-dimensional vector fields. These chiral adjoint fields obtain mass after the SUSY breaking. Therefore masses of them are not compactification scale, but the SUSY breaking scale.

We ignore the contribution of the  $SU(3)_C$  octet to physics at the TeV scale although our model predicts the existence of  $SU(3)_C$  octet,  $SU(2)_L$  triplet and singlet because this chiral adjoint octet superfield is  $\mathcal{O}(10)$  TeV due to the radiative correction. Therefore we focus on the Higgs sector with the triplet and singlet chiral superfield in addition to the MSSM doublets.

The Higgs sector is constructed by the displayed fields in Tab. 4.1. Here  $H_u$  ( $H_d$ ) gives up-type quark (down-type quark and charged lepton) masses. The superpotential in the effective theory is given by

$$W = \mu H_u \cdot H_d + \mu_\Delta \text{tr}(\Delta^2) + \frac{\mu_S}{2} S^2 + \lambda_\Delta H_u \cdot \Delta H_d + \lambda_S S H_u \cdot H_d, \quad (4.13)$$

where  $\Delta = \Delta^a \sigma^a / 2$  with the Pauli matrices  $\sigma^a$  ( $a = 1, 2, 3$ ). Notice that trilinear self-couplings are absent among  $\Delta$  and  $S$  although these couplings are not forbidden by the gauge symmetries because  $\Delta$  and  $S$  originate from the gauge supermultiplet. Furthermore, the new two Higgs couplings  $\lambda_\Delta$  and  $\lambda_S$  are unified with the gauge couplings  $g_{\text{GUT}}$  at the GUT scale as  $\lambda_\Delta = 2\sqrt{5/3}\lambda_S = g_{\text{GUT}}$ . Masses of fermionic components of  $\Delta$  and  $S$  are written as  $m_\Delta$  and  $m_S$ , respectively, and their magnitudes are of order of the TeV scale because they are generated by the SUSY breaking [44]. Similarly, the parameter of the supersymmetric tadpole of  $S$  is of order of the SUSY breaking scale. This tadpole term is removed by field redefinition with generality. The soft SUSY breaking terms are written as

$$V_{\text{soft}} = \tilde{m}_d^2 |H_d|^2 + \tilde{m}_u^2 |H_u|^2 + 2\tilde{m}_\Delta^2 \text{tr}(\Delta^\dagger \Delta) + \tilde{m}_S^2 |S|^2 + \left[ B\mu H_u \cdot H_d + \eta S + B_\Delta \mu_\Delta \text{tr}(\Delta^2) + \frac{1}{2} B_S \mu_S S^2 + \lambda_\Delta A_\Delta H_u \cdot \Delta H_d + \lambda_S A_S S H_u \cdot H_d + \text{h.c.} \right]. \quad (4.14)$$

The values of these parameters are obtained by solving the RGEs at the low energy scale. Notice that the VEV of the neutral component  $v_\Delta$  of the triplet field has to be smaller than  $\simeq 10$  GeV in order to satisfy the electroweak rho parameter.

## 4.2 RGE Analysis

In this section, we discuss the mass parameters and RGE of coupling constants in our model. First, we focus on the unification of the three gauge coupling constants. The light adjoint chiral multiplets, triplet and singlet, destroy unification of gauge couplings, which is achieved in the minimal SUSY  $SU(5)$  GUT. Therefore the gauge couplings unification need additional extra matter in the our model. Then, solving the RGEs, we derive the values of parameters at the TeV scale. Finally, we show the some benchmark points consistent with the observed value of the mass of the Higgs boson.

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$H_u$	<b>1</b>	<b>2</b>	+1/2
$H_d$	<b>1</b>	<b>2</b>	-1/2
$S$	<b>1</b>	<b>1</b>	0
$\Delta$	<b>1</b>	<b>3</b>	0

Table 4.1: Superfields in the Higgs sector.

### 4.2.1 Coupling Unification

The coefficients of the beta functions of the gauge couplings in the MSSM are given by

$$b_{\text{MSSM}} = (33/5, 1, -3), \quad (4.15)$$

while contributions from the adjoint chiral multiplets are

$$\delta_{\text{adj}}b = (0, 2, 3). \quad (4.16)$$

One way to recover the gauge coupling unification is to introduce incomplete  $SU(5)$  multiplets whose contributions are

$$\delta_{\text{add}}b = (3 + n, 1 + n, n), \quad (4.17)$$

with  $n$  being a natural number. However, too large  $n$  may cause violation of perturbativity around the GUT scale. We here take  $n = 1$ , and the unified gauge coupling is in a perturbative region:  $\alpha_G \simeq 0.3$ . This case is realized by adding two vector-like pairs of  $(\mathbf{1}, \mathbf{2})_{-1/2}$ , one of  $(\mathbf{3}, \mathbf{1})_{-2/3}$  and one of  $(\mathbf{1}, \mathbf{1})_1$ , where the values denote  $SU(3)_C$ ,  $SU(2)_L$  and  $U(1)_Y$  quantum numbers. Fig. 4.1 shows evolution of the gauge coupling constants in the MSSM (black lines), the MSSM with the adjoint multiplets (red), and the MSSM with the adjoint and additional chiral multiplets (blue).

In this model, the strong interaction is not asymptotically free irrelevantly to the choice of the additional fields to recover the gauge coupling unification. Thus, the QCD corrections are large, and the masses of the colored particles tend to be large at the TeV scale, as compared to those in the MSSM. It is interesting to examine the extraordinary pattern of the mass spectrum of the colored particles for the hadron colliders. We, however, focus on the colorless fields; the  $SU(2)$  triplet and singlet Higgs multiplets. These additional fields couple to the two MSSM Higgs doublets. Their coupling constants push up the SM-like Higgs boson mass due to the tree level  $F$ -term contribution, and thus the correct value of the Higgs boson mass (around 126 GeV) can be easily realized.

Furthermore, they cause mixing between the MSSM doublet Higgs fields and the additional Higgs fields, which results in modification of the coupling constants of the SM-like Higgs field. When such corrections are large enough to be detected at collider experiments, we can discriminate our model by precisely measuring the pattern of the deviations in the Higgs coupling constants. In the next section, we will discuss these issues in more detail.

One of the characteristic features of this model is that the trilinear coupling constants between the MSSM Higgs doublets and the additional triplet and singlet are unified with the

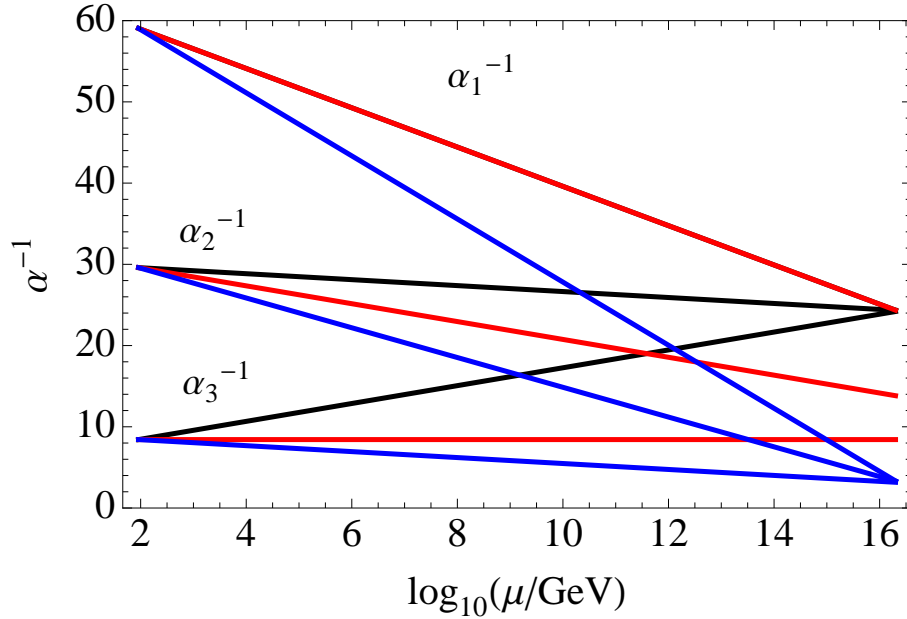


Figure 4.1: Evolution of the gauge coupling constants in the MSSM (black lines), the MSSM with the adjoint multiplets (red), and the MSSM with the adjoint and additional chiral multiplets (blue) [45].

SM gauge coupling constants at the GUT scale. Thus, the low-energy values of these coupling constants in the Higgs sector are apparently determined by the RG running once the extra matters are specified.

For instance, taking the above example of the additional chiral matter multiplets to recover the gauge coupling unification, the Higgs sector coupling constants  $\lambda_\Delta$  (red line) and  $\lambda_S$  (blue), and the gauge coupling constants (green) evolve as shown in Fig. 4.2. Here, we normalize the singlet and  $U(1)$  gauge couplings as  $\lambda'_S = (2\sqrt{5/3})\lambda_S$  and  $g_1 = (\sqrt{5/3})g_Y$ , respectively, and one-loop renormalization group equations are used. Since the  $SU(2)$  gauge coupling is strong around the GUT scale,  $\lambda_\Delta$  grows as the energy decreases. After the  $SU(2)$  gauge coupling becomes weak,  $\lambda_\Delta$  decreases as the energy decreases due to large trilinear couplings in the superpotential. We note that the triplet coupling  $\lambda_\Delta$  remains in a perturbative region down to the TeV scale. At the TeV scale, we obtain

$$\lambda_\Delta = 1.1, \quad \lambda_S = 0.25. \quad (4.18)$$

Similarly, the  $\mu$ -parameters of the adjoint chiral multiplets are unified at the GUT scale, and their ratio at the TeV scale is determined as  $\mu_S : \mu_\Delta : \mu_O = 1 : 2.9 : 230$ , where  $\mu_O$  stands for the octet  $\mu$ -parameter. The mass scale of the octet is far beyond the reach of collider experiments, as discussed qualitatively above.

Let us turn to the running of the soft SUSY breaking parameters. Since the unified gauge coupling is strong, the gaugino masses around the GUT scale must be large in order to avoid the experimental gluino mass limit [46]. For instance, for the unified gaugino mass of  $M_{1/2} = 3600$  GeV, the gluino mass is pushed down to  $m_{\tilde{g}} = 1400$  GeV. As a result, soft mass parameters at the TeV scale are typically as large as 4-7 TeV for colored particles and 1-2 TeV for colorless particles. As in the MSSM, the soft mass squared of the up-type Higgs boson,  $|\tilde{m}_{h_u}^2|$  is enhanced due to the large top Yukawa interaction. Therefore, some tuning is needed to realize electroweak

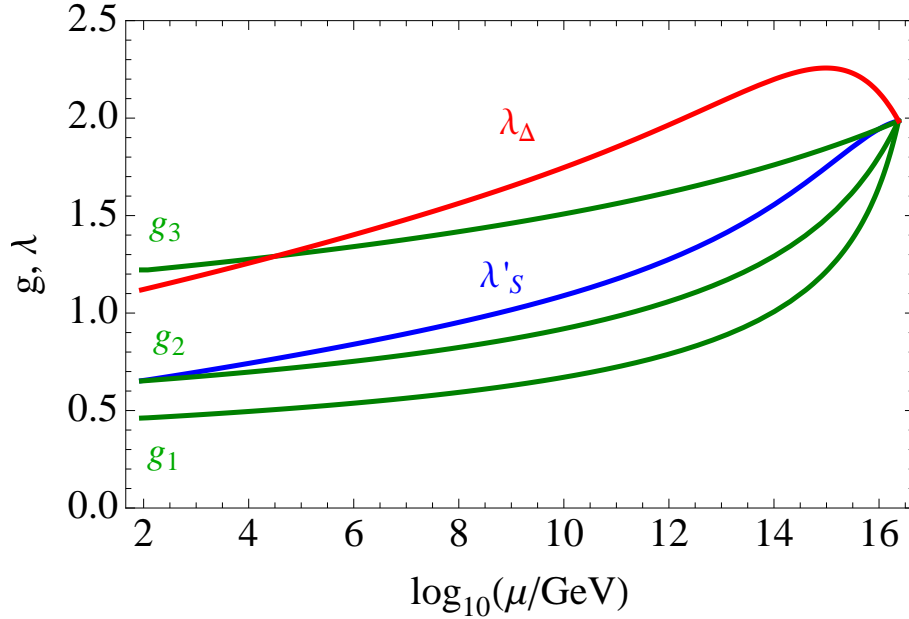


Figure 4.2: An example of running of the Higgs triplet and singlet coupling constants  $\lambda_\Delta$  (red line) and  $\lambda'_S$  (blue) as well as the gauge coupling constants  $g_3$ ,  $g_2$  and  $g_1$  (green) from the top to the bottom. The horizontal axis is the common logarithm of the energy scale in units of GeV. Here, we normalize the singlet and  $U(1)_Y$  gauge couplings as  $\lambda'_S = (2\sqrt{5}/3)\lambda_S$  and  $g_1 = (\sqrt{5}/3)g_Y$ , respectively, and one loop RGEs are used [45].

symmetry breaking. The higgsino mass parameter  $\mu$  and the CP-odd Higgs boson mass  $m_A$  also tend to be 1-4 TeV. In order to realize scenarios where some of the extra Higgs boson masses are of the order of  $\mathcal{O}(100)$  GeV, further tuning is required among the input parameters.

### 4.2.2 Benchmark Points and the Mass of the SM-like Higgs boson

After the electroweak symmetry breaking, four CP-even, three CP-odd and three charged Higgs bosons appear as physical states in the Higgs sector, as well as six neutralinos and three charginos. Since the Higgs sector is extended by new additional particles as compared with the MSSM, the predictions of our model is different from the MSSM. We here focus on the mass of the SM-like Higgs boson, which is determined by low energy soft SUSY breaking parameters obtained by solving the RGEs discussed above.

First, we exemplify rough predictions of our low energy effective theory without solving the RGEs. For relatively large triplet and singlet scalar masses, the SM-like Higgs boson mass is approximately written as [47]

$$m_h^2 \simeq m_Z^2 \cos^2 \beta + \frac{3m_t^4}{2\pi^2 v^2} \left( \ln \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2} \left( 1 - \frac{X_t^2}{12m_{\tilde{t}}^2} \right) \right) + \frac{1}{8}\lambda_\Delta^2 v^2 \sin^2 2\beta + \frac{1}{2}\lambda_S^2 v^2 \sin^2 2\beta, \quad (4.19)$$

where  $m_Z$  is the  $Z$ -boson mass,  $m_t$  is the top quark mass,  $m_{\tilde{t}}$  is the average of the two stop masses, and  $X_t = A_t - \mu \cot \beta$  parametrizes mixing between the two stop quarks. The first two terms correspond to the MSSM prediction. The last two terms originate from the existence of the trilinear couplings between the MSSM Higgs doublets and the additional triplet and singlet.



Within the MSSM, at the tree level the SM-like Higgs boson mass is smaller than the  $Z$ -boson mass. In order to reach 126 GeV using the effect of the stop loop correction, the mass scale of the stops or the mixing parameter  $X_t$  should be large. For  $X_t = 0$ , the stop mass should be of the order of  $O(10)$  TeV. Even in the maximum mixing case where  $X_t = \pm\sqrt{6}m_{\tilde{t}}$ , the stop mass is required to be as large as  $O(1)$  TeV [7]. We also note that preferable range for  $\tan\beta$  is larger than 10.

In our model, on the contrary, the predicted Higgs boson mass tends to be larger than that in the MSSM thanks to the tree level F-term contributions, in particular, for small  $\tan\beta$  region. Such a result is reminiscent of the NMSSM [5], where the SM-like Higgs boson mass is raised also by the coupling with a singlet superfield.

For calculation of the masses of the Higgs scalars and superparticles, we have used the public numerical code `SuSpect` [48], which takes the  $\overline{\text{DR}}$  renormalization scheme, instead of the approximate formula Eq.(4.19). We have appropriately modified `SuSpect` to add the new contributions from the Higgs trilinear couplings. Here, for the simplicity, we have taken the limit  $v_\Delta \rightarrow 0$ . The computation of the SM-like Higgs boson mass including these triplet and singlet contributions is described in Appendix B.1. Notice that the formula given in Eq. (4.19) is valid when the neutral components of the triplet and singlet are heavier than the MSSM-like CP-even Higgs bosons. In general, the CP-even Higgs bosons mix with each other and the formulas for their mass eigenvalues are rather complicated.

Then let us consider the mass of the SM-like Higgs boson including the radiative effects. As we mentioned, in order to have a successful electroweak symmetry breaking, fine-tuning for input parameters at the GUT scale is required. Therefore, we will show some benchmark points that reproduce the mass of the SM-like Higgs boson, instead of scanning the parameter space. We focus on the following three different cases:

- (A) All the Higgs bosons other than the SM-like Higgs boson are heavy.
- (B) The new Higgs bosons other than the MSSM-like Higgs bosons are heavy.
- (C) The new Higgs bosons affect the SM-like Higgs boson couplings.

Bearing the fact that there are a few GeV uncertainties in the numerical computation of the SM-like Higgs boson mass, we take the range of  $122 \text{ GeV} < m_h < 129 \text{ GeV}$  as its allowed region. Examples of successful benchmark points of input parameters at the GUT scale are listed in Tab. 4.2. Here,  $\mu$  and  $B$  parameters for the extra matters have insignificant effects on Higgs sector parameters, and are omitted from the list. Values of parameters of the TeV-scale effective theory are obtained after RG running and shown in Tab. 4.3.

### 4.3 Impact on Higgs Properties

In this section, we discuss properties of the particles in the Higgs sector. We will show that our model can be discriminative from others by measuring the coupling constants and the masses of the Higgs sector particles at the LHC and future electron-positron collider [49–52]. Even in the cases where the additional Higgs bosons are beyond the reach of direct discovery at these colliders, the existence of these new particles can be indirectly probed by precise measurements of the coupling constants of the discovered SM-like Higgs boson.

Case	$\tan \beta$	$M_{1/2}$	$\mu_\Sigma$	
(A)(B)(C)	3	3600 GeV	-300 GeV	
Case	$A_0$	$\tilde{m}_0^2$	$\tilde{m}_{H_u}^2$	$\tilde{m}_{H_d}^2$
(A)	5500 GeV	(1000 GeV) <sup>2</sup>	(10375 GeV) <sup>2</sup>	(8570 GeV) <sup>2</sup>
(B)	1000 GeV	(1800 GeV) <sup>2</sup>	(12604 GeV) <sup>2</sup>	(10381.5 GeV) <sup>2</sup>
(C)	8000 GeV	(3000 GeV) <sup>2</sup>	(10605.1 GeV) <sup>2</sup>	(8751.4 GeV) <sup>2</sup>
Case	$\tilde{m}_5^2$	$\tilde{m}_{10}^2$	$\tilde{m}_\Sigma^2$	
(A)	-(6300 GeV) <sup>2</sup>	-(2000 GeV) <sup>2</sup>	-(570 GeV) <sup>2</sup>	
(B)	-(7700 GeV) <sup>2</sup>	-(1960 GeV) <sup>2</sup>	-(670 GeV) <sup>2</sup>	
(C)	-(6418 GeV) <sup>2</sup>	-(1638.5 GeV) <sup>2</sup>	-(400 GeV) <sup>2</sup>	

Table 4.2: Benchmark points of input parameters at the GUT scale [45].

Case	$M_1$	$M_2$	$M_3$	$\mu_\Delta$	$\mu_S$		
(A)(B)(C)	194 GeV	388 GeV	1360 GeV	-252 GeV	-85.8 GeV		
Case	$\mu$	$B\mu$	$\tilde{m}_{u_3}$	$\tilde{m}_{q_3}$	$y_t A_t$		
(A)	205 GeV	41400 GeV <sup>2</sup>	3290 GeV	4830 GeV	4030 GeV		
(B)	177 GeV	40800 GeV <sup>2</sup>	1730 GeV	4480 GeV	6050 GeV		
(C)	174 GeV	42000 GeV <sup>2</sup>	4220 GeV	5550 GeV	2910 GeV		
Case	$\tilde{m}_\Delta$	$\tilde{m}_S$	$\lambda_\Delta A_\Delta$	$\lambda'_S A_S$	$B_\Delta \mu_\Delta$	$B_S \mu_S$	$m_h$
(A)	607 GeV	805 GeV	662 GeV	683 GeV	92000 GeV <sup>2</sup>	-78700 GeV <sup>2</sup>	123 GeV
(B)	784 GeV	612 GeV	1340 GeV	1110 GeV	30700 GeV <sup>2</sup>	-110000 GeV <sup>2</sup>	123 GeV
(C)	521 GeV	216 GeV	284 GeV	446 GeV	207000 GeV <sup>2</sup>	-33600 GeV <sup>2</sup>	122 GeV

Table 4.3: Parameters of the TeV-scale effective theory obtained after RG running [45].

### 4.3.1 Vertices of the SM-like Higgs boson

First, we investigate the couplings between the SM-like Higgs boson and SM particles, which have been already measured to some extent at the LHC. So far, the SM predictions has been consistent. In the future, precision of these observables will be significantly improved by the high-luminosity LHC and the ILC, and therefore this method serves as a powerful tool in discriminating beyond-the-SM models.

In our discussion, we treat the vacuum expectation value of the triplet field such as zero due to the electroweak rho parameter with  $\rho \sim 1$ . At the leading order, the Higgs coupling with the  $W$ - or  $Z$ -boson is given by

$$g_{hVV} = g_V m_V (R_{11}^S \cos \beta + R_{12}^S \sin \beta), \quad (V = W, Z) \quad (4.20)$$

and those with the up-type quarks, down-type quarks and charged leptons by

$$g_{huu} = \frac{\sqrt{2} m_u}{v} \frac{R_{12}^S}{\sin \beta}, \quad g_{hdd} = \frac{\sqrt{2} m_d}{v} \frac{R_{11}^S}{\cos \beta}, \quad g_{h\ell\ell} = \frac{\sqrt{2} m_\ell}{v} \frac{R_{11}^S}{\cos \beta}, \quad (4.21)$$

where  $R^S$  is mixing of CP-even Higgs boson matrix as shown Appendix B.1. The Higgs self-coupling is

$$g_{hhh} = R_{1a}^S R_{1b}^S R_{1c}^S \lambda_{s_a s_b s_c}. \quad (4.22)$$



	ILC(250)	ILC(500)	ILC(1000)	ILC(LumUp)
$\sqrt{s}$ (GeV)	250	250+500	250+500+1000	250+500+1000
L (fb <sup>-1</sup> )	250	250+500	250+500+1000	1150+1600+2500
$\gamma\gamma$	18 %	8.4 %	4.0 %	2.4 %
$gg$	6.4 %	2.3 %	1.6 %	0.9 %
$WW$	4.8 %	1.1 %	1.1 %	0.6 %
$ZZ$	1.3 %	1.0 %	1.0 %	0.5 %
$t\bar{t}$	–	14 %	3.1 %	1.9 %
$b\bar{b}$	5.3 %	1. %	1.3 %	0.7 %
$\tau^+\tau^-$	5.7 %	2.3 %	1.6 %	0.9 %
$c\bar{c}$	6.8 %	2.8 %	1.8 %	1.0 %
$\mu^+\mu^-$	91 %	91 %	16 %	10 %
$\Gamma_T(h)$	12 %	4.9 %	4.5 %	2.3 %
$hhh$	–	83 %	21 %	13 %
BR(invis.)	< 0.9 %	< 0.9 %	< 0.9 %	< 0.4 %

Table 4.4: This table was reported in Ref. [52]. Summary of expected accuracies  $\Delta g_i/g_i$  for model independent determinations of the Higgs boson couplings. The theory errors are  $\Delta F_i/F_i = 0.1\%$ . For the invisible branching ratio, the numbers quoted are 95% confidence upper limits.

where  $\lambda_{s_a s_b s_c}$  are tree-level couplings among CP-even Higgs bosons in the gauge basis. The effective vertex of  $h\gamma\gamma$  is given by [53]

$$\begin{aligned}
\Gamma(h \rightarrow \gamma\gamma) &= \frac{G_F \alpha^2 m_h^3}{128 \sqrt{2} \pi^3} \\
&\times \left| \sum_f N_C Q_f^2 g_{hff} A_{1/2}^h(\tau_f) + g_{hVV} A_1^h(\tau_W) + \frac{m_W^2 \lambda_{hh_i^+ h_i^-}}{2c_W^2 m_{h_i^\pm}^2} A_0^h(\tau_{h_i^\pm}) \right. \\
&\left. + \sum_{\chi_i^\pm} \frac{2m_W}{m_{\chi_i^\pm}} \lambda_{h\chi_i^+ \chi_i^-} A_{1/2}^h(\tau_{\chi_i^\pm}) + \sum_{\tilde{f}_i} \frac{1}{m_{\tilde{f}_i}^2} g_{h\tilde{f}_i \tilde{f}_i} N_C Q_{\tilde{f}_i}^2 A_0^h(\tau_{\tilde{f}_i}) \right|^2, \quad (4.23)
\end{aligned}$$

where  $A_i^h$  are amplitudes at lowest-order for spin-0, spin-1/2 and spin-1 particle contributions, the number of color is  $N_C = 3$ ,  $Q_f$  denote the electric charges of fermions  $f$  and  $\tau_X = m_h^2/4m_X^2$  with loop mass  $m_X$ . For the definitions of the amplitudes  $A_i^h$ , see, for example, Ref. [53]. The Higgs boson couplings with the charged Higgs bosons are given by

$$\lambda_{hh_i^+ h_j^-} = R_{1a}^S R_{ib}^C R_{jc}^C \lambda_{s_a w_b^+ w_c^-}. \quad (4.24)$$

The corresponding in the SM are

$$g_{hVV}|_{\text{SM}} = g_V m_V, \quad g_{huu}|_{\text{SM}} = \frac{\sqrt{2}m_u}{v}, \quad g_{hdd}|_{\text{SM}} = \frac{\sqrt{2}m_d}{v}, \quad g_{h\ell\ell}|_{\text{SM}} = \frac{\sqrt{2}m_\ell}{v}, \quad g_{hhh}|_{\text{SM}} = \frac{m_Z^2}{v}. \quad (4.25)$$

It is useful to define deviation parameters that

$$\kappa_X = \frac{g_{hXX}}{g_{hXX}|_{\text{SM}}}, \quad (4.26)$$

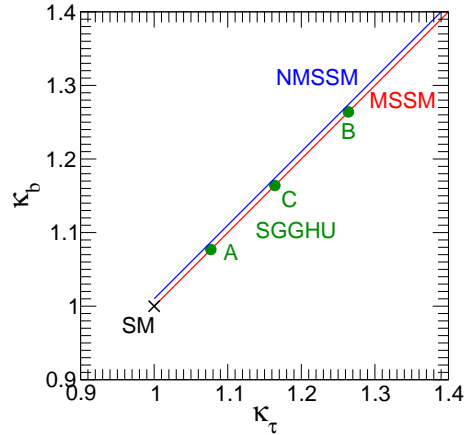


Figure 4.3: The deviations in the Higgs boson coupling with the tau lepton  $\kappa_\tau$  and that with the bottom quark  $\kappa_b$  from the SM predictions are plotted. The predictions of the three benchmark points (A), (B) and (C) in the SGGHU are shown with green blobs. The MSSM and NMSSM predictions are shown with red and blue lines, respectively. For the purpose of illustration, the NMSSM line is slightly displaced from  $\kappa_\tau = \kappa_b$  [45].

where  $X$  denotes SM particles. Such deviations are extracted from measurements of the decay widths of the Higgs boson.

In Fig. 4.3, the deviations in the Higgs boson coupling with the tau lepton  $\kappa_\tau$  and that with the bottom quark  $\kappa_b$  from the SM predictions are plotted. The predictions of the three benchmark points (A), (B) and (C) in the SGGHU are shown with green blobs. The MSSM and NMSSM predictions are shown with red and blue lines, respectively. Here, we simply adjust the stop masses and mixing so that the observed Higgs boson mass is reproduced. In our model, the Higgs boson couplings to the down-type quarks and charged leptons are common and fall in the type-II Yukawa interactions of the THDM. Therefore, the predicted SGGHU deviations lie on the MSSM and NMSSM lines, as is evident from Eq.(4.21). The present LHC data has already constrained the parameter space from the measurement of the Higgs boson coupling with the tau lepton as  $\kappa_\tau < 1.3$  [54]. At the ILC with  $\sqrt{s} = 500$  GeV, expected accuracies for the deviations  $\kappa_\tau$  and  $\kappa_b$  are 2.3% and 1.6%, respectively [52] (in Tab. 4.4).

In Fig. 4.4, the deviations in the Higgs boson coupling with the weak gauge bosons  $\kappa_V$  and that with the bottom quark  $\kappa_b$  from the SM predictions are plotted. The predictions of the three benchmark points (A), (B) and (C) in the SGGHU are shown with green blobs. The MSSM predictions are shown with red lines for  $\tan\beta = 10$  (thick line) and  $\tan\beta = 3$  (dashed). The NMSSM predictions are shown with blue grid lines, which indicate mixings between the SM-like and singlet like Higgs bosons of 10%, 20% and 30% from the right to the left. As is reported in Ref. [52] (in Tab. 4.4), the ILC with  $\sqrt{s} = 500$  GeV can reach accuracy of 1.0% (1.1%) for the Higgs boson coupling with the  $Z$ -boson (the  $W$ -boson). Therefore, signatures different from the MSSM and its variants are expected to be observed using  $\kappa_V$  at the ILC. Notice that the VEV of the neutral component of the triplet Higgs boson  $v_\Delta$  is small compared to those of the doublet Higgs bosons. Therefore, the mixing between the SM-like Higgs boson and the CP-even component of the Higgs singlet dominates over that between the SM-like Higgs boson and the triplet component. In this sense, our model is similar to the NMSSM. It will be difficult to distinguish our model only from these observables.

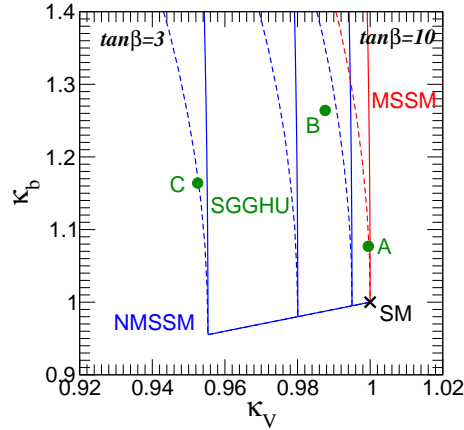


Figure 4.4: The deviations in the Higgs boson coupling with the weak gauge bosons  $\kappa_V$  and that with the bottom quark  $\kappa_b$  from the SM predictions are plotted. The predictions of the three benchmark points (A), (B) and (C) in the SGGHU are shown with green blobs. The MSSM predictions are shown with red lines for  $\tan\beta = 10$  (thick line) and  $\tan\beta = 3$  (dashed). The NMSSM predictions are shown with blue grid lines, which indicate mixings between the SM-like and singlet like Higgs bosons of 10%, 20% and 30% from the right to the left [45].

In Fig. 4.5, the deviations in the Higgs boson coupling with the charm quark  $\kappa_c$  and that with the bottom quark  $\kappa_b$  from the SM predictions are plotted. As in Fig. 4.4, the predictions of the three benchmark points (A), (B) and (C) in the SGGHU are shown with green blobs, and the MSSM and NMSSM predictions are shown with red and blue lines, respectively. In sharp contrast to the  $\kappa_V$ - $\kappa_b$  relation, correlations between  $\kappa_c$  and  $\kappa_b$  strongly depend on the value of  $\tan\beta$ . For example, the benchmark point (C) with  $\tan\beta = 3$  is not covered by the NMSSM predictions with  $\tan\beta = 10$ , and the deviation can be measured at the ILC with  $\sqrt{s} = 500$  GeV, which aims to measure  $\kappa_c$  with accuracy of 2.8% (in Tab. 4.4). Independent  $\tan\beta$  measurement using decay of the Higgs boson at the ILC [55,56] will also play an important role in discriminating models. Although it will be difficult to completely distinguish our model from the NMSSM from the precision measurements of Higgs boson couplings, if the deviation pattern of the Higgs couplings is found to be close to our benchmark points, there is a fair possibility that the SGGHU is realized. The ILC is absolutely necessary for investigating the Higgs properties and distinguishing particle physics models.

As for other Higgs boson couplings, the deviations of the Higgs boson coupling with the photon are  $0.94 < \kappa_\gamma < 1.0$ , and those of the Higgs self-coupling  $0.82 < \kappa_h < 0.93$  for the benchmark points we show. To observe deviations in these observables from the SM predictions one needs more precise measurements at the ILC with  $\sqrt{s} = 1$  TeV [52] (in Tab. 4.4).

### 4.3.2 Additional Higgs bosons

Finally, we mention the additional MSSM-like Higgs bosons. Since four-point couplings in the Higgs sector are expressed in terms of gauge couplings and F-term couplings in SUSY models, differences of the masses of the additional MSSM-like Higgs bosons is useful measures in probing

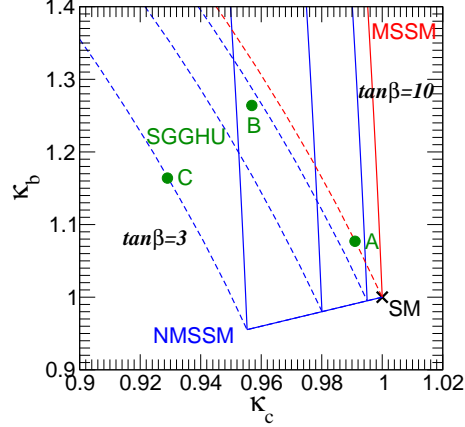


Figure 4.5: The deviations in the Higgs boson coupling with the charm quark  $\kappa_c$  and that with the bottom quark  $\kappa_b$  from the SM predictions are plotted. See the caption of Fig. 4.4 for details [45].

more fundamental physics. The charged Higgs boson mass  $m_{H^\pm}$  is given by

$$\begin{aligned} m_{H^\pm}^2 &= m_{H^\pm}^2|_{\text{MSSM}}(1 + \delta_{H^\pm})^2 \\ &\simeq m_A^2 + m_W^2 + \frac{1}{8}\lambda_\Delta^2 v^2 - \frac{1}{2}\lambda_S^2 v^2, \end{aligned} \quad (4.27)$$

where  $\delta_{H^\pm}$  is the deviation in  $m_{H^\pm}$  from the MSSM and  $m_A$  is the CP-odd Higgs boson mass. The sign of the singlet contribution is opposite to the triplet one due to the group theory. From Eq. (4.18),  $m_{H^\pm}$  becomes large as compared to the MSSM. We emphasize that these  $\lambda_S$  and  $\lambda_\Delta$  couplings are determined by the RGEs and predicted  $m_{H^\pm}$  is always large as compared with the MSSM that in this model. Since  $m_{H^\pm}|_{\text{MSSM}}$  is the sum of  $m_A$  and  $m_W$ , we can obtain  $\delta_{H^\pm}$  by measuring  $m_A$  and  $m_{H^\pm}$  precisely. Fig. 4.6 shows the deviation in  $m_{H^\pm}$  from the MSSM as a function of  $m_A$  in the large soft mass scenario. The green, blue and red lines show the NMSSM, the triplet-extended MSSM and our model, respectively. The mass deviation is  $O(1)\%$  –  $O(10)\%$ . On the other hand, the deviation in the heavy CP-even Higgs boson mass  $m_H$  from the MSSM prediction is less than  $O(1)\%$ . Since the charged Higgs mass can be determined with an accuracy of a few percent at the LHC [51], we can test our model.

When the masses of the triplet-like and singlet-like scalar bosons are below 500 GeV, the ILC and CLIC [50] have capability to directly produce these new particles. For example, the benchmark point (C) gives mass spectrum of the Higgs sector particles shown in Tab. 4.5. In this case, the mass of the lighter triplet-like Higgs boson  $\Delta^\pm$  is less than 500 GeV, and we can probe  $\Delta^\pm$  using the channel  $e^+e^- \rightarrow \Delta^+\Delta^- \rightarrow t\bar{t}\bar{b}$ , which proceeds via the mixing between the MSSM-like and triplet-like charged Higgs bosons.

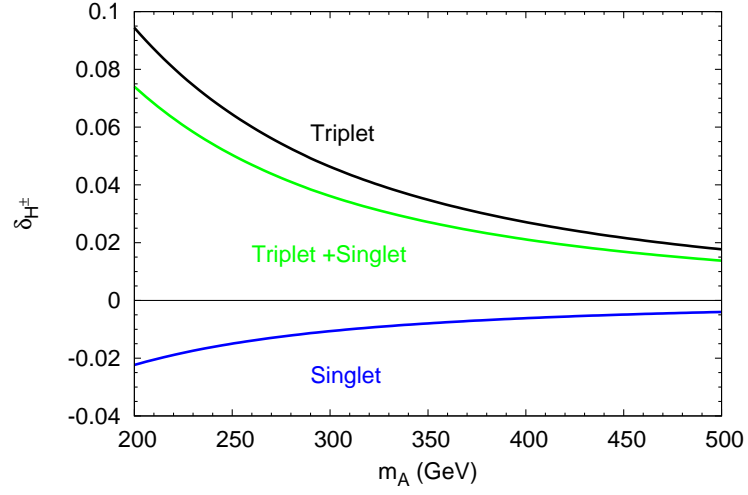


Figure 4.6: The deviation in  $m_{H^\pm}$  from the MSSM as a function of the CP-odd Higgs boson mass  $m_A$  in the large soft mass scenario. The green, blue and red line correspond to the NMSSM, the MSSM with triplet and our model case, respectively [45].

CP-even	CP-odd	Charged
122 GeV	–	–
139 GeV	171 GeV	204 GeV
370 GeV	304 GeV	496 GeV
745 GeV	497 GeV	745 GeV

Table 4.5: Mass spectrum of the Higgs scalars for the benchmark point (C) [45].



# Chapter 5

## Conclusion

We have discussed the phenomenology of the Higgs sector. In particular, we have focused on the low energy effective theory of the supersymmetric grand Gauge-Higgs unification and investigated the testability of this model at collider experiments.

The Higgs boson couples to many standard model particles and plays a role of triggering spontaneous breaking of the electroweak gauge symmetry. The discovered new boson at the Large Hadron Collider whose coupling strength to the other particles seems to be consistent with the prediction in the SM within the error of the current data. Therefore, the SM is very successful model. However, no one believes that the SM is a fundamental theory of particle physics. The first reason is that the gauge symmetry of the SM is not unified. From viewpoint of the history of unification of law in physics, the SM is nothing but low energy effective theory of a more fundamental theory such as the grand unified theory. Another reason is that, the SM has phenomenological and theoretical problems which cannot be explained in this model. The hierarchy problem is the fine-tuning problem. Since quadratic divergences appear in the radiative correction to the mass of the Higgs boson, a huge cancelation between its bare mass and loop contribution is required for renormalization of the Higgs boson mass in the SM. These quadratic divergences could be canceled and the hierarchy problem could disappear in a new physics model beyond the SM.

In these situations, SUSY-GUTs are attractive, because in these models, the SM gauge groups are unified and the hierarchy problem disappears. Furthermore, the gauge couplings are unified at the GUT scale. However, SUSY-GUTs also have a fine-tuning problem of the mass difference between the colored triplet Higgs field and the  $SU(2)_L$  doublet Higgs field, which are originated from common multiplet, so-called doublet-triplet splitting. We need a new physics model which can solve this problem.

In the chapter 3, we have reviewed the two Higgs doublet model as a one of the simplest extended Higgs model. The THDM contains two  $SU(2)_L$  doublet scalar fields. It is interesting that different physics depending on the assignment of the  $Z_2$  charge appear in this model. We can restrict parameters of the Higgs sector which are independent of assignment of  $Z_2$  charge from the vacuum stability, the tree-level unitarity and the precision measurement. We have shown the allowed region of parameters of the Higgs sector.

In the chapter 4, we have discussed the SGGHU. In this model, the  $SU(5)$  grand unified gauge symmetry is broken by the Hosotani mechanism. This model can provide a natural solution to the DT-splitting thanks to the phase nature of the Hosotani mechanism and predicts the existence of the new adjoint chiral superfields whose quantum numbers are equal to the gauge

bosons in the SM and masses are at the SUSY breaking scale. The Higgs sector is extended from the MSSM one by the  $SU(2)_L$  triplet and singlet chiral superfields. We have evaluated the masses and couplings of the Higgs sector in the our model. These are different from the SM, the MSSM and other models. For some benchmark points, we have calculated deviations of couplings between the SM-like Higgs boson and SM particles from the SM predictions. The deviations of the couplings from the SM values prove to be  $\mathcal{O}(1)\%$  when the triplet-like and singlet-like Higgs boson masses are below  $\simeq 1$  TeV. In the case of light Higgs bosons, for instance, benchmark point (C), Higgs bosons will be directly produced by the ILC and the CLIC. The deviation of the charged Higgs boson mass from that of the MSSM is  $\mathcal{O}(1)\% - \mathcal{O}(10)\%$  when their masses are below  $\simeq 500$  GeV. Such a deviation is within the scope of the LHC. The extended Higgs sector in the SUSY models means that the neutralino and chargino sectors are also extended. In our model, there are six neutralinos and three charginos. We have specified these mass matrices and couplings to the Higgs bosons in Appendix B.3.2.

We emphasize that our supersymmetric grand gauge-Higgs unification model is a good example of the grand unified theory that is verifiable at collider experiments.



# Appendix A

## NMSSM

In this appendix, we specify the Higgs sector in the NMSSM as the simplest extended of the MSSM with  $Z_3$  symmetry.

### A.1 Higgs potential

The Higgs sector of superpotential  $W$  and soft-SUSY terms  $V_{\text{soft}}$  are written as

$$W = +\lambda_S S H_1 H_2 + \frac{\kappa}{3} S^3, \quad (\text{A.1})$$

$$V_{\text{soft}} = m_{H_1}^2 H_1^\dagger H_1 + m_{H_2}^2 H_2^\dagger H_2 + m_S^2 S^\dagger S + (-\lambda_S A_S S H_1 H_2 + \frac{\kappa}{3} A_S S^3 + \text{h.c.}). \quad (\text{A.2})$$

The  $SU(2)$ -doublet superfields are contracted by the antisymmetric tensor  $\epsilon_{ab}$ , with  $\epsilon_{12} = 1$ . The neutral component of Higgs fields can be decomposed into their vevs, CP-even and CP-odd fluctuations as,

$$H_i^0 = \frac{1}{\sqrt{2}}(v_i + S_i + iP_i), \quad S = \frac{1}{\sqrt{2}}(v_S + S_3 + iP_3). \quad (\text{A.3})$$

The CP-even and the CP-odd mass matrices are diagonalized by an orthogonal matrices  $R^S$  and  $R^P$ , respectively

$$\begin{aligned} h_i &= R_{ij}^S S_j, \\ a_i &= R_{ij}^P P_j. \end{aligned} \quad (\text{A.4})$$

### A.2 Definitions of the couplings

#### A.2.1 Higgs self-couplings

Here, we use for  $s_i$  stands for  $\{h_d, h_u, s_0\}$ ,  $h_i$  stands for  $\{h, H, s_0\}$ ,  $p_i$  stands for  $\{z_d, z_u, z_s\}$ ,  $a_i$  stands for  $\{G^0, A, z_s\}$  and  $h_i^\pm$  stands for  $\{G^\pm, H^\pm\}$ . Also,  $v_i$  stands for  $\{v_d, v_u\}$  with  $\tan\beta = v_2/v_1$  and  $(v_1^2 + v_2^2)^2 = (246\text{GeV})^2$ . The terms of neutral Higgs sector in the NMSSM Lagrangian can be written as

$$\begin{aligned} \mathcal{L} \supset & -\lambda_{s_i s_j s_k s_l} S_i S_j S_k S_l - \lambda_{p_i p_j p_k p_l} P_i P_j P_k P_l - \lambda_{s_i s_j p_k p_l} S_i S_j P_k P_l \\ & - \lambda_{s_i s_j s_k} S_i S_j S_k - \lambda_{s_i p_j p_k} S_i P_j P_k, \end{aligned} \quad (\text{A.5})$$

where  $\lambda_{s_1 s_1 s_2 s_2} = \lambda_{s_1 s_2 s_1 s_2} = \lambda_{s_2 s_1 s_1 s_2} = \lambda_{s_1 s_2 s_2 s_1} = \lambda_{s_2 s_1 s_2 s_1} = \lambda_{s_2 s_2 s_1 s_1}$ . The trilinear neutral-Higgs self couplings are written as

$$\begin{aligned}
\lambda_{s_1 s_1 s_1} &= \lambda_{s_1 p_1 p_1} = \frac{1}{8}(g_2^2 + g_Y^2)v_1, & \lambda_{s_2 s_2 s_2} &= \lambda_{s_2 p_2 p_2} = \frac{1}{8}(g_2^2 + g_Y^2)v_2, \\
\lambda_{s_1 p_2 p_2} &= 3\lambda_{s_1 s_2 s_2} = -\frac{1}{8}(g_2^2 + g_Y^2 - 4\lambda_S^2)v_1, & \lambda_{s_2 p_1 p_1} &= 3\lambda_{s_2 s_1 s_1} = -\frac{1}{8}(g_2^2 + g_Y^2 - 4\lambda_S^2)v_2, \\
\lambda_{s_3 p_1 p_1} &= \lambda_{s_3 p_2 p_2} = 3\lambda_{s_1 s_1 s_3} = 3\lambda_{s_1 s_1 s_3} = \frac{\lambda_S^2}{2}v_S, \\
\lambda_{s_3 s_3 s_3} &= \frac{A_\kappa \kappa}{3\sqrt{2}} + \kappa^2 v_S, & \lambda_{s_3 p_3 p_3} &= -\frac{A_\kappa \kappa}{\sqrt{2}} + \kappa^2 v_S, \\
\lambda_{s_1 s_3 s_3} &= \frac{3\lambda_S}{2}(-\kappa v_2 + \lambda_S v_1), & \lambda_{s_2 s_3 s_3} &= \frac{3\lambda_S}{2}(-\kappa v_1 + \lambda_S v_2), \\
\lambda_{s_1 p_3 p_3} &= \frac{\lambda_S}{2}(\kappa v_2 + \lambda_S v_1), & \lambda_{s_2 p_3 p_3} &= \frac{\lambda_S}{2}(\kappa v_1 + \lambda_S v_2), \\
\lambda_{s_3 p_1 p_3} &= -\kappa \lambda_S v_2, & \lambda_{s_3 p_2 p_3} &= -\kappa \lambda_S v_1, & \lambda_{s_1 s_2 s_3} &= -\frac{\lambda_S}{6}\left(\frac{A_S}{\sqrt{2}} + \kappa v_S\right), \\
\lambda_{s_1 p_2 p_3} &= \lambda_{s_2 p_1 p_3} = \lambda_S\left(\frac{A_S}{\sqrt{2}} - \kappa v_S\right), & \lambda_{s_3 p_1 p_2} &= \lambda_S\left(\frac{A_S}{\sqrt{2}} + \kappa v_S\right).
\end{aligned} \tag{A.6}$$

The quartic neutral-Higgs self couplings are written as

$$\begin{aligned}
\lambda_{s_1 s_1 s_1 s_1} &= \lambda_{s_2 s_2 s_2 s_2} = \lambda_{p_1 p_1 p_1 p_1} = \lambda_{p_2 p_2 p_2 p_2} = \frac{1}{32}(g_2^2 + g_Y^2), & \lambda_{s_3 s_3 s_3 s_3} &= \lambda_{p_3 p_3 p_3 p_3} = \frac{\kappa^2}{4}, \\
\lambda_{s_1 s_1 s_2 s_2} &= \lambda_{p_1 p_1 p_2 p_2} = -\frac{1}{96}(g_2^2 + g_Y^2 - 4\lambda_S^2), & \lambda_{s_1 s_2 s_3 s_3} &= \lambda_{p_1 p_2 p_3 p_3} = -\frac{\kappa \lambda_S}{24}, \\
\lambda_{s_1 s_1 s_3 s_3} &= \lambda_{s_2 s_2 s_3 s_3} = \lambda_{p_1 p_1 p_3 p_3} = \lambda_{p_2 p_2 p_3 p_3} = \frac{\lambda_S^2}{24}, \\
\lambda_{s_1 s_1 p_1 p_1} &= \lambda_{s_2 s_2 p_2 p_2} = \frac{1}{16}(g_2^2 + g_Y^2), & \lambda_{s_1 s_1 p_2 p_2} &= \lambda_{s_2 s_2 p_1 p_1} = -\frac{1}{16}(g_2^2 + g_Y^2 - 4\lambda_S^2), \\
\lambda_{s_1 s_1 p_3 p_3} &= \lambda_{s_2 s_2 p_3 p_3} = \lambda_{s_3 s_3 p_1 p_1} = \lambda_{s_3 s_3 p_2 p_2} = \frac{\lambda_S^2}{4}, \\
\lambda_{s_1 s_2 p_3 p_3} &= \lambda_{s_3 s_3 p_1 p_2} = -\lambda_{s_1 s_3 p_2 p_3} = -\lambda_{s_2 s_3 p_1 p_3} = \frac{\kappa \lambda_S}{4}, \\
\lambda_{s_3 s_3 p_3 p_3} &= \frac{\kappa^2}{2}.
\end{aligned} \tag{A.7}$$

These couplings can be rewritten as

$$\begin{aligned}
\lambda_{s_i s_j h_k h_l} &= 6R_{ka}^S R_{lb}^S \lambda_{s_i s_j s_a s_b}, & \lambda_{s_i s_j h_k h_l} &= R_{ka}^P R_{lb}^P \lambda_{s_i s_j p_a p_b}, \\
\lambda_{s_i h_k h_l} &= 3R_{ka}^S R_{lb}^S \lambda_{s_i s_a s_b}, & \lambda_{s_i a_k a_l} &= R_{ka}^P R_{lb}^P \lambda_{s_i p_a p_b}.
\end{aligned} \tag{A.8}$$

The terms of the charged and neutral Higgs sector in the NMSSM Lagrangian can be written as

$$\begin{aligned} \mathcal{L} \supset & - \sum_{i,j,k,l} \lambda_{s_i s_j h_k^+ h_l^-} S_i S_j h_k^+ h_l^- - \sum_{i,j,k,l} \lambda_{p_i p_j h_k^+ h_l^-} P_i P_j h_k^+ h_l^- \\ & - \sum_{i,k,l} \lambda_{s_i h_k^+ h_l^-} S_i h_k^+ h_l^- - \sum_{i,k,l} i \lambda_{p_i h_k^+ h_l^-} P_i h_k^+ h_l^-, \end{aligned} \quad (\text{A.9})$$

where  $h_i^\pm$  is the mass eigenstates and stands for  $(G^\pm, H^\pm)$ . The trilinear neutral-charged coupling are written as

$$\begin{aligned} \lambda_{s_1 h_1^+ h_1^-} &= \frac{1}{4} \left( (g_2^2 + g_Y^2 \cos 2\beta) v_1 + (2\lambda_S^2 - g_2^2) v_2 \sin 2\beta \right), \\ \lambda_{s_1 h_2^+ h_2^-} &= \frac{1}{4} \left( (g_2^2 - g_Y^2 \cos 2\beta) v_1 - (2\lambda_S^2 - g_2^2) v_2 \sin 2\beta \right), \\ \lambda_{s_1 h_1^+ h_2^-} &= \frac{1}{4} \left( g_Y^2 \sin 2\beta v_1 + (\lambda_S^2 - g_2^2) v_2 \cos 2\beta \right), \\ \lambda_{s_2 h_1^+ h_1^-} &= \frac{1}{4} \left( (g_2^2 - g_Y^2 \cos 2\beta) v_2 + (2\lambda_S^2 - g_2^2) v_2 \sin 2\beta \right), \\ \lambda_{s_2 h_2^+ h_2^-} &= \frac{1}{4} \left( (g_2^2 + g_Y^2 \cos 2\beta) v_2 - (2\lambda_S^2 - g_2^2) v_2 \sin 2\beta \right), \\ \lambda_{s_2 h_1^+ h_2^-} &= \frac{1}{4} \left( g_Y^2 \sin 2\beta v_2 + (2\lambda_S^2 - g_2^2) v_2 \cos 2\beta \right), \\ \lambda_{s_3 h_1^+ h_1^-} &= \frac{\lambda_S}{2} \left( 2\lambda_S v_S - \left( \frac{1}{\sqrt{2}} A_S + \kappa v_S \right) \right), \\ \lambda_{s_3 h_2^+ h_2^-} &= \frac{\lambda_S}{2} \left( 2\lambda_S v_S + \left( \frac{1}{\sqrt{2}} A_S + \kappa v_S \right) \right), \\ \lambda_{s_3 h_1^+ h_2^-} &= \frac{\lambda_S}{\sqrt{2}} (A_S + \kappa v_S) \cos 2\beta, \\ \lambda_{p_1 h_1^+ h_2^-} &= \frac{1}{4} (2\lambda_S^2 - g_2^2) v_1, \quad \lambda_{p_2 h_1^+ h_2^-} = \frac{1}{4} (2\lambda_S^2 - g_2^2) v_2, \\ \lambda_{p_3 h_1^+ h_2^-} &= \frac{\lambda_S}{\sqrt{2}} (A_S - \sqrt{2} \kappa v_S), \end{aligned} \quad (\text{A.10})$$

$$\begin{aligned} \lambda_{s_1 s_1 h_1^+ h_1^-} &= \lambda_{s_2 s_2 h_2^+ h_2^-} = \lambda_{p_1 p_1 h_1^+ h_1^-} = \lambda_{p_2 p_2 h_2^+ h_2^-} = \frac{1}{8} (g_2^2 + g_Y^2 \cos 2\beta), \\ \lambda_{s_1 s_1 h_2^+ h_2^-} &= \lambda_{s_2 s_2 h_1^+ h_1^-} = \lambda_{p_1 p_1 h_2^+ h_2^-} = \lambda_{p_2 p_2 h_1^+ h_1^-} = \frac{1}{8} (g_2^2 - g_Y^2 \cos 2\beta), \\ \lambda_{s_1 s_2 h_1^+ h_1^-} &= \lambda_{p_1 p_2 h_2^+ h_2^-} = -\lambda_{s_1 s_2 h_2^+ h_2^-} = \lambda_{p_1 p_2 h_1^+ h_1^-} = \frac{1}{8} (2\lambda_S^2 - g_2^2) \sin 2\beta, \\ \lambda_{s_1 s_1 h_1^+ h_2^-} &= \lambda_{p_1 p_1 h_1^+ h_2^-} = -\lambda_{s_2 s_2 h_1^+ h_2^-} = \lambda_{p_2 p_2 h_1^+ h_2^-} = -\frac{g_Y^2}{8} \sin 2\beta, \\ \lambda_{s_1 s_2 h_1^+ h_2^-} &= -\lambda_{p_1 p_2 h_1^+ h_2^-} = \frac{1}{8} (2\lambda_S^2 - g_2^2) \cos 2\beta, \quad \lambda_{s_3 s_3 h_1^+ h_2^-} = -\lambda_{p_3 p_3 h_1^+ h_2^-} = \frac{\kappa \lambda_S}{2} \cos 2\beta, \\ \lambda_{s_3 s_3 h_1^+ h_1^-} &= -\lambda_{p_3 p_3 h_2^+ h_2^-} = \frac{\lambda_S}{2} (\lambda_S - \kappa \sin 2\beta), \quad \lambda_{s_3 s_3 h_2^+ h_2^-} = -\lambda_{p_3 p_3 h_1^+ h_1^-} = \frac{\lambda_S}{2} (\lambda_S + \kappa \sin 2\beta). \end{aligned} \quad (\text{A.11})$$

### A.2.2 Higgs-neutralino couplings

In the NMSSM, the Lagrangian contain Higgs-neutralino couplings as

$$\mathcal{L} \supset - \sum_{ikl} \lambda_{s_i \psi_k^0 \psi_l^0} S_i \psi_k^0 \psi_l^0 - i \sum_{ikl} \lambda_{p_i \psi_k^0 \psi_l^0} P_i \psi_k^0 \psi_l^0 + \text{h.c.}, \quad (\text{A.12})$$

where  $\psi_i^0 = \{\tilde{B}^0, \tilde{W}^0, \tilde{h}_1^0, \tilde{h}_2^0, \tilde{s}^0\}$ . The couplings in Eq. (A.12) are symmetric for exchanging the neutralino indices  $k$  and  $l$ . The non-zero Higgs-neutralino couplings are following [57]

$$\begin{aligned} \lambda_{s_1 \psi_1^0 \psi_3^0} &= -\lambda_{p_1 \psi_1^0 \psi_3^0} = -\frac{g_Y}{4}, & \lambda_{s_1 \psi_2^0 \psi_3^0} &= -\lambda_{p_1 \psi_2^0 \psi_3^0} = +\frac{g_2}{4}, \\ \lambda_{s_1 \psi_4^0 \psi_5^0} &= \lambda_{p_1 \psi_4^0 \psi_5^0} = +\frac{\lambda_S}{2\sqrt{2}}, & \lambda_{s_2 \psi_1^0 \psi_4^0} &= -\lambda_{p_2 \psi_1^0 \psi_4^0} = +\frac{g_Y}{4}, \\ \lambda_{s_2 \psi_2^0 \psi_4^0} &= -\lambda_{p_2 \psi_2^0 \psi_4^0} = -\frac{g_2}{4}, & \lambda_{s_2 \psi_3^0 \psi_5^0} &= \lambda_{p_2 \psi_3^0 \psi_5^0} = -\frac{\lambda_S}{2\sqrt{2}}, \\ \lambda_{s_3 \psi_3^0 \psi_4^0} &= \lambda_{p_3 \psi_3^0 \psi_4^0} = -\frac{\lambda_S}{2\sqrt{2}}, & \lambda_{s_3 \psi_5^0 \psi_5^0} &= \lambda_{p_3 \psi_5^0 \psi_5^0} = \frac{\kappa}{\sqrt{2}}. \end{aligned} \quad (\text{A.13})$$

### A.2.3 Higgs-chargino couplings

The Lagrangian also contain the Higgs-chargino couplings as

$$\mathcal{L} \supset - \sum_{ikl} \lambda_{s_i \psi_k^+ \psi_l^-} S_i \psi_k^+ \psi_l^- - i \sum_{ikl} \lambda_{p_i \psi_k^+ \psi_l^-} P_i \psi_k^+ \psi_l^- + \text{h.c.}, \quad (\text{A.14})$$

where  $\psi_i^+ = \{\tilde{W}^+, \tilde{h}_u^+\}$  and  $\psi_i^- = \{\tilde{W}^-, \tilde{h}_d^-\}$ . The non-zero Higgs-chargino couplings are following [57]

$$\begin{aligned} \lambda_{s_1 \psi_1^+ \psi_2^-} &= -\lambda_{p_1 \psi_1^+ \psi_2^-} = +\frac{g_2}{\sqrt{2}}, & \lambda_{s_2 \psi_2^+ \psi_1^-} &= -\lambda_{p_2 \psi_2^+ \psi_1^-} = +\frac{g_2}{\sqrt{2}}, \\ \lambda_{s_3 \psi_2^+ \psi_2^-} &= \lambda_{p_3 \psi_2^+ \psi_2^-} = +\frac{\lambda_S}{\sqrt{2}}. \end{aligned} \quad (\text{A.15})$$

## A.3 One-loop self energies and tadpole

### A.3.1 Scalar self energies

The Higgs contributions to the scalar self energies

$$\begin{aligned} 16\pi^2 \Pi_{s_i s_j}^H(p^2) &= \sum_k^3 2\lambda_{s_i s_j h_k h_k} A(m_{h_k}) + \sum_{k,l}^3 2\lambda_{s_i h_k h_l} \lambda_{s_j h_k h_l} B_0(p^2; m_{h_k}, m_{h_l}) \\ &+ \sum_k^3 2\lambda_{s_i s_j a_k a_k} A(m_{a_k}) + \sum_{k,l}^3 2\lambda_{s_i a_k a_l} \lambda_{s_j a_k a_l} B_0(p^2; m_{a_k}, m_{a_l}) \\ &+ \sum_k^2 2\lambda_{s_i s_j h_k^+ h_k^-} A(m_{h_k^\pm}) + \sum_{k,l}^3 \lambda_{s_i h_k^+ h_l^-} \lambda_{s_j h_k^+ h_l^-} B_0(p^2; m_{h_k^\pm}, m_{h_l^\pm}). \end{aligned} \quad (\text{A.16})$$

Here,  $A$  and  $B_0$  are the Passarino-Veltman functions [58].

The neutralino and chargino contributions to the scalar self energies

$$\begin{aligned}
16\pi^2\Pi_{s_i s_j}^\chi &= \sum_{k,l}^5 4\lambda_{s_i\chi_k^0\chi_l^0}\lambda_{s_j\chi_k^0\chi_l^0}^* \left[ (p^2 - m_{\chi_k^0}^2 - m_{\chi_l^0}^2)B_0(m_{\chi_k^0}, m_{\chi_l^0}) - A(m_{\chi_k^0}) - A(m_{\chi_l^0}) \right. \\
&\quad \left. - 2m_{\chi_k^0}m_{\chi_l^0}B_0(m_{\chi_k^0}, m_{\chi_l^0}) \right] \\
&+ \sum_{k,l}^2 2\lambda_{s_i\chi_k^+\chi_l^-}\lambda_{s_j\chi_k^+\chi_l^-}^* \left[ (p^2 - m_{\chi_k^\pm}^2 - m_{\chi_l^\pm}^2)B_0(m_{\chi_k^\pm}, m_{\chi_l^\pm}) - A(m_{\chi_k^\pm}) - A(m_{\chi_l^\pm}) \right. \\
&\quad \left. - 2m_{\chi_k^\pm}m_{\chi_l^\pm}B_0(m_{\chi_k^\pm}, m_{\chi_l^\pm}) \right]. \tag{A.17}
\end{aligned}$$

### A.3.2 Tadpole

The contributions to the tadpoles from neutralinos and charginos

$$16\pi^2T_i^\chi = -4 \sum_k^5 \lambda_{s_i\chi_k\chi_k} m_{\chi_i} A(m_{\chi_i}) - 4 \sum_k^2 \lambda_{s_i\chi_k^+\chi_k^-} m_{\chi_i} A(m_{\chi_i}). \tag{A.18}$$

The contributions to the tadpoles from Higgs bosons

$$16\pi^2T_i^\phi = \sum_\phi^{h,a,h^\pm} \sum_k \lambda_{s_i\phi_k\phi_k} A(m_{\phi_i}). \tag{A.19}$$



# Appendix B

## SGGHU

In this Appendix, we specify the Higgs sector in the SGGHU.

### B.1 Higgs potential

#### F-term

In the Gauge-Higgs unification model, there are two Higgs doublet  $H_u = (H_u^+, H_u^0)^T \sim (2, 1)$  and  $H_d = (H_d^0, H_d^-)^T \sim (2, -1)$ , a Higgs singlet  $S \sim (1, 0)$  and a Higgs triplet

$$\Delta = \Delta^a T^a = \frac{1}{2} \begin{pmatrix} \Delta^3 & \sqrt{2}\Delta^+ \\ \sqrt{2}\Delta^- & -\Delta^3 \end{pmatrix} \sim (3, 0), (\text{SU}(2) \times \text{U}(1)),$$

where  $\Delta^+ = (\Delta^1 - i\Delta^2)/\sqrt{2}$  and  $\Delta^- = (\Delta^1 + i\Delta^2)/\sqrt{2}$ . The low-energy superpotential is written as following:

$$\begin{aligned} W &= \mu(H_{u_i}(i\tau_2)_{ij}H_{d_j}) + \frac{1}{2}\mu_\Delta\Delta^a\Delta^a + \frac{1}{2}\mu_S S^2 + \lambda_\Delta H_{u_i}(i\tau_2\frac{\sigma^a}{2})_{ij}\Delta^a H_{d_j} + \lambda_S S H_{u_i}(i\tau_2)_{ij}H_{d_j} \\ &= \mu\{H_u^+ H_d^- - H_u^0 H_d^0\} + \frac{1}{4}\mu_\Delta\{(\Delta^3)^2 + 2\Delta^+\Delta^-\} \\ &\quad + \frac{\lambda_\Delta}{2}\{H_d^- \Delta^3 H_u^+ + H_d^0 \Delta^3 H_u^0 - \sqrt{2}H_d^0 \Delta^- H_u^+ + \sqrt{2}H_d^- \Delta^+ H_u^0\} \\ &\quad + \lambda_S\{H_u^+ H_d^- S - H_u^0 H_d^0 S\} + \frac{1}{2}\mu_S S^2. \end{aligned} \tag{B.1}$$

The F-term contributions are given by

$$V_F = |F_{H_d}|^2 + |F_{H_u}|^2 + |F_S|^2 + |F_{\Delta_0}|^2 + |F_{H_u^+}|^2 + |F_{H_d^-}|^2 + |F_{\Delta^+}|^2 + |F_{\Delta^-}|^2, \tag{B.2}$$

where

$$\begin{aligned}
F_{H_u^0} &= -\frac{\partial W}{\partial H_u^0} = +\mu H_d^0 + \frac{\lambda_\Delta}{2}(H_d^0 \Delta^3 + \sqrt{2} H_d^- \Delta^+) + \lambda_S H_d^0 S, \\
F_{H_d^0} &= -\frac{\partial W}{\partial H_d^0} = +\mu H_u^0 + \frac{\lambda_\Delta}{2}(\Delta^3 H_u^0 - \sqrt{2} \Delta^- H_u^+) + \lambda_S H_u^0 S, \\
F_S &= -\frac{\partial W}{\partial S} = -\mu_S S - \lambda_S H_u^+ H_d^- + \lambda_S H_u^0 H_d^0, \\
F_{\Delta^3} &= -\frac{\partial W}{\partial \Delta^3} = -\mu_\Delta \Delta^3 + \frac{\lambda_\Delta}{2}(H_d^- H_u^+ + H_d^0 H_u^0), \\
F_{H_u^+} &= -\frac{\partial W}{\partial H_u^+} = -\mu H_d^- + \frac{\lambda_\Delta}{2}(H_d^- \Delta^3 - \sqrt{2} H_d^0 \Delta^-) - \lambda_S H_d^- S, \\
F_{H_d^-} &= -\frac{\partial W}{\partial H_d^-} = -\mu H_u^+ + \frac{\lambda_\Delta}{2}(\Delta^3 H_u^+ + \sqrt{2} \Delta^+ H_u^0) - \lambda_S H_u^+ S, \\
F_{\Delta^+} &= -\frac{\partial W}{\partial \Delta^+} = -\mu_\Delta \Delta^- + \frac{\lambda_\Delta}{\sqrt{2}} H_d^- H_u^0, \\
F_{\Delta^-} &= -\frac{\partial W}{\partial \Delta^-} = -\mu_\Delta \Delta^+ - \frac{\lambda_\Delta}{\sqrt{2}} H_d^0 H_u^+.
\end{aligned} \tag{B.3}$$

## D-term

The Higgs singlet "S" has  $\{1, 0\}$ , that is, the D-term of this model is equivalent the D-term of MSSM + triplet model.

$$\begin{aligned}
D^a &= -g_2 \sum_i (\phi_{(i)}^\dagger t_{(i)}^a \phi_{(i)}) \\
&= -g_2 H_{u_i}^* \left(\frac{\sigma^a}{2}\right)_{ij} H_{u_j} - g_2 H_{d_i}^* \left(\frac{\sigma^a}{2}\right)_{ij} H_{d_j} - g_2 \Delta^{b*} i \epsilon^{abc} \Delta^c
\end{aligned} \tag{B.4}$$

$$\begin{aligned}
D_Y &= -g_Y \sum_i (\phi_{(i)}^\dagger Y_{(i)} \phi_{(i)}) \\
&= -\frac{g_Y}{2} H_{u_i}^* H_{u_i} + \frac{g_Y}{2} H_{d_i}^* H_{d_i}
\end{aligned} \tag{B.5}$$



The D-term is written as

$$\begin{aligned}
V_D &= \frac{1}{2}(D^a)^2 + \frac{1}{2}(D_Y)^2 \\
&= \frac{g_Y^2}{8} [ (|H_u^+|^2 + |H_u^0|^2)^2 - 2(|H_u^+|^2 + |H_u^0|^2)(|H_d^0|^2 + |H_d^-|^2) + (|H_d^0|^2 + |H_d^-|^2)^2 ] \\
&\quad + \frac{g_2^2}{8} [ |H_u^+|^4 + 2|H_u^+ H_u^0|^2 + |H_u^0|^4 + |H_d^0|^4 + 2|H_d^{-*} H_d^0|^2 + |H_d^-|^4 \\
&\quad + \{ -8\bar{\Delta}^+ \bar{\Delta}^- (\Delta^3)^2 + 8(\Delta^+ \bar{\Delta}^- + \bar{\Delta}^+ \Delta^-) |\Delta^3|^2 - 8\Delta^+ \Delta^- (\Delta^{3*})^2 \} \\
&\quad - 4\sqrt{2}(H_u^{0*} H_u^+) (\Delta^{3*} \Delta^- - \bar{\Delta}^- \Delta^3) - 4\sqrt{2}(H_u^{+*} H_u^0) (\Delta^3 \bar{\Delta}^+ - \Delta^{3*} \Delta^+) \\
&\quad - 2(|H_u^+|^2 - |H_u^0|^2) (\bar{\Delta}^- \Delta^+ - \bar{\Delta}^+ \Delta^-) \\
&\quad - 4\sqrt{2}(H_d^{-*} H_d^0) (\Delta^{3*} \Delta^- - \bar{\Delta}^- \Delta^3) - 4\sqrt{2}(H_d^{0*} H_d^-) (\Delta^3 \bar{\Delta}^+ - \Delta^{3*} \Delta^+) \\
&\quad - 2(|H_u^0|^2 - |H_d^-|^2) (\bar{\Delta}^- \Delta^+ - \bar{\Delta}^+ \Delta^-) \\
&\quad + 4H_u^{0*} H_d^{0*} H_u^+ H_d^- + 4H_u^0 H_d^0 H_u^{+*} H_d^{-*} + 2|H_u^+|^2 |H_d^0|^2 - |H_u^0| |H_d^0|^2 + 2|H_d^0|^2 |H_d^-|^2 ] \tag{B.6}
\end{aligned}$$

### Soft-term

The Soft-term is written as

$$\begin{aligned}
\tilde{V} &= \tilde{m}_d^2 |H_d|^2 + \tilde{m}_u^2 |H_u|^2 + \tilde{m}_\Delta^2 (|\Delta^3|^2 + |\Delta^-|^2 + |\Delta^+|^2) + \tilde{m}_S^2 |S|^2 \\
&\quad + [B\mu H_u \cdot H_d + \bar{\eta} S + B_\Delta \mu_\Delta \text{tr}(\Delta^2) + \frac{1}{2} B_S \mu_S S^2 \\
&\quad + \lambda_\Delta A_\Delta H_u \cdot \Delta H_d + \lambda_S A_S H_u \cdot H_d S + \text{h.c.}] \\
&= \tilde{m}_d^2 (|H_d^-|^2 + |H_d^0|^2) + \tilde{m}_u^2 (|H_u^+|^2 + |H_u^0|^2) + \tilde{m}_\Delta^2 (|\Delta^3|^2 + |\Delta^-|^2 + |\Delta^+|^2) + \tilde{m}_S^2 |S|^2 \\
&\quad + [B\mu \{ H_u^+ H_d^- - H_u^0 H_d^0 \} + \bar{\eta} S + \frac{1}{2} B_\Delta \mu_\Delta \{ (\Delta^3)^2 + 2\Delta^+ \Delta^- \} + \frac{1}{2} B_S \mu_S S^2 \\
&\quad - \frac{\lambda_\Delta}{2} A_\Delta \{ H_d^- \Delta^3 H_u^+ + H_d^0 \Delta^3 H_u^0 - \sqrt{2} H_d^0 \Delta^- H_u^+ + \sqrt{2} H_d^- \Delta^+ H_u^0 \} \\
&\quad + \lambda_S A_S \{ H_u^+ H_d^- S - H_u^0 H_d^0 S \} + \text{h.c.}] \tag{B.7}
\end{aligned}$$

## B.2 Tadpole

The fields component is written as

$$\begin{aligned}
H_d &= \begin{pmatrix} \frac{1}{\sqrt{2}}(h_d + v_d + iz_d) \\ w_d^- \end{pmatrix}, H_u = \begin{pmatrix} w_u^+ \\ \frac{1}{\sqrt{2}}(h_u + v_u + iz_u) \end{pmatrix}, \\
S &= \frac{1}{\sqrt{2}}(s_0 + v_S + iz_S), \Delta = \frac{1}{2} \begin{pmatrix} \frac{1}{\sqrt{2}}(\Delta_0 + v_\Delta + iz_\Delta) & \sqrt{2}\Delta^+ \\ \sqrt{2}\Delta^- & -\frac{1}{\sqrt{2}}(\Delta_0 + v_\Delta + iz_\Delta) \end{pmatrix}. \tag{B.8}
\end{aligned}$$

We obtain  $V_{\text{tot}}$  from  $V_F$ ,  $V_D$ ,  $\tilde{V}$ . The minimisation equations are given by

$$\begin{aligned} \left\langle \frac{\partial V}{\partial H_d} \right\rangle = & v_d \left\{ \mu^2 + \frac{\lambda_S^2}{2} (v_S^2 + v_u^2) + \sqrt{2} \lambda_S \mu v_S + \frac{1}{8} (g_2^2 + g_Y^2) (v_d^2 - v_u^2) \right. \\ & + \frac{\lambda_\Delta^2}{8} (v_\Delta^2 + v_u^2) + \frac{\lambda_\Delta}{\sqrt{2}} \mu v_\Delta + \frac{\lambda_\Delta \lambda_S}{2} v_\Delta v_S + \tilde{m}_d^2 \left. \right\} \\ & + v_u \left\{ -\frac{\lambda_S}{\sqrt{2}} \mu_S v_S - B\mu - \frac{\lambda_S}{2\sqrt{2}} A_S v_S - \frac{\lambda_\Delta}{2\sqrt{2}} \mu_\Delta v_\Delta - \frac{\lambda_\Delta}{2\sqrt{2}} A_\Delta v_\Delta \right\} = 0, \end{aligned} \quad (\text{B.9})$$

$$\begin{aligned} \left\langle \frac{\partial V}{\partial H_u} \right\rangle = & v_u \left\{ \mu^2 + \frac{\lambda_S^2}{2} (v_S^2 + v_u^2) + \sqrt{2} \lambda_S \mu v_S + \frac{1}{8} (g_2^2 + g_Y^2) (v_d^2 - v_u^2) \right. \\ & + \frac{\lambda_\Delta^2}{8} (v_\Delta^2 + v_u^2) + \frac{\lambda_\Delta}{\sqrt{2}} \mu v_\Delta + \frac{\lambda_\Delta \lambda_S}{2} v_\Delta v_S + \tilde{m}_u^2 \left. \right\} \\ & + v_d \left\{ -\frac{\lambda_S}{\sqrt{2}} \mu_S v_S - B\mu - \frac{\lambda_S}{2\sqrt{2}} A_S v_S - \frac{\lambda_\Delta}{2\sqrt{2}} \mu_\Delta v_\Delta - \frac{\lambda_\Delta}{2\sqrt{2}} A_\Delta v_\Delta \right\} = 0, \end{aligned} \quad (\text{B.10})$$

$$\begin{aligned} \left\langle \frac{\partial V}{\partial S} \right\rangle = & v_S \left\{ \mu_S^2 + \frac{\lambda_S^2}{2} v^2 + B_S \mu_S + \tilde{m}_S^2 \right\} \\ & + \lambda_S \left\{ \frac{1}{\sqrt{2}} \mu v^2 - \frac{1}{\sqrt{2}} \mu_S v_d v_u - \frac{1}{\sqrt{2}} A_S v_d v_u + \frac{\lambda_\Delta}{4} v^2 v_\Delta \right\} - \sqrt{2} \bar{\eta} = 0, \end{aligned} \quad (\text{B.11})$$

$$\begin{aligned} \left\langle \frac{\partial V}{\partial \Delta} \right\rangle = & v_\Delta \left\{ \mu_\Delta^2 + B_\Delta \mu_\Delta + \frac{\lambda_\Delta^2}{8} v^2 + \tilde{m}_\Delta^2 \right\} \\ & + \lambda_\Delta \left\{ \frac{1}{2\sqrt{2}} \mu v^2 + \frac{\lambda_S}{4} v^2 v_S - \frac{1}{2\sqrt{2}} \mu_\Delta v_d v_u - \frac{1}{2\sqrt{2}} A_\Delta v_d v_u \right\} = 0. \end{aligned} \quad (\text{B.12})$$

Eq.(B.9) and Eq.(B.10) lead to  $\hat{m}_3^2$  and  $\mu_{\text{eff}}^2$ :

$$\hat{m}_3^2 = B\mu + \frac{\lambda_S}{\sqrt{2}} (\mu_S + A_S) v_S + \frac{\lambda_\Delta}{2\sqrt{2}} (\mu_\Delta + A_\Delta) v_\Delta, \quad (\text{B.13})$$

$$\mu_{\text{eff}} = \mu + \frac{\lambda_S}{\sqrt{2}} v_S + \frac{\lambda_\Delta}{2\sqrt{2}} v_\Delta. \quad (\text{B.14})$$

Tadpole is written as Eq.(B.13) and Eq.(B.14):

$$\left\langle \frac{\partial V}{\partial H_d} \right\rangle = v_d \left\{ \mu_{\text{eff}}^2 + \frac{\lambda_S^2}{2} v_u^2 + \frac{\lambda_\Delta^2}{8} v_u^2 + \frac{g_2^2 + g_Y^2}{8} (v_d^2 - v_u^2) + \tilde{m}_d^2 \right\} - v_u \hat{m}_3^2 = 0, \quad (\text{B.15})$$

$$\left\langle \frac{\partial V}{\partial H_u} \right\rangle = v_u \left\{ \mu_{\text{eff}}^2 + \frac{\lambda_S^2}{2} v_d^2 + \frac{\lambda_\Delta^2}{8} v_d^2 + \frac{g_2^2 + g_Y^2}{8} (v_u^2 - v_d^2) + \tilde{m}_u^2 \right\} - v_d \hat{m}_3^2 = 0, \quad (\text{B.16})$$

$$\begin{aligned} \left\langle \frac{\partial V}{\partial S} \right\rangle = & v_S (\mu_S^2 + \tilde{m}_S^2 + B_S \mu_S) \\ & + \frac{\lambda_S}{\sqrt{2}} \{ \mu_{\text{eff}} v^2 - (A_S + \mu_S) v^2 s_\beta c_\beta + 2(\mu_S + \bar{\eta}) \} = 0, \end{aligned} \quad (\text{B.17})$$

$$\begin{aligned} \left\langle \frac{\partial V}{\partial \Delta} \right\rangle = & v_\Delta (\mu_\Delta^2 + \tilde{m}_\Delta^2 + B_\Delta \mu_\Delta) \\ & + \frac{\lambda_\Delta}{2\sqrt{2}} \{ +\mu_{\text{eff}} v^2 - (A_\Delta + \mu_\Delta) v^2 s_\beta c_\beta \} = 0. \end{aligned} \quad (\text{B.18})$$

We obtain  $A_\Delta$  and  $v_S$  from tadpole conditions:

$$A_\Delta = \frac{4(\tilde{m}_\Delta^2 + \mu_\Delta^2)}{\sqrt{2}v_d v_u} v_\Delta + \frac{\mu_{\text{eff}}}{s_\beta c_\beta} - \mu_\Delta, \quad (\text{B.19})$$

$$v_S = \frac{\lambda_S \{(v_d^2 + v_u^2)\mu_{\text{eff}} + (A_S + \mu_S)v_d v_u\}}{\sqrt{2}(\tilde{m}_S^2 + \mu_S^2)}. \quad (\text{B.20})$$

## B.3 Mass matrices

In this thesis, we ignore  $v_\Delta$  because of the electroweak rho parameter.

### B.3.1 Higgs mass matrices

#### CP-odd

In the CP-odd basis  $\{z_d, z_u, z_s, z_\Delta\}$ , the mass matrix  $M_{\text{odd}}^2$  is written as

$$M_{\text{odd}}^2 = \begin{pmatrix} \hat{m}_3^2 t_\beta & \hat{m}_3^2 & \frac{\lambda_S}{\sqrt{2}}(A_S - \mu_S)v s_\beta & \frac{\lambda_\Delta}{2\sqrt{2}}(\mu_{\text{eff}} - \mu_\Delta s_{2\beta})v c_\beta^{-1} \\ \cdots & \hat{m}_3^2 t_\beta^{-1} & \frac{\lambda_S}{\sqrt{2}}(A_S - \mu_S)v c_\beta & \frac{\lambda_\Delta}{2\sqrt{2}}(\mu_{\text{eff}} - \mu_\Delta s_{2\beta})v s_\beta^{-1} \\ \cdots & \cdots & \tilde{m}_S^2 + \mu_S^2 - m_{3S}^2 + \frac{\lambda_S^2}{2}v^2 & \frac{\lambda_S \lambda_\Delta}{4}v^2 \\ \cdots & \cdots & \cdots & \tilde{m}_\Delta^2 + \mu_\Delta^2 - m_{3\Delta}^2 + \frac{\lambda_\Delta^2}{8}v^2 \end{pmatrix}, \quad (\text{B.21})$$

where  $m_{3S}^2 \equiv B_S \mu_S$ ,  $m_{3\Delta}^2 \equiv B_\Delta \mu_\Delta$ ,  $s_\beta = \sin \beta$ ,  $c_\beta = \cos \beta$ ,  $t_\beta = \tan \beta$ , and subscript  $-1$  describes inverse number. We obtain the CP-odd Higgs boson mass:

$$m_A^2 \sim \frac{\hat{m}_3^2}{\sin \beta \cos \beta} \quad (\text{B.22})$$

#### CP-even

In the basis  $\{h_d, h_u, s_0, \Delta_o\}$ , the mass matrix  $M_{\text{even}}^2$  is written by

$$M_{\text{even}}^{(0)2} = \begin{pmatrix} \frac{g_2^2 + g_Y^2}{4} v^2 c_\beta^2 + \hat{m}_3^2 t_\beta & -M^2 s_\beta c_\beta & \frac{\lambda_S}{\sqrt{2}}(2\mu_{\text{eff}} c_\beta - (A_S + \mu_S)s_\beta)v & \frac{\lambda_\Delta}{2\sqrt{2}}\mu_{\text{eff}} c_{2\beta} c_\beta^{-1} \\ \cdots & \frac{g_2^2 + g_Y^2}{4} v^2 s_\beta^2 + \hat{m}_3^2 t_\beta^{-1} + \epsilon & \frac{\lambda_S}{\sqrt{2}}(2\mu_{\text{eff}} s_\beta - (A_S + \mu_S)c_\beta)v & -\frac{\lambda_\Delta}{2\sqrt{2}}\mu_{\text{eff}} c_{2\beta} s_\beta^{-1} \\ \cdots & \cdots & \tilde{m}_S^2 + \mu_S^2 + m_{3S}^2 + \frac{\lambda_S^2}{2}v^2 & \frac{\lambda_S \lambda_\Delta}{4}v^2 \\ \cdots & \cdots & \cdots & \tilde{m}_\Delta^2 + \mu_\Delta^2 + m_{3\Delta}^2 + \frac{\lambda_\Delta^2}{8}v^2 \end{pmatrix}, \quad (\text{B.23})$$

where

$$M^2 = \frac{\hat{m}_3^2}{\sin \beta \cos \beta} + \left( \frac{g_Y^2}{4} + \frac{g_2^2}{4} - \lambda_S^2 - \frac{\lambda_\Delta^2}{4} \right) v^2. \quad (\text{B.24})$$

The mass of light CP-even Higgs boson  $m_h$  and the heavy CP-even Higgs boson  $m_H$  are written as following

$$m_h^2 \sim m_Z^2 \cos^2 2\beta + \left( \frac{\lambda_S^2}{2} + \frac{\lambda_\Delta^2}{8} \right) v^2 \sin^2 2\beta, \quad (\text{B.25})$$

$$m_H^2 \sim \frac{\hat{m}_3^2}{\sin \beta \cos \beta} + m_Z^2 \sin^2 2\beta - \left( \frac{\lambda_S^2}{2} + \frac{\lambda_\Delta^2}{8} \right) v^2 \sin^2 2\beta. \quad (\text{B.26})$$

## Charged

In the basis  $\{w_d^+, w_u^+, \Delta^+, \bar{\Delta}^+\}$ , the mass matrix  $M_{\text{charged}}^2$  is written by

$$M_{\text{charged}}^2 = \begin{pmatrix} M_C^2 \sin^2 \beta & M_C^2 \sin \beta \cos \beta & \frac{\lambda_\Delta}{2}(\mu_{\text{eff}} c_\beta - A_\Delta s_\beta)v & \frac{\lambda_\Delta}{2}(\mu_{\text{eff}} c_\beta - \mu_\Delta s_\beta)v \\ \cdots & M_C \cos^2 \beta & -\frac{\lambda_\Delta}{2}(\mu_{\text{eff}} s_\beta - \mu_\Delta c_\beta)v & -\frac{\lambda_\Delta}{2}(\mu_{\text{eff}} s_\beta - A_\Delta c_\beta)v \\ \cdots & \cdots & \frac{\lambda_\Delta^2}{4} s_\beta^2 v^2 + \frac{g_2^2}{8} c_{2\beta} v^2 + \mu_\Delta^2 + \tilde{m}_\Delta^2 & 0 \\ \cdots & \cdots & \cdots & \frac{\lambda_\Delta^2}{4} c_\beta^2 v^2 - \frac{g_2^2}{8} c_{2\beta} v^2 + \mu_\Delta^2 + \tilde{m}_\Delta^2 \end{pmatrix}, \quad (\text{B.27})$$

where

$$M_C^2 = \frac{\hat{m}_3^2}{\sin \beta \cos \beta} + \left( \frac{g_2^2}{4} - \frac{\lambda_S^2}{2} + \frac{\lambda_\Delta^2}{8} \right) v^2. \quad (\text{B.28})$$

We obtain the charged Higgs mass

$$m_{H^\pm}^2 \sim m_W^2 + m_A^2 + \left( -\frac{\lambda_s^2}{2} + \frac{\lambda_\Delta^2}{8} \right) v^2. \quad (\text{B.29})$$

## B.3.2 Neutralinos and charginos

### Neutralinos

In the basis  $\psi^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{s}_0, \tilde{\Delta}_0)$ , the neutralino mass part of the Lagrangian is

$$\mathcal{L}_{\text{neutralino mass}} = -\frac{1}{2}(\psi^0)^T M_{\tilde{N}} \psi^0 + \text{h.c.}, \quad (\text{B.30})$$

where

$$M_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -\frac{g_Y}{2} v c_\beta & \frac{g_Y}{2} v s_\beta & 0 & 0 \\ \cdots & M_2 & \frac{g_2}{2} v c_\beta & -\frac{g_2}{2} v s_\beta & 0 & 0 \\ \cdots & \cdots & 0 & -\mu_{\text{eff}} & -\frac{\lambda_S}{\sqrt{2}} v s_\beta & \frac{\lambda_\Delta}{2\sqrt{2}} v s_\beta \\ \cdots & \cdots & \cdots & 0 & -\frac{\lambda_S}{\sqrt{2}} v c_\beta & \frac{\lambda_\Delta}{2\sqrt{2}} v c_\beta \\ \cdots & \cdots & \cdots & \cdots & \mu_S & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \mu_\Delta \end{pmatrix}, \quad (\text{B.31})$$

where the  $M_1$  and  $M_2$  come from soft Lagrangian. If

$$m_Z \ll |\mu \pm M_1|, |\mu \pm M_2|, \quad (\text{B.32})$$

then the neutralino masses are given by

$$m_{\tilde{N}_1} = M_1 + \frac{m_Z^2 s_W^2 (M_1 + \mu_{\text{eff}} s_{2\beta})}{M_1^2 - \mu_{\text{eff}}^2}, \quad (\text{B.33})$$

$$m_{\tilde{N}_2} = M_2 + \frac{m_W^2 (M_2 + \mu_{\text{eff}} s_{2\beta})}{M_2^2 - \mu_{\text{eff}}^2}, \quad (\text{B.34})$$

$$m_{\tilde{N}_3} = m_{\tilde{N}_3}|_{\text{MSSM}} + \frac{(1 - s_{2\beta})v^2}{(\mu_{\text{eff}} - \mu_S)(\mu_{\text{eff}} - \frac{1}{2}\mu_\Delta)} \left\{ \left( \frac{\lambda_S^2}{2} + \frac{\lambda_\Delta^2}{8} \right) \mu_{\text{eff}} - \frac{\lambda_S^2}{2} \mu_\Delta - \frac{\lambda_\Delta^2}{8} \mu_S \right\}, \quad (\text{B.35})$$

$$m_{\tilde{N}_4} = m_{\tilde{N}_4}|_{\text{MSSM}} - \frac{(1 + s_{2\beta})v^2}{(\mu_{\text{eff}} + \mu_S)(\mu_{\text{eff}} + \frac{1}{2}\mu_\Delta)} \left\{ \left( \frac{\lambda_S^2}{2} + \frac{\lambda_\Delta^2}{8} \right) \mu_{\text{eff}} + \frac{\lambda_S^2}{2} \mu_\Delta + \frac{\lambda_\Delta^2}{8} \mu_S \right\}, \quad (\text{B.36})$$

$$m_{\tilde{N}_5} = \mu_S + \frac{\lambda_S^2 v^2 (\mu_S - \mu_{\text{eff}} s_{2\beta})}{(\mu_S - \mu_{\text{eff}})(\mu_S + \mu_{\text{eff}})}, \quad (\text{B.37})$$

$$m_{\tilde{N}_6} = \mu_\Delta + \frac{\lambda_\Delta^2 v^2 (\mu_\Delta - \mu_{\text{eff}} s_{2\beta})}{(\mu_\Delta - \mu_{\text{eff}})(\mu_\Delta + \mu_{\text{eff}})}. \quad (\text{B.38})$$

## Charginos

In the basis  $\psi^+ = (\tilde{W}^+, \tilde{H}_u^+, \tilde{\Delta}^+)$  and  $\psi^- = (\tilde{W}^-, \tilde{H}_d^-, \tilde{\Delta}^-)$ , the chargino mass part of the Lagrangian is

$$\mathcal{L}_{\text{chargino mass}} = -\frac{1}{2}(\psi^+)^T M_{\tilde{C}}^T \psi^- - \frac{1}{2}(\psi^-)^T M_{\tilde{C}} \psi^+, \quad (\text{B.39})$$

where

$$M_{\tilde{C}} = \begin{pmatrix} M_2 & \frac{g_2}{\sqrt{2}} v s_\beta & 0 \\ \frac{g_2}{\sqrt{2}} v c_\beta & \mu_{\text{eff}} & \frac{\lambda_\Delta}{2} v s_\beta \\ 0 & -\frac{\lambda_\Delta}{2} v c_\beta & \mu_\Delta \end{pmatrix}. \quad (\text{B.40})$$

The chargino masses correspond to the positive square roots of the eigenvalues of  $M_{\tilde{C}}^\dagger M_{\tilde{C}}$ . In the limit of Eq.(B.32) and  $v_\Delta \rightarrow 0$  with real  $M_2$ ,  $\mu_{\text{eff}}$  and  $\mu_\Delta$ , the chargino masses are given by

$$m_{\tilde{C}_1} \sim M_2 + m_W^2 \frac{M_2 + \mu_{\text{eff}} s_{2\beta}}{M_2^2 - \mu_{\text{eff}}^2}, \quad (\text{B.41})$$

$$m_{\tilde{C}_2} \sim \mu_{\text{eff}} + m_W^2 \frac{\mu_{\text{eff}} + M_2 s_{2\beta}}{\mu_{\text{eff}}^2 - M_2^2} + \frac{\lambda_\Delta^2}{8} v^2 \frac{\mu_{\text{eff}} + M_2 s_{2\beta}}{\mu_{\text{eff}}^2 - \mu_\Delta^2}, \quad (\text{B.42})$$

$$m_{\tilde{C}_3} \sim \mu_\Delta + \frac{\lambda_\Delta^2}{8} v^2 \frac{\mu_\Delta + \mu_{\text{eff}} s_{2\beta}}{\mu_\Delta^2 - \mu_{\text{eff}}^2}. \quad (\text{B.43})$$

## B.4 Definitions of the couplings

### B.4.1 Higgs self-couplings

As A.2.1, we use for  $s_i$  stands for  $\{h_d, h_u, s_0, \Delta_o\}$ ,  $h_i$  stands for  $\{h, H, s_0, \Delta_0\}$ ,  $p_i$  stands for  $\{z_d, z_u, z_s, z_\Delta\}$ ,  $a_i$  stands for  $\{G^0, A, z_s, z_\Delta\}$ ,  $w_i^\pm$  stands for  $\{w_d^\pm, w_u^\pm, \Delta^\pm, \bar{\Delta}^\pm\}$  and  $h_i^\pm$  stands for  $\{G^\pm, H^\pm, \Delta^\pm, \bar{\Delta}^\pm\}$ . Also,  $v_i$  stands for  $\{v_d, v_u\}$  with  $\tan \beta = v_2/v_1$  and  $(v_1^2 + v_2^2)^2 = (246\text{GeV})^2$ .

The trilinear neutral-Higgs self couplings are written as

$$\begin{aligned}
\lambda_{s_1 s_1 s_1} &= \lambda_{s_1 p_1 p_1} = \frac{1}{8}(g_2^2 + g_Y^2)v_1, \\
\lambda_{s_2 s_2 s_2} &= \lambda_{s_2 p_2 p_2} = \frac{1}{8}(g_2^2 + g_Y^2)v_2, \\
\lambda_{s_1 p_2 p_2} &= 3\lambda_{s_1 s_2 s_2} = -\frac{1}{8}(g_2^2 + g_Y^2 - 4\lambda_S^2 - \lambda_\Delta^2)v_1, \\
\lambda_{s_2 p_1 p_1} &= 3\lambda_{s_1 s_1 s_2} = -\frac{1}{8}(g_2^2 + g_Y^2 - 4\lambda_S^2 - \lambda_\Delta^2)v_2, \\
\lambda_{s_1 s_1 s_3} &= \lambda_{s_2 s_2 s_3} = \frac{\lambda_S}{3\sqrt{2}}\mu_{\text{eff}}, \quad \lambda_{s_1 s_1 s_4} = \lambda_{s_2 s_2 s_4} = \frac{\lambda_\Delta}{6\sqrt{2}}\mu_{\text{eff}}, \\
\lambda_{s_1 p_3 p_3} &= 3\lambda_{s_1 s_3 s_3} = \frac{\lambda_S^2}{2}v_1, \quad \lambda_{s_2 p_3 p_3} = 3\lambda_{s_2 s_3 s_3} = \frac{\lambda_S^2}{2}v_2, \\
\lambda_{s_1 p_3 p_4} &= 3\lambda_{s_1 s_3 s_4} = \frac{\lambda_S \lambda_\Delta}{4}v_1, \quad \lambda_{s_2 p_3 p_4} = 3\lambda_{s_2 s_3 s_4} = \frac{\lambda_S \lambda_\Delta}{4}v_2, \\
\lambda_{s_1 p_4 p_4} &= 3\lambda_{s_1 s_4 s_4} = \frac{\lambda_\Delta^2}{8}v_1, \quad \lambda_{s_2 p_4 p_4} = 3\lambda_{s_2 s_4 s_4} = \frac{\lambda_\Delta^2}{8}v_2, \\
\lambda_{s_3 p_1 p_1} &= \lambda_{s_3 p_2 p_2} = \frac{\lambda_S}{\sqrt{2}}\mu_{\text{eff}}, \quad \lambda_{s_4 p_1 p_1} = \lambda_{s_4 p_2 p_2} = \frac{\lambda_\Delta}{2\sqrt{2}}\mu_{\text{eff}}, \\
\lambda_{s_1 s_2 s_3} &= -\frac{\lambda_S}{6\sqrt{2}}(A_S + \mu_S), \quad \lambda_{s_3 p_1 p_2} = \frac{\lambda_S}{2\sqrt{2}}(A_S + \mu_S), \\
\lambda_{s_1 p_2 p_3} &= \lambda_{s_2 p_1 p_3} = \frac{\lambda_S}{2\sqrt{2}}(A_S - \mu_S), \quad \lambda_{s_1 s_2 s_4} = -\frac{\lambda_\Delta}{12\sqrt{2}}(A_\Delta + \mu_\Delta), \\
\lambda_{s_4 p_1 p_2} &= \frac{\lambda_\Delta}{4\sqrt{2}}(A_\Delta + \mu_\Delta), \quad \lambda_{s_1 p_2 p_4} = \lambda_{s_2 p_1 p_4} = \frac{\lambda_\Delta}{4\sqrt{2}}(A_\Delta - \mu_\Delta),
\end{aligned} \tag{B.44}$$

where unspecified quartic couplings are zero.

The quartic neutral-Higgs self couplings are written as

$$\begin{aligned}
\lambda_{s_1 s_1 s_1 s_1} &= \lambda_{p_1 p_1 p_1 p_1} = \lambda_{s_2 s_2 s_2 s_2} = \lambda_{p_2 p_2 p_2 p_2} = \frac{1}{32}(g_2^2 + g_Y^2), \\
\lambda_{s_1 s_1 p_1 p_1} &= \lambda_{s_2 s_2 p_2 p_2} = \frac{1}{16}(g_2^2 + g_Y^2), \\
\lambda_{s_1 s_1 s_2 s_2} &= \lambda_{p_1 p_1 p_2 p_2} = -\frac{1}{96}(g_2^2 + g_Y^2 - 4\lambda_S^2 - \lambda_\Delta^2), \\
\lambda_{s_2 s_2 p_1 p_1} &= \lambda_{s_1 s_1 p_2 p_2} = -\frac{1}{16}(g_2^2 + g_Y^2 - 4\lambda_S^2 - \lambda_\Delta^2), \\
\lambda_{s_1 s_1 s_3 s_3} &= \lambda_{p_1 p_1 p_3 p_3} = \lambda_{s_2 s_2 s_3 s_3} = \lambda_{p_2 p_2 p_3 p_3} = \frac{\lambda_S^2}{24}, \\
\lambda_{s_1 s_1 p_3 p_3} &= \lambda_{s_2 s_2 p_3 p_3} = \lambda_{s_3 s_3 p_1 p_1} = \lambda_{s_3 s_3 p_2 p_2} = \frac{\lambda_S^2}{4}, \\
\lambda_{s_1 s_1 s_3 s_4} &= \lambda_{p_1 p_1 p_3 p_4} = \lambda_{s_2 s_2 s_3 s_4} = \lambda_{p_2 p_2 p_3 p_4} = \frac{\lambda_S \lambda_\Delta}{48}, \\
\lambda_{s_1 s_1 p_3 p_4} &= \lambda_{s_2 s_2 p_3 p_4} = \frac{\lambda_S \lambda_\Delta}{4}, \\
\lambda_{s_1 s_1 s_4 s_4} &= \lambda_{p_1 p_1 p_4 p_4} = \lambda_{s_2 s_2 s_4 s_4} = \lambda_{p_2 p_2 p_4 p_4} = \frac{\lambda_\Delta^2}{96}, \\
\lambda_{s_1 s_1 p_4 p_4} &= \lambda_{s_2 s_2 p_4 p_4} = \lambda_{s_4 s_4 p_1 p_1} = \lambda_{s_4 s_4 p_2 p_2} = \frac{\lambda_\Delta^2}{16},
\end{aligned} \tag{B.45}$$

where unspecified quartic couplings are zero.

The trilinear neutral-charged coupling are written as

$$\begin{aligned}
\lambda_{s_1 w_1^+ w_1^-} &= \frac{1}{4}(g_2^2 + g_Y^2)v_d, & \lambda_{s_1 w_2^+ w_2^-} &= \frac{1}{4}(g_2^2 - g_Y^2 + 2\lambda_\Delta^2)v_d, \\
\lambda_{s_1 w_3^+ w_3^-} &= \frac{g_2^2}{2}v_d, & \lambda_{s_1 w_4^+ w_4^-} &= -\frac{1}{2}(g_2^2 - \lambda_\Delta^2)v_d, \\
\lambda_{s_2 w_1^+ w_1^-} &= \frac{1}{4}(g_2^2 - g_Y^2 + 2\lambda_\Delta^2)v_u, & \lambda_{s_2 w_2^+ w_2^-} &= \frac{1}{4}(g_2^2 + g_Y^2)v_u, \\
\lambda_{s_2 w_3^+ w_3^-} &= -\frac{1}{2}(g_2^2 - \lambda_\Delta^2)v_u, & \lambda_{s_2 w_4^+ w_4^-} &= \frac{g_2^2}{2}v_u, \\
\lambda_{s_3 w_1^+ w_1^-} &= \lambda_{s_3 w_2^+ w_2^-} = -\lambda_S(\lambda_\Delta v_\Delta - \sqrt{2}\mu_{\text{eff}}), \\
\lambda_{s_3 w_3^+ w_3^-} &= \lambda_{s_3 w_4^+ w_4^-} = 0, \\
\lambda_{s_4 w_1^+ w_1^-} &= \lambda_{s_4 w_2^+ w_2^-} = \frac{\lambda_\Delta}{2}(\lambda_\Delta v_\Delta - \sqrt{2}\mu_{\text{eff}}), \\
\lambda_{s_4 w_3^+ w_3^-} &= \lambda_{s_4 w_4^+ w_4^-} = g_2^2 v_\Delta, \\
\lambda_{s_1 w_1^+ w_2^-} &= \frac{1}{8}(2g_2^2 - 4\lambda_S^2 + \lambda_\Delta^2)v_u, \\
\lambda_{s_1 w_1^+ w_3^-} &= \frac{1}{4}(-\sqrt{2}g_2^2 v_\Delta + 2\lambda_\Delta \mu_{\text{eff}}), \\
\lambda_{s_1 w_1^+ w_4^-} &= \frac{1}{2\sqrt{2}}(g_2^2 - \lambda_\Delta^2)v_\Delta + \frac{\lambda_\Delta}{2}\mu_{\text{eff}}, \\
\lambda_{s_1 w_2^+ w_3^-} &= \frac{\lambda_\Delta}{2}\mu_\Delta, & \lambda_{s_1 w_2^+ w_4^-} &= \frac{\lambda_\Delta}{2}A_\Delta, \\
\lambda_{s_1 w_3^+ w_4^-} &= 0, \\
\lambda_{s_2 w_1^+ w_2^-} &= \frac{1}{8}(2g_2^2 - 4\lambda_S^2 + \lambda_\Delta^2)v_d, \\
\lambda_{s_2 w_1^+ w_3^-} &= -\frac{\lambda_\Delta}{2}A_\Delta, & \lambda_{s_2 w_1^+ w_4^-} &= -\frac{\lambda_\Delta}{2}\mu_\Delta, \\
\lambda_{s_2 w_2^+ w_3^-} &= -\frac{1}{2\sqrt{2}}(g_2^2 - \lambda_\Delta^2)v_\Delta - \frac{\lambda_\Delta}{2}\mu_{\text{eff}}, \\
\lambda_{s_2 w_2^+ w_4^-} &= -\frac{1}{4}(-\sqrt{2}g_2^2 v_\Delta + 2\lambda_\Delta \mu_{\text{eff}}), \\
\lambda_{s_2 w_3^+ w_4^-} &= 0, \\
\lambda_{s_3 w_1^+ w_2^-} &= \frac{\lambda_S}{\sqrt{2}}(A_S + \mu_S), \\
\lambda_{s_3 w_1^+ w_3^-} &= \lambda_{s_3 w_1^+ w_4^-} = \frac{\lambda_S \lambda_\Delta}{2\sqrt{2}}v_d \\
\lambda_{s_3 w_2^+ w_3^-} &= \lambda_{s_3 w_2^+ w_4^-} = -\frac{\lambda_S \lambda_\Delta}{2\sqrt{2}}v_u \\
\lambda_{s_3 w_3^+ w_4^-} &= 0, \\
\lambda_{s_4 w_1^+ w_2^-} &= -\frac{\lambda_\Delta}{2\sqrt{2}}(A_\Delta + \mu_\Delta), \\
\lambda_{s_4 w_1^+ w_3^-} &= -\lambda_{s_4 w_1^+ w_4^-} = -\frac{1}{4\sqrt{2}}(2g_2^2 - \lambda_\Delta^2)v_d, \\
\lambda_{s_4 w_2^+ w_3^-} &= -\lambda_{s_4 w_2^+ w_4^-} = -\frac{1}{4\sqrt{2}}(2g_2^2 - \lambda_\Delta^2)v_u, \\
\lambda_{s_4 w_3^+ w_4^-} &= -g_2^2 v_\Delta,
\end{aligned} \tag{B.46}$$



The quartic neutral-charged coupling are written as

$$\begin{aligned}
 \lambda_{s_1 s_1 w_1^+ w_1^-} &= \frac{1}{8}(g_2^2 + g_Y^2), & \lambda_{s_1 s_1 w_2^+ w_2^-} &= \frac{1}{8}(g_2^2 - g_Y^2 + 2\lambda_\Delta^2), \\
 \lambda_{s_1 s_1 w_3^+ w_3^-} &= \frac{g_2^2}{4}, & \lambda_{s_1 s_1 w_4^+ w_4^-} &= -\frac{1}{4}(g_2^2 - \lambda_\Delta^2), \\
 \lambda_{s_1 s_2 w_1^+ w_2^-} &= \frac{1}{8}(2g_2^2 - 4\lambda_S^2 + \lambda_\Delta^2), \\
 \lambda_{s_1 s_3 w_1^+ w_3^-} &= \lambda_{s_1 s_3 w_1^+ w_4^-} = \frac{\lambda_S \lambda_\Delta}{2\sqrt{2}}, & \lambda_{s_1 s_4 w_1^+ w_3^-} &= -\lambda_{s_1 s_4 w_1^+ w_4^-} = -\frac{1}{4\sqrt{2}}(2g_2^2 - \lambda_\Delta^2), \\
 \lambda_{s_2 s_2 w_1^+ w_1^-} &= \frac{1}{8}(g_2^2 - g_Y^2 + 2\lambda_\Delta^2), & \lambda_{s_2 s_2 w_2^+ w_2^-} &= \frac{1}{8}(g_2^2 + g_Y^2), \\
 \lambda_{s_2 s_2 w_3^+ w_3^-} &= -\frac{1}{4}(g_2^2 - \lambda_\Delta^2), & \lambda_{s_2 s_2 w_4^+ w_4^-} &= \frac{g_2^2}{4}, \\
 \lambda_{s_2 s_3 w_2^+ w_3^-} &= \lambda_{s_2 s_3 w_2^+ w_4^-} = -\frac{\lambda_S \lambda_\Delta}{2\sqrt{2}}, & \lambda_{s_2 s_4 w_2^+ w_3^-} &= -\lambda_{s_2 s_4 w_2^+ w_4^-} = -\frac{1}{4\sqrt{2}}(2g_2^2 - \lambda_\Delta^2), \\
 \lambda_{s_3 s_3 w_1^+ w_1^-} &= \lambda_{s_3 s_3 w_2^+ w_2^-} = \frac{\lambda_S^2}{2}, & \lambda_{s_3 s_4 w_1^+ w_1^-} &= \lambda_{s_3 s_4 w_2^+ w_2^-} = -\frac{\lambda_S \lambda_\Delta}{2}, \\
 \lambda_{s_4 s_4 w_1^+ w_1^-} &= \lambda_{s_4 s_4 w_2^+ w_2^-} = \frac{\lambda_\Delta^2}{8}, & \lambda_{s_4 s_4 w_3^+ w_3^-} &= -\lambda_{s_4 s_4 w_3^+ w_4^-} = \lambda_{s_4 s_4 w_4^+ w_4^-} = \frac{g_2^2}{2}.
 \end{aligned} \tag{B.47}$$

## B.4.2 Higgs-neutralino couplings

In Gauge-Higgs Unification model, the Lagrangian contain Higgs-neutralino couplings as

$$\mathcal{L} \supset - \sum_{ikl} \lambda_{s_i \psi_k^0 \psi_l^0} S_i \psi_k^0 \psi_l^0 - i \sum_{ikl} \lambda_{p_i \psi_k^0 \psi_l^0} P_i \psi_k^0 \psi_l^0 + \text{h.c.}, \tag{B.48}$$

where  $\psi_i^0 = \{\tilde{B}^0, \tilde{W}^0, \tilde{h}_1^0, \tilde{h}_2^0, \tilde{s}^0, \tilde{\Delta}^0\}$ . The couplings in Eq. (B.48) are symmetric for exchanging the neutralino indices  $k$  and  $l$ . The non-zero Higgs-neutralino couplings are following:

$$\begin{aligned}
 \lambda_{s_1 \psi_1^0 \psi_3^0} &= -\lambda_{p_1 \psi_1^0 \psi_3^0} = -\frac{g_Y}{4}, & \lambda_{s_1 \psi_2^0 \psi_3^0} &= -\lambda_{p_1 \psi_2^0 \psi_3^0} = +\frac{g_2}{4}, \\
 \lambda_{s_1 \psi_4^0 \psi_5^0} &= \lambda_{p_1 \psi_4^0 \psi_5^0} = +\frac{\lambda_S}{2\sqrt{2}}, & \lambda_{s_1 \psi_4^0 \psi_6^0} &= \lambda_{p_1 \psi_4^0 \psi_6^0} = +\frac{\lambda_\Delta}{4\sqrt{2}}, \\
 \lambda_{s_2 \psi_1^0 \psi_4^0} &= -\lambda_{p_2 \psi_1^0 \psi_4^0} = +\frac{g_Y}{4}, & \lambda_{s_2 \psi_2^0 \psi_4^0} &= -\lambda_{p_2 \psi_2^0 \psi_4^0} = -\frac{g_2}{4}, \\
 \lambda_{s_2 \psi_3^0 \psi_5^0} &= \lambda_{p_2 \psi_3^0 \psi_5^0} = -\frac{\lambda_S}{2\sqrt{2}}, & \lambda_{s_2 \psi_3^0 \psi_6^0} &= \lambda_{p_2 \psi_3^0 \psi_6^0} = +\frac{\lambda_\Delta}{4\sqrt{2}}, \\
 \lambda_{s_3 \psi_3^0 \psi_4^0} &= \lambda_{p_3 \psi_3^0 \psi_4^0} = -\frac{\lambda_S}{2\sqrt{2}}, & \lambda_{s_4 \psi_3^0 \psi_4^0} &= \lambda_{p_4 \psi_3^0 \psi_4^0} = +\frac{\lambda_\Delta}{4\sqrt{2}}.
 \end{aligned} \tag{B.49}$$

The couplings of the Higgs bosons to the neutralino mass eigenstates  $\chi_i^0$  are related to the neutralino interaction eigenstates as

$$\lambda_{\phi_i \chi_k^0 \chi_l^0} = N_{ka}^* N_{lb}^* \lambda_{\phi_i \psi_a^0 \psi_b^0}, \tag{B.50}$$

where  $\phi_i$  represent  $s_i$  or  $p_i$ ,  $N$  is the diagonalization matrix for neutralino mass matrix.

### B.4.3 Higgs-chargino couplings

The Lagrangian also contain the Higgs-chargino couplings as

$$\mathcal{L} \supset - \sum_{ikl} \lambda_{s_i \psi_k^+ \psi_l^-} S_i \psi_k^+ \psi_l^- - i \sum_{ikl} \lambda_{p_i \psi_k^+ \psi_l^-} P_i \psi_k^+ \psi_l^- + \text{h.c.}, \quad (\text{B.51})$$

where  $\psi_i^+ = \{\tilde{W}^+, \tilde{h}_u^+, \tilde{\Delta}^+\}$  and  $\psi_i^- = \{\tilde{W}^-, \tilde{h}_d^-, \tilde{\Delta}^-\}$ . The non-zero Higgs-chargino couplings are following

$$\begin{aligned} \lambda_{s_1 \psi_1^+ \psi_2^-} &= -\lambda_{p_1 \psi_1^+ \psi_2^-} = +\frac{g_2}{\sqrt{2}}, & \lambda_{s_1 \psi_2^+ \psi_3^-} &= \lambda_{p_1 \psi_2^+ \psi_3^-} = +\frac{\lambda_\Delta}{2}, \\ \lambda_{s_2 \psi_3^+ \psi_2^-} &= \lambda_{p_2 \psi_3^+ \psi_2^-} = +\frac{\lambda_\Delta}{2}, & \lambda_{s_2 \psi_2^+ \psi_1^-} &= -\lambda_{p_2 \psi_2^+ \psi_1^-} = +\frac{g_2}{\sqrt{2}}, \\ \lambda_{s_3 \psi_2^+ \psi_2^-} &= \lambda_{p_3 \psi_2^+ \psi_2^-} = +\frac{\lambda_S}{\sqrt{2}}, & \lambda_{s_4 \psi_1^+ \psi_3^-} &= -\lambda_{p_4 \psi_1^+ \psi_3^-} = +\frac{g_2}{4}, \\ \lambda_{s_4 \psi_2^+ \psi_2^-} &= \lambda_{p_4 \psi_2^+ \psi_2^-} = +\frac{\lambda_\Delta}{2\sqrt{2}}, & \lambda_{s_4 \psi_3^+ \psi_1^-} &= -\lambda_{p_4 \psi_3^+ \psi_1^-} = -\frac{g_2}{4}. \end{aligned} \quad (\text{B.52})$$

The mass matrix  $M_{\tilde{C}}$  for the charginos is diagonalized by a bi-unitary transformation

$$\text{diag}(m_{\chi_i^\pm}) = U^* M_{\tilde{C}} V^\dagger, \quad (\text{B.53})$$

where the unitary matrices  $U$  and  $V$  rotate the negative and positive chargino interaction eigenstates, respectively, into the corresponding mass eigenstates

$$\chi_i^- = U_{ij} \psi_j^-, \quad \chi_i^+ = V_{ij} \psi_j^+. \quad (\text{B.54})$$

## B.5 One-loop self energies and tadpole

### B.5.1 Scalar self energies

The Higgs contributions to the scalar self energies

$$\begin{aligned} 16\pi^2 \Pi_{s_i s_j}^H(p^2) &= \sum_k^4 2\lambda_{s_i s_j h_k h_k} A(m_{h_k}) + \sum_{k,l}^4 2\lambda_{s_i h_k h_l} \lambda_{s_j h_k h_l} B_0(p^2; m_{h_k}, m_{h_l}) \\ &+ \sum_k^4 2\lambda_{s_i s_j a_k a_k} A(m_{a_k}) + \sum_{k,l}^4 2\lambda_{s_i a_k a_l} \lambda_{s_j a_k a_l} B_0(p^2; m_{a_k}, m_{a_l}) \\ &+ \sum_k^4 2\lambda_{s_i s_j h_k^+ h_k^-} A(m_{h_k^\pm}) + \sum_{k,l}^4 \lambda_{s_i h_k^+ h_l^-} \lambda_{s_j h_k^+ h_l^-} B_0(p^2; m_{h_k^\pm}, m_{h_l^\pm}). \end{aligned} \quad (\text{B.55})$$

The neutralino and chargino contributions to the scalar self energies

$$\begin{aligned}
16\pi^2\Pi_{s_i s_j}^\chi &= \sum_{k,l}^6 4\lambda_{s_i\chi_k^0\chi_l^0}\lambda_{s_j\chi_k^0\chi_l^0}^* \left[ (p^2 - m_{\chi_k^0}^2 - m_{\chi_l^0}^2)B_0(m_{\chi_k^0}, m_{\chi_l^0}) - A(m_{\chi_k^0}) - A(m_{\chi_l^0}) \right. \\
&\quad \left. - 2m_{\chi_k^0}m_{\chi_l^0}B_0(m_{\chi_k^0}, m_{\chi_l^0}) \right] \\
&+ \sum_{k,l}^3 2\lambda_{s_i\chi_k^+\chi_l^-}\lambda_{s_j\chi_k^+\chi_l^-}^* \left[ (p^2 - m_{\chi_k^\pm}^2 - m_{\chi_l^\pm}^2)B_0(m_{\chi_k^\pm}, m_{\chi_l^\pm}) - A(m_{\chi_k^\pm}) - A(m_{\chi_l^\pm}) \right. \\
&\quad \left. - 2m_{\chi_k^\pm}m_{\chi_l^\pm}B_0(m_{\chi_k^\pm}, m_{\chi_l^\pm}) \right]. \tag{B.56}
\end{aligned}$$

### B.5.2 Tadpole

The contributions to the tadpoles from neutralinos and charginos

$$16\pi^2T_i^\chi = -4\sum_k^6 \lambda_{s_i\chi_k\chi_k} m_{\chi_i} A(m_{\chi_i}) - 4\sum_k^3 \lambda_{s_i\chi_k^+\chi_k^-} m_{\chi_i} A(m_{\chi_i}). \tag{B.57}$$

The contributions to the tadpoles from Higgs bosons

$$16\pi^2T_i^\phi = \sum_\phi \sum_k^{h,a,h^\pm} \lambda_{s_i\phi_k\phi_k} A(m_{\phi_i}). \tag{B.58}$$



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