Chaotic Plasma Concentration in Superconducting Cosmic String Wakes

Jun-ichi Sakai

Department of Applied Mathematics and Physics,
Faculty of Engineering, Toyama University,
Toyama 930 JAPAN

Abstract

It is shown that there occurs chaotic plasma concentration in the wakes formed by superconducting cosmic strings, which may be an important process for formation of large scale structures in the Universe. The maximum plasma density can be enhanced by several hundred times of the initial state due to the combined effect of the magnetic force and the self-gravity. The plasma concentration can be repeated in the X-type magnetic field configuration and the quasi-periodic pulsation may drive strong shock waves.

1. Introduction

Witten (1985) has demonstrated that in some grand unified models cosmic strings could behave as superconducting wires and thus could have strong electromagnetic interactions with the surrounding cosmic plasmas. Since then, the superconducting cosmic strings have been investigated by many people, associated with some astrophysical effects such as galaxy formation (Ostriker et al., 1986), extragalactic radio sources (Chudnofsky et al., 1986) and a mechanism for generating cosmic magnetic fields (Spergel et al., 1989).

The gravitational perturbations due to cosmic strings (for a review of strings and their cosmological implications, see Vilenkin, 1985) may induce galaxy formation (Silk and Vilenkin, 1984). Rees (1986) suggested that in the wake formed by long cosmic strings there occurs baryon concentration to make galaxies. If we consider the interaction of the superconducting cosmic strings with cosmic plasmas, the magntic field structure near the string is quite similar to the earth's magnetosphere and its tail as shown in Fig. 1. Chudnofsky et al., (1986) investigated some physical processes (shock acceleration and magnetic reconnection) to lead particle acceleration which may occur in the front of the string magnetosphere.

In the present paper, we investgate the dynamics of the baryon (plasma) concentration in the wake formed by long superconducting cosmic strings. There appears X-type magnetic configuration near the baryon concentration region as shown in Fig. 1. Therefore the magnetic interaction as well as local gravitational collapse due to self-gravity of plasmas may become important for the dynamics of the plasma concentration. It is shown that there occurs chaotic plasma concentration in the wake region where there appears X-type magnetic field structure. The maximum density can be enhanced by several hundred times of the initial state. The plasma concentration can be repeated and the chaotic pulsation near the X-type magnetic field can drive strong shock waves into the intergalactic space.

In section 2 we present a model describing the plasma dynamics near the X-type magnetic field configuration. In section 3 we show results of the numerical analysis of the equations derived in the previous section. In section 4 we summarize results and discuss some implications related to the chaotic plasma dynamics.

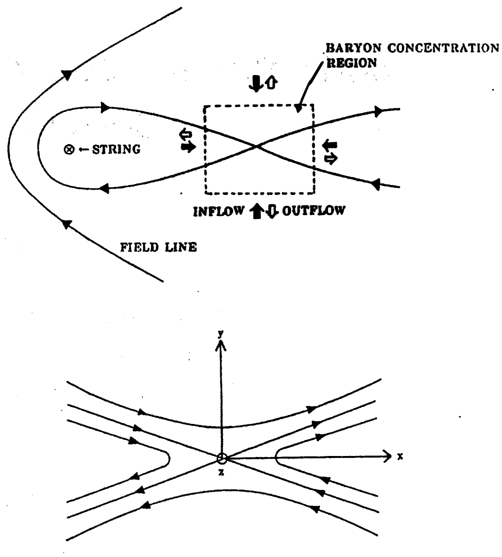

Fig. 1. A shematic diagram of the magnetosphere and the wake formed by superconducting cosmic strings and the local coordinates. The plasma concentration can occur near the X-type magnetic field configuration of the wake.

2. Basic Equations

As we are interested in the local plasma dynamics near the X-type magnetic field configuration in the wake formed by superconducting cosmic strings shown in Fig. 1, we take into account both effects of the magnetic force and the self-gravity. Then the relevant basic equations are MHD equations with the self-gravity as follows,

$$\frac{\partial \rho}{\partial t}+\mathrm{div}(\rho\mathbf{v})=0, \tag{1}$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+\frac{1}{4\pi}\mathrm{rot}\mathbf{B}\times\mathbf{B}-\rho\nabla\Phi, \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t}=\mathrm{rot}(\mathbf{v}\times\mathbf{B}), \tag{3}$$

$$\Delta\Phi=4\pi G\rho, \tag{4}$$

$$\frac{\partial p}{\partial t}+\mathbf{v}\cdot\nabla p+\gamma p\,\mathrm{div}\mathbf{v}=0, \tag{5}$$

whcre ρ, $\mathbf{v}$, $\mathbf{B}$, p, Φ are the density, the velocity, the magnetic field, the pressure, and the gravitational potential, respectively.

G is the gravitational constant and γ is the adiabatic ratio, which will be taken to be $\gamma=5/3$. The coordinate is taken as shown in Fig. 1. As we are interested in the local plasma behaviour near the X-type magnetic configuration, we assume that the flow velocities are given by

$$v_x=\frac{\dot{b}}{b}x, \tag{6}$$

$$v_y = \frac{\dot{a}}{a} y,$$

where the time dependent scale factors, a(t) and b(t) are determined self-consistently later. The dot means time devivative. From Eqs. (1) and (6) we obtain the plasma density as follows,

$$\rho = \frac{\rho_0}{ab}, \tag{7}$$

where ρ_0 is a constant. The plasma density is homogenous in space near the X-type magnetic configuration and depends only on time. The gravitational potential Φ is taken to be

$$\Phi = \pi G \rho (x^2 + y^2), \tag{8}$$

which can satisfy Eq. (4).

We assume that the magnetic fields are taken to be

$$\begin{aligned} B_x &= B_{x0}(t) \frac{y}{\lambda}, \\ B_y &= B_{y0}(t) \frac{x}{\lambda}, \\ B_z &= B_{z0}(t), \end{aligned} \tag{9}$$

where B_{x0}, B_{y0}, and B_{z0} are time-dependent functions. λ is a scale characterizing the thickness of the wake sheet. By using Eqs. (3), (6), (9), we can determine the unknown functions B_{x0}, B_{y0}, $B_{z0}(t)$, as

$$\begin{aligned} B_{x0}(t) &= \frac{B_0}{a^2}, \\ B_{y0}(t) &= \frac{B_0}{b^2}, \\ B_{z0}(t) &= \frac{B_{z0}}{ab}, \end{aligned} \tag{10}$$

where B_0 and B_{z0} are constants.

If the plasma pressure p is taken to be

$$P(x, y, t) = p_0(t) - \left(p_{x0}(t)\frac{x^2}{\lambda^2} + p_{y0}(t)\frac{y^2}{\lambda^2}\right), \tag{11}$$

$p_0(t)$, $p_{x0}(t)$ and $p_{y0}(t)$ can be determined from Eqs. (5) and (6). as

$$\begin{aligned} p_0(t) &= \frac{p_0}{(ab)^{\gamma}}, \\ p_{x0}(t) &= \frac{p_0}{a^{\gamma} b^{\gamma+2}}, \\ p_{y0}(t) &= \frac{p_0}{a^{\gamma+2} b^{\gamma}}, \end{aligned} \tag{12}$$

where p_0 is a constant.

Finally we obtain the basic equations for the scale factors a(t) and b(t) from Eq. (2) as

$$\frac{d^2a}{dt^2} = \frac{c_S^2}{\lambda^2 a^{\gamma} b^{\gamma-1}} + \frac{v_A^2}{\lambda^2}\left(\frac{1}{b} - \frac{b}{a^2}\right) - \frac{2\pi\rho_0 G}{b}, \tag{13}$$

$$\frac{d^2a}{dt^2} = \frac{c_S^2}{\lambda^2 a^{\gamma-1} b^{\gamma}} - \frac{v_A^2}{\lambda^2}\left(\frac{a}{b^2} - \frac{1}{a}\right) - \frac{2\pi\rho_0 G}{a}, \tag{14}$$

where $c_S^2 = p_0/\rho_0$ and $v_A^2 = B_0^2/4\pi\rho_0$.

The above equations (6), (7), (8), (9), (10), (11) and (12) are exact solutions of Eqs. (1)~(6), if the scale factors a(t) and b(t) can be solved from Eqs. (13) and (14). The non-dimensional form of Eqs. (13) and (14) is given by

$$\frac{d^2a}{dt^2}=\frac{\beta}{a^{\gamma}b^{\gamma-1}}+\left(\frac{1}{b}-\frac{b}{a^2}\right)-\frac{G_0}{b}, \tag{15}$$

$$\frac{d^2b}{dt^2}=\frac{\beta}{a^{\gamma-1}b^{\gamma}}-\left(\frac{a}{b^2}-\frac{1}{a}\right)-\frac{G_0}{a}, \tag{16}$$

where $\beta=c_S^2/v_A^2$, is the plasma beta ratio, $G_0=2\pi\rho_0 G\lambda^2/v_A^2=(\tau_A/\tau_G)^2$, where τ_A is the Alfvén transit time defined by $\tau_A=\lambda/v_A$ and τ_G is the free-fall time defined by $\tau_G=\frac{1}{\sqrt{2\pi\rho_0 G}}$. The time is normalized by τ_A. The above equations have equilibrium solutions a_0 and b_0 which can satisfy the following equations,

$$\beta(a_0b_0)^{1/3}+(1-G_0)a_0^2-b_0^2=0, \tag{17}$$

$$\beta(a_0b_0)^{1/3}+(1-G_0)b_0^2-a_0^2=0, \tag{18}$$

The special equilibrium solutions, $a_0=b_0=(\beta/G_0)^{3/4}$ means tnat the magnetic force is absent and the pressure can balance with the gravitational force. The other solutions which show $a_0=b_0$ imply that the three forces can balance each other.

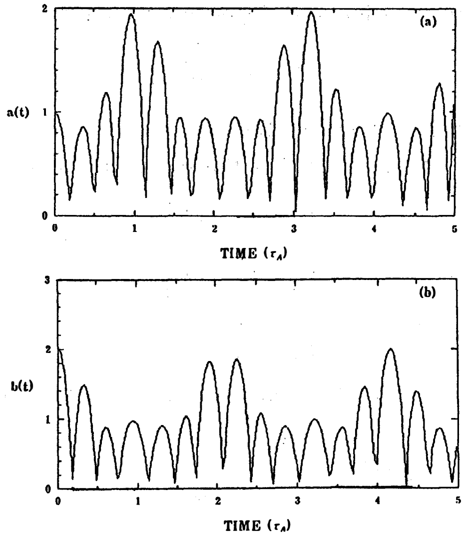

Fig. 2. Time evolution of the scale factors a(t) and b(t). The time is normarized by the Alfvén transit time $\tau_A=\lambda/v_A$. The initial values are a(0)=1 and b(0)=2.

3. Chaotic Plasma Concentration

We show here some numerical results obtained from the basic equations (15) and (16). If we consider the characteristic scale λ as $\lambda=10^{22}$ cm, the average density $n_0=1\ cm^{-3}$, the magnetic field $B_0\sim3\times10^{-6}$G, we get $v_A=10^6$cm/s, $\tau_G\sim10^{15}$ sec, and $\tau_A=10^{16}$sec. If we assume plasma temperature as $T\sim3\cdot10^4$K, the sound velocity becomes $c_S\sim3\times10^6$cm/s. Then the plasma beta ratio β is 10 and $G_0=(\tau_A/\tau_G)^2=100$.

We take the initial values a(t) and b(t) as a(0)=1, b(0)=2, which means that the initial magnetic fields are $B_x(0)=B_0\frac{y}{\lambda}$, $B_y(0)=\frac{B_0}{4}\frac{x}{\lambda}$. We also take $\dot{a}(0)=-0.1$, $\dot{b}(0)=0$, which corresponds

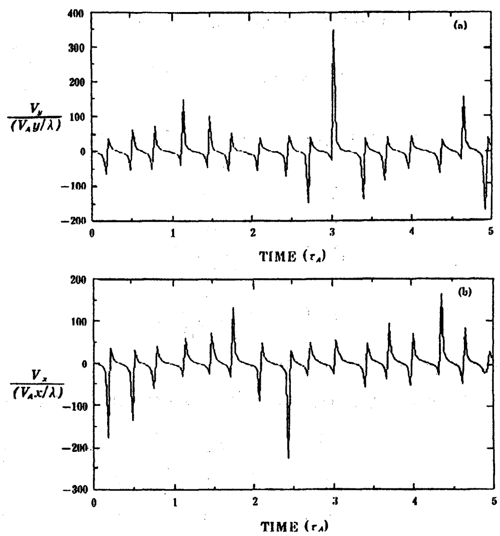

Fig. 3. Time evolution of the plasma velocities, v_y and v_x.

that the initial plasma flow velocities are $v_y(0) = -0.1 v_A\ y/\lambda$ and $v_x(0)=0$. The initial density is $\rho(0)=\rho_0/2$.

Fig. 2 shows the time evolution of the scale factor a(t) and b(t). As seen in Fig. 2. two scale factors can behave quasi-periodic (chaotic) oscillations in phase. This means that the plasmas can make concentration from both directions of x and y axis and expand. This chaotic pulsation can be seen in Fig. 3, where the time evolution of the velocity components is shown. The negative value of the velocity corresponds to the collapse to the X-type point and positive value corresponds to the expansion of the plasmas. The maximum local velocity can exceed the local Alfvȇn velocity. Fig. 4. shows the time evolution of the density. The density enhancement can become up to about 600 times of the initial density, because the initial density was 0.5 ρ_0. The time evolution of the magnetic fields is shown in Fig. 5. The observed strong plasma concentration which can lead to several hundred times of the initial state is important for the formation of large scale structures like galaxy in the wake formed by superconducting cosmic strings. If we take into account the cooling effect, the cooling time can be reduced to 10^{-2}, because of the density enhancement of 10^2 times. During the collapse phase, the magnetic fields can be also compressed up to 10^{-4}G from the initial value $\sim 10^{-6}$G. The magnetic field compression associated with the plasma concentration may be also important for the explanation of the observed galactic magnetic fields. This observed strong chaotic pulsation can drive shock waves, which may be important for the high energy particle acceleration and formation of other large scale strucures (Ostriker et. al. 1986).

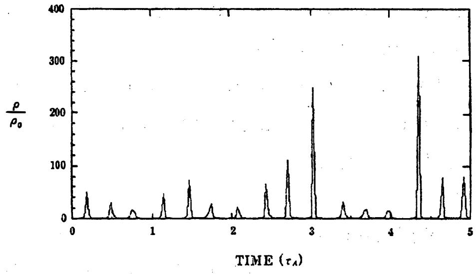

Fig. 4. Time evolution of the plasma density. The initial density is $\rho/\rho_0=0.5$.

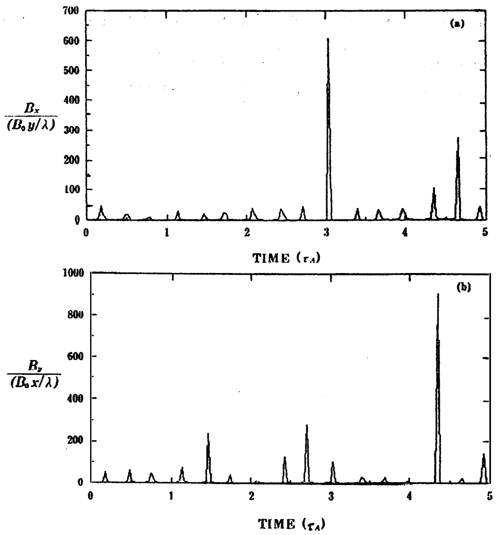

Fig. 5. Time evolution of the magnetic fields, B_x and B_y.

4. Summary and Discussion

We have shown that there occurs chaotic plasma concentration in the wake formed by superconducting cosmic strings. The maximum density can be enhanced by several hundred times of the initial state due to the combination of the magnetic force and the self-gravity. The chaotic plasma pulsation near the X-type magnetic field can drive strong shock waves. The density enhancement may lead to radiative cooling and further to strong gravitational collapse.

We need more detailed analysis for the question of making small scale structures by the local gravitational instability.

The plasma concentration in the X-type magnetic field configuration can also become important for the interaction region where two magnetized molecular clouds can collide and coalesce to make stars.

References

Chudnofsky, E.M., Field, G.E., Spergel, D.M. and Vilenkin, A., 1986. Phys. Rev. D34, 944.
Ostriker, J.P., Thompson, C., Witten, E., 1986. Phys. Lett. B180, 231.
Rees, M.J., 1986. Mon. Not. R. astr. Soc. 222, 27.
Silk, J.I. and Vilenkin, A., 1984. Phys. Rev. Lett., 53, 1700.
Spergel, D.M., Press, W.H. and Scherrer, R.H., 1989. Phys. Rev. D39, 379.
Vilenkin, A., 1985. Phys. Rep. 121, 263.
Witten, E., 1985. Nucl. Phys. B249, 557.