Explosive Transverse Electric Field and Particle Acceleration During Magnetic Collapse

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ABSTRACT

It is shown that ion streams across the magnetic field moving with electrons cause explosive transverse electrostatic field during magnetic collapse such as coalescence instability of current loops. The electrostatic field can explosively accelerate ions and electrons perpendicular both to the magnetic field and ion streams. Ions and electrons are almost simultaneously accelerated to the opposite direction, respectively. The results obtained here well explain the simulation results of collisionless coalescence instability of current loops. The simultaneous acceleration mechanismof ions and electrons is applied to the origin of explosive high energy particles in cosmic plasmas.

Keywords: Particle Acceleration, Explosive Transeverse Electrostatic Field, Magnetic Collapse, Coalescence Instability, Solar Flare

1. INTRODUCTION

The acceleration mechanisms of charged particles to suprathermal energies in cosmic plasmas are of fundamental importance. A various kinds of acceleration mechanisms[1] in the use of electric and magnetic fields in plasmas are proposed in connection with specific examples of observed high energy particles since the Fermi-type acceleration.

Recent observations[2] of solar flares showed that in the impulsive phase ions and electrons can be almost simultaneously accelerated .within a few seconds in a quasi-periodic manner. This observation implies the improvement of widely accepted stochastic acceleration of particles. Tajima et al.[3] explained the quasi-periodic acceleration of ions and eleotrons by nonlinear coalescence instability of two parallel current loops. In their computer simulation of the coalescence instability they showed the existence of strong explosive electrostatic field across the magnetic field which may cause the production of high energy ions and electrons.

Sugihara et al.[4] found a mechanism($\vec{V}_p \times \vec{B}$ acceleration) of charged particles trapped by strong electrostatic waves propagating across the magnetic field. In the present paper we show theoretically the existence of explosive electrostaic field across the magnetic field moving with magnetized electrons during magnetic field collapse such as the coalescence instability of current loops. The transverse electrostatic field grows explosively with a velocity relative to the magnitic field. The charged particles (ions and electrons) driven by the electrostatic field can be accelerated in the direction perpendicular both to the magnetic and electric field. This acceleration mechanismis thought to be an extension of $\vec{V}_P \times \vec{B}$ acceleration.

2. BASIC EQUATIONS FOR 1-D MODEL OF MAGNETIC COLLAPSE

In this section we present basic equations for one dimensional model of magnetic collapse such as coalescence instability of current loops. Before complete merging state of current loops ion streams across the poloidal magnetic field $\vec{B}$ produced by the loop current $\vec{J}$ itself are driven by $\vec{J} \times \vec{B}$ force. For simplicity we choose the drirection of ion streams as x-coordinate and the poloidal magnetic field as y-coordinate, while the loop currents flow in the z direction. We assume that all physical quantities depend only on the x-coordinate (1-D model of magnetic collapse). During magnetic collapse besides complete merging state of two current blobs the pressure effect of plasmas are negligible compared with the Lorentz forces.

We start from two fluid equations of plasmas, neglecting the pressure terms as well as displaeement currrent in the Maxwell equation. We obtain the following equations for electron fluid:

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_{ex}) = 0, \tag{1}$$

$$\frac{\partial v_{ex}}{\partial t} + v_{ex}\frac{\partial}{\partial x} v_{ex} = -\frac{e}{m_e}(E_x - \frac{v_{ez}}{c} B_y), \tag{2}$$

$$\frac{\partial v_{ez}}{\partial t} + v_{ex}\frac{\partial}{\partial x} v_{ez} = -\frac{e}{m_e}(E_z + \frac{v_{ex}}{c} B_y). \tag{3}$$

As the ions flowalmost across the magnetic field because of $J_z \times B_y$ force, we may treat ions as unmagnetized fluid i.e. we neglect the term $eB_y v_{iz}/c$ compared with eE_x in the ion equation of motion. Therefore we obtain the following equations for ion fluid:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_{ix}) = 0, \tag{4}$$

$$\frac{\partial v_{ix}}{\partial t} + v_{ix}\frac{\partial}{\partial x} v_{ix} = \frac{e}{m_i} E_x, \tag{5}$$

$$\frac{\partial v_{iz}}{\partial t} + v_{ix}\frac{\partial}{\partial x} v_{iz} = \frac{e}{m_i}(E_z + \frac{v_{ix}}{c} B_y). \tag{6}$$

Equations (1)-(6) are colosed by the Maxwell equations,

$$\frac{\partial B_y}{\partial t} = c\frac{\partial E_z}{\partial x}, \tag{7}$$

$$\frac{\partial E_x}{\partial x} = 4\pi e(n_i - n_e), \tag{8}$$

$$v_{ez} = -\frac{c}{4\pi e n_e}\frac{\partial B_y}{\partial x}, \tag{9}$$

where we assumed that current in the z–direction can be maintained only by electrons in Eq. (9).

We try to seek local solutions of Eq.(1)−(9) representing magnetic collapse, where there is no definite scale–length i.e. the scale–leneth characterizing a distance between two current loop may continously vary in time during magnetic collapse. Such a physical situation may be well described by self–similar solutions, in which scale factors a(t) and b(t) varying continously in time are introduced as follows,

$$v_{ex} = \frac{\dot{a}}{a} x , \tag{10}$$

$$v_{ix} = \frac{\dot{b}}{b} x , \tag{11}$$

where the linear dependence on x means the fact that ion and electron streams are flowing in the opposite direction around x=0, which is the center of two current loops. In Eq. (10) and (11) $\dot{a}$ and $\dot{b}$ mean the time derivative. The unknown scale factors a and b will be determined from the above basic equations. From the continuity equations (1) and (4), we obtain

$$n_e = n_0 / a, \tag{12}$$

$$n_i = n_0 / b, \tag{13}$$

where n_0 is a constant. Equations(12) and(13) show that the density of ions and electrons is homogenous in space and varies only in time during magnetic collapse. It is reasonable to assume that the poloidal magnetic field may have linear dependence on x as $B_y = B_0(t)x/\lambda$, because the poloidal magnetic field reverses its sign at x=0. λ is a characteristic length of magnetic inhomogeneity. Above poloidal magnetic field means that the current along z–direction is constant in the range of λ.

From Eqs.(7) and (9) we obtain

$$E_z = \frac{\dot{B}_0(t) x^2}{2\lambda c} + E_{z0}(t) , \tag{14}$$

$$v_{ez} = -\frac{c a(t) B_0(t)}{4\pi e n_0 \lambda} . \tag{15}$$

Substituting Eqs.(10),(14) and (15) into Eq.(3), we found

$$B_y = \frac{B_{00} x}{a^2 \lambda} , \tag{16}$$

$$E_z = -\frac{B_{00} \dot{a} x^2}{c a^3 \lambda} - \frac{B_{00} c m_e \dot{a}}{4\pi n_0 e^2 \lambda a^2} , \tag{17}$$

$$v_{ez} = -\frac{c B_{00}}{4\pi e n_0 \lambda a} . \tag{18}$$

Assuming the electrostatic field E_x as $E_x = E_0(t)\ x/\lambda$ and making use of Eqs. (2), (5) and (8), we found

$$\frac{\ddot{b}}{b} = \frac{e E_0}{m_i \lambda} , \tag{19}$$

$$\frac{\ddot{a}}{a} = -\frac{eE_0}{m_e\lambda} - \frac{B_{00}^2}{4\pi n_0 m_e \lambda^2 a^3} , \tag{20}$$

$$E_0 = 4\pi e n_0 \lambda \left(\frac{1}{b} - \frac{1}{a}\right) . \tag{21}$$

Finally we obtain the coupled nonlinear equations for a(t) and b(t) from above equations;

$$\ddot{a} = -\omega_{pe}^2 \left(\frac{a}{b} - 1\right) - \frac{B_{00}^2}{4\pi n_0 m_e \lambda^2 a^2} , \tag{22}$$

$$\ddot{b} = \omega_{pi}^2 \left(1 - \frac{b}{a}\right) , \tag{23}$$

where $\omega_{pe}^2 = \dfrac{4\pi n_0 e^2}{m_e}$, and $\omega_{pi}^2 = \dfrac{4\pi n_0 e^2}{m_i}$.

3. EXPLOSIVE ELECTROSTATIC FIELD DURING MAGNETIC COLLAPSE

In the previous section we derived basic equations (22) and (23) describing one-dimensional magnetic collapse. In Eqs. (22) and (23) the two first terms of right hand sides represent the effect of charge separation. While the second term of Eq. (22) comes from the force $J_z \times B_y$, which can drive magnetic collapse. When this driving term is dominant, it is good approximation to assume quasi-neutrality ($n_i = n_e$) which means $a = b$. This was confirmed by the numerical calculation of Eq. (22) and (23). In a quasi—neutral plasma we obtain the following equation for a from the summation of Eqs. (19) and (20),

$$\ddot{a} = -\frac{v_A^2}{\lambda^2 a^2} , \tag{24}$$

where $v_A^2 = B_{00}^2 / 4\pi n_0 (m_i + m_e)$.

The equation (24) is similar to the gravitational collapse in the Newtonian dynamics of a large gas cloud. The solution of Eq. (24) repesenting magnetic collapse is given by

$$a(t) = \alpha (t_0 - t)^{2/3} , \tag{25}$$

where α is gixen by $\alpha = \left(\dfrac{9}{2}\right)^{1/3} (v_A/\lambda)^{2/3}$ and t_0 is an explosion time.

The electrostatic field can be determined from Eq. (19), not from the Possion equation (21) as

$$E_0(t) = \frac{m_i \lambda}{e} \frac{\ddot{a}}{a} = -\frac{m_i v_A^2}{\lambda e a^3} , \tag{26}$$

By means of the solution (25) we obtain various kinds of physical quantities as follows;

$$v_{ex} = v_{ix} = -\frac{2}{3} \frac{x}{(t_0 - t)} , \tag{27}$$

$$n = n_i = n_e = \left(\frac{2}{9}\right)^{1/3} \frac{\lambda^{2/3} n_0}{v_A^{2/3}(t_0 - t)^{2/3}} , \tag{28}$$

$$E_x = -\frac{2}{9}\frac{m_i}{e} \frac{x}{(t_0 - t)^2} , \tag{29}$$

$$B_y = \left(\frac{2}{9}\right)^{2/3} \frac{B_{00}\lambda^{1/3} x}{v_A^{4/3}(t_0 - t)^{4/3}} , \tag{30}$$

$$E_z = \frac{2}{3}\left(\frac{2}{9}\right)^{2/3} \frac{B_{00}\lambda^{1/3} x^2}{v_A^{4/3} c(t_0 - t)^{7/3}} + \frac{2}{3}\left(\frac{2}{9}\right)^{1/3} \frac{B_{00} c}{\omega_{pe}^2 \lambda^{1/3} v_A^{2/3}(t_0 - t)^{5/3}} , \tag{31}$$

We found that during magnetic collapse the electromagnetic fields can grow explosively in time as seen from above relations. Particular interest in above solutions is that the explosive transverse electrostatic field E_x can grow faster than the magnetic field B_y, because of the fast explosiveness in time i.e. $E_x \propto (t_0 - t)^{-2}$, $B_y \propto (t_0 - t)^{-4/3}$ in the limit of $t \rightarrow t_0$. From this fact we may expect the particle acceleration driven by the electrostatic field as well as $V_x \times B_y$ acceleration in the z-direction. In the next section we examine the particle acceleration under the above explosive electromagnetic fields during magnetic collapse.

4. SIMULTANEOUS ACCELERATION OF IONS AND ELECTRONS DURING MAGNETIC COLLAPSE

In this section we examine a test particle motion under the explosive electromagnetic fields during magnetic collapse given in the previous section. The equations of motion of a test ion are given by

$$\frac{dv_x}{dt} = -\frac{eE_{x0}}{m_i} \frac{x}{(t_0 - t)^2} - \frac{eB_{y0} x v_z}{m_i c(t_0 - t)^{4/3}} , \tag{32}$$

$$\frac{dv_z}{dt} = \frac{eE_{z0}}{m_i} \frac{x^2}{(t_0 - t)^{7/3}} + \frac{9eE_{z0} c^2 v_A^{2/3}}{2m_i \lambda^{2/3} \omega_{pe}^2 (t_0 - t)^{5/3}} + \frac{eB_{y0}}{m_i c} \frac{x\dot{x}}{(t_0 - t)^{4/3}} , \tag{33}$$

where $E_{x0} = 2m_i/9e$, $E_{z0} = \frac{2}{3}\left(\frac{2}{9}\right)^{2/3} \frac{B_{00}\lambda^{1/3}}{c v_A^{4/3}}$,

and $B_{y0} = \left(\frac{2}{9}\right)^{2/3} \frac{B_{00}\lambda^{1/3}}{v_A^{4/3}}$.

Transforming the time t to τ by $\tau = t_0 - t$, we obtain

$$\frac{dv_x}{d\tau} = -\frac{eE_{x0}}{m_i} \frac{x}{\tau^2} - \frac{eB_{y0}}{m_i c} \frac{x v_z}{\tau^{4/3}} , \tag{34}$$

$$\frac{dv_z}{d\tau} = -\frac{eB_{y0}}{m_i c} \left[\frac{2}{3} \frac{x^2}{\tau^{7/3}} + \frac{\triangle}{\tau^{5/3}} - \frac{x}{\tau^{4/3}} \frac{dx}{d\tau} \right] , \tag{35}$$

where $\Delta = 9c^2 v_A^{2/3}/2\omega_{pe}^2 \lambda^{2/3}$ and we used the relation $E_{z0} = 2B_{y0}/3c$ in Eq. (35).

Here we take particular attention to the ion which can be almostly accelerated by the transverse electrostatic field E_x across the magnetic field. This ion can be accelerated without gyration. The equation of this ion is approximately given by

$$\frac{dv_x}{d\tau} = -\frac{eE_{x0}}{m_i}\frac{x}{\tau^2} , \tag{36}$$

because of $eE_x \gg ev_z B_y/c$.
The solution of Eq. (36) is given by

$$x = c_1 \tau^{2/3} + c_2 \tau^{1/3} , \tag{37}$$

$$v_x = \dot{x} = \frac{2}{3}c_1 \tau^{-1/3} + \frac{1}{3}c_2 \tau^{-2/3} , \tag{38}$$

where c_1 and c_2 are determined from initial conditions,

$$\left.\begin{aligned} c_1 &= 3(v_0 t_0^{1/3} - x_0 t_0^{-2/3}), \\ c_2 &= 3(x_0 t_0^{-1/3} - v_0 t_0^{2/3}), \end{aligned}\right\} \tag{39}$$

The equation (38) shows that the ion for nonzero c_1 and c_2 can be explosively accelerated in the x-direction. By means of the solutions (37) and (38) we find the solution for v_z from Eq. (35)

$$v_z = \frac{eB_{y0}}{m_i c}\left(\frac{c_2^2 + 3\Delta}{2\tau^{2/3}} + \frac{c_1 c_2}{\tau^{1/3}}\right) , \tag{40}$$

which shows that the ion for a particular initial condition, i.e. $c_2 \neq 0$ can be explosively accelerated in the z-direction. The dominant term in Eq. (40) is the first term including c_2 in the limit of $\tau \to 0$. The solution (40) also shows that ions and electrons can be accelerated in the opposite direction each other, because v_z is proportjonal to the sign of the charge e. Actually we obtain for a test electron

$$v_{ez} = -\frac{eB_{y0}}{m_e c}\left\{\frac{d_1^2}{2}\tau^{\mu-\frac{2}{3}} + \frac{3}{2}\Delta\tau^{-\frac{2}{3}} + d_1 d_2 \tau^{-\frac{1}{3}} + \frac{d_2^2}{2}\tau^{-\mu-\frac{1}{3}}\right\} , \tag{41}$$

where

$$\mu = (1 + 8m_i/9m_e)^{1/2} ,$$

$$d_1 = \mu^{-1}\left[v_0 t_0^{\frac{1-\mu}{2}} - \frac{(1-\mu)}{2} x_0 t_0^{-\frac{1+\mu}{2}}\right] ,$$

$$d_2 = \mu^{-1}\left[\frac{(1+\mu)}{2} x_0 t_0^{\frac{\mu-1}{2}} - v_0 t_0^{\frac{1+\mu}{2}}\right] .$$

The main term in Eq. (41) is the last one, which shows that the electron with nonzero d_2 can be explosively accelerated in the opposite direction compared with the ion.

The above acceleration can continue until the Lorentz force $v_z B_y/c$ exceeds the electrostatic force. The ion with $\Delta \ll c_2^2 \leq (v_A^{8/3}/2\omega_{i1}^2 \lambda^{2/3})$ can be accelerated until the explosion time t=

t_0, because the electrostatic force always exceeds the magnetic force $v_z \times B_y$. On the other hand, the electron can be detrapped from the clectrostatic field at $\tau = \tau_A$ before the explosion time,

$$\tau_A = \left[\frac{2}{d_2^2} \left(\frac{eE_{x0}}{m_e} \right) \left(\frac{eB_{y0}}{m_e c} \right)^{-2} \right]^{\frac{3}{1-3\mu}} \tag{42}$$

At $\tau = \tau_A$ the electron can gain the maximum velocity v_{ze}^{max},

$$\begin{aligned} v_{ze}^{max} &= -\frac{d_2^2}{2} \left(\frac{eB_{y0}}{m_e c} \right) \tau_A^{-\mu-\frac{1}{3}} \\ &= -\frac{d_2^2}{2} \left(\frac{eB_{y0}}{m_e c} \right) \left[\frac{2}{d_2^2} \left(\frac{eE_{x0}}{m_e} \right) \left(\frac{eB_{y0}}{m_e c} \right)^{-2} \right]^{\frac{-1-3\mu}{1-3\mu}} . \end{aligned} \tag{43}$$

The explosiveness ($v_{ez} \propto \tau^{-\mu-\frac{1}{3}}$) of electron acceleration in the z-direction is stronger than that of ion acceleration ($v_{iz} \propto \tau^{-2/3}$). Anyway ions and electrons can be almost simultaueously accelerated during magnetic collapse.

5. SUMMARY

We found that ion streams across the collapsing magnetic field can generate explosive electromagnetic fields. The transverse electrostatic field can grow faster than the magnetic field. The ions and electrons can be almost simultaneously accelerated in the opposite direction along the current floweach other. The computer simulation of collisionless coalescence instability showed[3] all above features. The dynamical magnetic collapse may occur in every plasma of astrophysical interest. The magnetic collapse can be driven in a system of many current loops such as in active star atomosphere. It may also be driven in accretion disk with mgnetized plasma by strong gravitational force. The explostve acceleration of high energy particles can be observed during such magnetic collapse.

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