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Influence of the Fluid Inertia Forces on the Dynamic Characteristics of Externally Pressurized Thrust Bearings*

(1st Report , Influence of the Fluid Inertia Forces Generated at the Restricted Part of Externally Pressurized Circular Thrust Bearings with Capillary Restriction)

By Yoshio HARUYAMA**, Tsuneji KAZAMAKI*** and Haruo MORI****

In this paper , the authors investigated theoretically and experimentally the influence of the fluid inertia forces generated at the restricted part on the dynamic characteristics of externally pressurized circular thrust bearings with capillary restriction.

From comperison with the experiment , it may be concluded that the influence on the dynamic characteristics should be considerable when the kinematic viscosity of the lubricant becomes too low , and that the presented analysis yields good predictions for both the bearing stiffness and the damping coefficient in a wide range of designing conditions.

1. Introduction

Many investigators have examined the influence of the fluid inertia forces on the static characteristics of bearings and found the influence to be small for usual applications. However, the significant influence of the fluid inertia forces on the dynamic characteristics of the externally pressurized bearings in which the speed of fluid in a bearing clearance is relatively high and of the self-acting bearings which are operated at high speed has been noted recently[(1)(2)], and there are written many papers[(3)~(5)] discussing the inertia effect in hydrodynamic lubrication.

In conventional studies of externally pressurized bearings, only the inertia forces generated at the bearing clearance have been considered but one generated at the restricted part is neglected. However, in the case of using the fluids of low kinematic viscosity, it may be possible to consider the influence of the inertia forces of externally pressurized bearings with a kind of restriction. In this paper, the authors investigated theoretically and experimentally the influence of the fluid inertia forces generated at the restricted part on the dynamic characteristics of externally pressurized circular thrust bearings with capillary restriction.

In recent years , the fluids of low kinematic viscosity , such as water and liquid metals , are much used as lubricants. In order to predict exactly the dynamic characteristics of externally pressurized bearings which are lubricated with the lubricants of low kinematic viscosity, these studies will be useful.

2. Nomenclature

$A=\dfrac{12\mu r_0\bar{U}}{p_a h_0^2}$

a : radius of capillary

$B=\dfrac{h_0\omega}{\pi r_0^2 p_a}b$: dimensionless damping coefficient

B_0 : dimensionless damping coefficient (neglect inertia force)

b : damping coefficient

e : amplitude of displacement

$H=h/h_0$: dimensionless clearance

h : clearance

h_0 : equilibrium clearance

$K=\dfrac{h_0}{\pi r_0^2 p_a}k$: dimensionless stiffness

K_0 : dimensionless stiffness (neglect inertia force)

k : stiffness

l : length of capillary

$P=p/p_a$: dimensionless pressure

P_r : dimensionless recess pressure

P_{r3} : amplitude of dimensionless recess pressure

P_s : dimensionless supply pressure

P_0 : dimensionless pressure at $H=1$

P_1 : dimensionless pressure proportional to displacement

P_2 : dimensionless pressure proportional to velocity

p : pressure

p_a : ambient pressure

p_s : supply pressure

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** Assistant, Faculty of Engineering, Toyama University, 1-1, Nakagawa-Sonomachi, Takaoka.

*** Professor, Faculty of Engineering, Toyama University, 1-1, Nakagawa-Sonomachi, Takaoka.

**** Professor, Faculty of engineering, Kyoto University, Yoshida-Honmachi, Sakyo-ku, Kyoto.

$Q=\dfrac{q}{2\pi r_0 h_0 \bar{U}}$: dimensionless flow rate
q : flow rate
$R=r/r_0$: dimensionless radial coordimate or dimensionless radius
$R_1=r_1/r_0$: dimensionless radius of recess
r : radial coordinate or radius
r_0 : outer radius of bearing
r_1 : radius of recess
t : time
$U=u/\bar{U}$: dimensionless radial velocity
$\bar{U}$: characteristic radial velocity
u : radial velocity
$Z=z/h_0$: dimensionless coordinate across the film thickness
z : coordinate across the film thickness
$\alpha=\dfrac{\rho a^2 p_a}{12\mu^2}\left(\dfrac{h_0}{r_0}\right)^2$: inertia parameter
$\Gamma=-\dfrac{3a^4\ln R_1}{4lh_0^3}$: feeding parameter
$\varepsilon=e/h_0$
$\zeta=a^2\omega/\nu$
θ : phase difference between displacement and recess pressure
$\lambda=(i\omega/\nu)^{1/2}$
μ : viscosity
ν : kinematic viscosity
ξ_1,ξ_2 : function of ζ, defined by Eq.(12)
ρ : density
$\sigma=\dfrac{12\mu\omega}{p_a}\left(\dfrac{r_0}{h_0}\right)^2$: squeeze number
$\tau=\omega t$: dimensionless time
Φ : integration constant
ω : angular speed of squeeze motion

Subscripts

in : inflow
out : outflow
st : steady
os : oscillatory

3. Theoretical analysis

The externally pressurized circular thrust bearing with capillary restriction analyzed here is shown in Fig.1. The capillary restriction is made of a straight circular pipe.

3.1 Assumptions

The following assumptions are used in order to simplify the analysis.

(1) The lubricant is an incompressible Newtonian fluid.
(2) The flow through the bearing gap is viscous so that the pressure follows Reynolds equation.
(3) The depth of the recess is large enough compared with the bearing clearance so that the recess pressure may be uniform and it may be equal to the bearing gap pressure at $r=r_1$.
(4) The flow through capillary restriction is laminar and the inlet is so short that it may be neglected.
(5) The thrust plate is parallel to the bearing surface.

3.2 Fundamental equations

The dimensionless bearing pressure follows the dimensionless Reynolds equation as

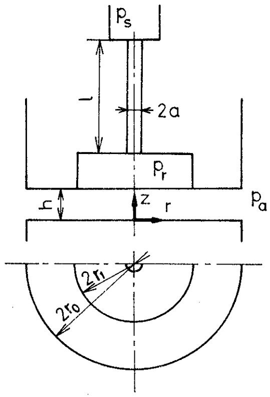


Fig.1 Coordinate system for thrust bearing

$$\frac{H^3}{R}\frac{\partial}{\partial R}\left(R\frac{\partial P}{\partial R}\right)=\sigma\frac{dH}{d\tau} \quad\cdots\cdots(1)$$

where

$$P=\frac{p}{p_a},\quad H=\frac{h}{h_0},\quad R=\frac{r}{r_0}$$

$$\tau=\omega t,\quad \sigma=\frac{12\mu\omega}{p_a}\left(\frac{r_0}{h_0}\right)^2$$

Introducing the boundary condition that $P=1(p=p_a)$ when $R=1(r=r_0)$, the dimensionless bearing pressure can be written in the form,

$$P=1-\frac{\sigma}{4H^3}\frac{dH}{d\tau}(1-R^2)-\frac{A\Phi}{H^3}\ln R \quad\cdots\cdots(2)$$

where $A=12\mu r_0\bar{U}/(p_a h_0^2)$, $\bar{U}$ is a characteristic radial velocity which is obtained theoretically from the mean steady radial velocity at the outer periphery of the bearing surface and the integration constant Φ is a function of τ which is determined from the continuity condition of flow rate described in section 3.3 .

Taking a small sinusoidal vibration, with amplitude e which occurs around the equilibrium state ($h=h_0$) of bearing into consideration, the dimensionless clearance, H, may be given by

$$H=1+\varepsilon\sin\tau,\quad \varepsilon=e/h_0 \quad\cdots\cdots(3)$$

P and Φ are assumed as

$$P=P_0(R)+\varepsilon\{P_1(R)\sin\tau+P_2(R)\cos\tau\}\cdots(4)$$

$$\Phi=\Phi_0+\varepsilon(\Phi_1\sin\tau+\Phi_2\cos\tau)\quad\cdots\cdots(5)$$

Substituting Eqs.(4) and (5) into Eq.(2), we obtain the following equations

$$\left.\begin{aligned}&P_0=1-A\Phi_0\ln R\\&P_1=A(3\Phi_0-\Phi_1)\ln R\\&P_2=-\frac{\sigma}{4}(1-R^2)-A\Phi_2\ln R\end{aligned}\right\}\cdots\cdots(6)$$

3.3 Continuity condition of flow rate

The dimensionless flow rate through the bearing gap is

$$Q_{out}=\frac{q_{out}}{2\pi r_0 h_0 \bar{U}}=\int_0^H URdZ=\Phi_0+\varepsilon(\Phi_1\sin\tau+\Phi_2\cos\tau)-\varepsilon\frac{r_0\omega}{2\bar{U}}R^2\cos\tau \quad \cdots(7)$$

We now consider the flow through the capillary restriction. For the case of a laminar flow through a straight circular pipe, in which the inlet length can be neglected, the flow rate is given by[6]

$$q_{st}=-\frac{\pi a^4}{8\mu}\frac{\partial p}{\partial x} \quad \cdots(8)$$

$$q_{os}=\frac{2\pi}{\omega\rho}\frac{\partial p}{\partial x}\left\{\frac{a^2}{2}i-\frac{a}{\lambda}\frac{J_1(\sqrt{-i\zeta})}{J_0(\sqrt{-i\zeta})}\right\}e^{i\omega t} \quad \cdots(9)$$

where

$$i=\sqrt{-1},\quad \lambda=(i\omega/\nu)^{1/2},\quad \zeta=a\omega^2/\nu$$

and J_0, J_1 are Bessel functions of the first kind. Using the assumption (4), the axial pressure gradient in the capillary restriction can be written as

$$p_a(P_r-P_s)/l=p_a\{P_{r0}+\varepsilon(P_{r1}\sin\tau+P_{r2}\cos\tau)-P_s\}/l$$

Then the dimensionless flow rate can be obtained from Eqs.(8) and (9) as

$$Q_{in}=\frac{q_{in}}{2\pi r_0 h_0\bar{U}}=\frac{a^4 p_a}{16\mu l r_0 h_0 \bar{U}}[(P_s-P_{r0})-\varepsilon\{(\xi_2P_{r1}+\xi_1P_{r2})\sin\tau-(-\xi_1P_{r1}+\xi_2P_{r2})\cos\tau\}] \quad \cdots(10)$$

where P_s is the dimensionless supply pressure, $P_{r0}\sim P_{r2}$ are the dimensionless recess pressures which can be given from Eq.(6) as

$$\left.\begin{aligned}P_{r0}&=1-A\Phi_0\ln R_1\\P_{r1}&=A(3\Phi_0-\Phi_1)\ln R_1\\P_{r2}&=-\frac{\sigma}{4}(1-R_1^2)-A\Phi_2\ln R_1\end{aligned}\right\} \quad \cdots(11)$$

ξ_1 and ξ_2 are real variables which are defined by

$$\frac{J_1(\sqrt{-i\zeta})}{J_0(\sqrt{-i\zeta})}\equiv\frac{\sqrt{-i\zeta}}{2}-\frac{\zeta\sqrt{-i\zeta}}{16}(\xi_1+i\xi_2) \quad \cdots(12)$$

In the case of $\zeta\ll1$, ξ_1 and ξ_2 are approximated by

$$\xi_1\doteqdot\frac{\zeta}{6},\quad \xi_2\doteqdot1-\frac{11\zeta^2}{384} \quad \cdots(13)$$

ζ is a dimensionless parameter which represents the influence of the fluid inertia forces on the velocity profile. Now we put

$$\zeta\equiv\alpha\sigma \quad \cdots(14)$$

where α is a dimensionless parameter which can be expressed by

$$\alpha=\frac{\rho a^2 p_a}{12\mu^2}\left(\frac{h_0}{r_0}\right)^2 \quad \cdots(15)$$

α is a new parameter which has never appeared in the conventional analysis. This parameter is concerned with the fluid inertia forces generated in the restricted part and may be named the inertia parameter in this paper.

The flow rate through the bearing recess is the sum of the flow rate through the restriction and the flow rate produced by the change of the recess clearance. Therefore, the continuity condition of flow rate at $R=R_1$ is given by

$$Q_{in}-\varepsilon\frac{r_0\omega}{2\bar{U}}R_1^2\cos\tau=Q_{out}|_{R=R_1} \quad \cdots(16)$$

Now the term "dimensionless parameter" is defined as

$$\Gamma\equiv-\frac{3a^4\ln R_1}{4lh_0^3} \quad \cdots(17)$$

Then, substituting Eqs.(7) and (10) into Eq.(16) and introducing Eq.(17), we get the following equations

$$\left.\begin{aligned}(1+\Gamma)(P_s-P_{r0})&=\Phi_0(P_s-1)\\(1+\Gamma)(\xi_2P_{r1}+\xi_1P_{r2})&=-\Phi_1(P_s-1)\\(1+\Gamma)(-\xi_1P_{r1}+\xi_2P_{r2})&=-\Phi_2(P_s-1)\end{aligned}\right\} \quad \cdots(18)$$

where Γ is called the feeding parameter which is equivalent to the ratio of the flow resistance within the restriction and the bearing gap. With this parameter, Γ, the term of A may be written as

$$A=-\frac{\Gamma}{1+\Gamma}\frac{P_s-1}{\ln R_1} \quad \cdots(19)$$

Substituting this equation into Eq.(11), and eliminating $P_{r0}\sim P_{r2}$ from Eqs.(11) and (18), the following relations are induced

$$\Phi_0=1 \quad \cdots(20)$$

$$\begin{bmatrix}1+\xi_2\Gamma & \xi_1\Gamma\\ -\xi_1\Gamma & 1+\xi_2\Gamma\end{bmatrix}\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix}=\begin{bmatrix}3\xi_2\Gamma+\frac{\sigma}{4}\xi_1(1+\Gamma)\frac{1-R_1^2}{P_s-1}\\ -3\xi_1\Gamma+\frac{\sigma}{4}\xi_2(1+\Gamma)\frac{1-R_1^2}{P_s-1}\end{bmatrix} \quad \cdots(21)$$

Substituting this $\Phi_0\sim\Phi_2$ into Eqs.(6) and (11), the bearing gap pressure and the recess pressure can be determined.

3.4 Stiffness and damping coefficient

The reaction force can be derived by integrating the pressure over the whole bearing surface. The stiffness and the damping coefficient are defined by a component of the reaction force proportional to the displacement and one proportional to the velocity of the displacement respectively. The dimensionless stiffness and the dimensionless damping coefficient normalized by $\pi r_0^2 p_a/h_0$ and $\pi r_0^2 p_a/(h_0\omega)$, respectively, are given by

$$K=\frac{h_0}{\pi r_0^2 p_a}k=-2\int_0^{R_1}P_{r1}RdR-2\int_{R_1}^1 P_1(R)RdR=-\frac{(3-\Phi_1)\Gamma}{2(1+\Gamma)}\frac{1-R_1^2}{\ln R_1}(P_s-1) \quad \cdots(22)$$

$$B=\frac{h_0\omega}{\pi r_0^2 p_a}b=-2\int_0^{R_1}P_{r2}RdR-2\int_{R_1}^{1}P_2(R)RdR$$
$$=\frac{\sigma}{8}(1-R_1^4)+\frac{\Phi_2\Gamma}{2(1+\Gamma)}\frac{1-R_1^2}{\ln R_1}(P_s-1) \quad \cdots\cdots\cdots\cdots(23)$$

The influence of the fluid inertia forces generated at the restricted part on K and B depends on the value of ζ. Finally, both K and B can be derived using the parameters: R_1 (the ratio of the radius of the inner periphery to one of the outer periphery), P_s (the supply pressure normalized by the ambient pressure), Γ (the feeding parameter), σ (the squeeze number) and α (the inertia parameter).

It is equivalent to assuming $\zeta=0$ ($\xi_1=0$, $\xi_2=1$) to neglect the inertia forces, and so the dimensionless stiffness and the dimensionless damping coefficient can be obtained as

$$K_0=-\frac{3}{2}\frac{\Gamma}{(1+\Gamma)^2}\frac{1-R_1^2}{\ln R_1}(P_s-1) \quad \cdots\cdots(24)$$

$$B_0=\frac{\sigma}{8}\left\{(1-R_1^4)+\frac{\Gamma}{1+\Gamma}\frac{(1-R_1^2)^2}{\ln R_1}\right\} \cdots\cdots(25)$$

In the case of $\sigma\to0$, the damping coefficient is obtained from Eqs.(13), (21) and (23) as follows,

$$B/\sigma|_{\sigma\to0}\doteqdot B_0/\sigma-\frac{\alpha}{4}\frac{\Gamma^2}{(1+\Gamma)^3}\frac{1-R_1^2}{\ln R_1}(P_s-1) \quad \cdots\cdots\cdots\cdots(26)$$

The influence of the inertia forces on the flow within the capillary is not so remarkable when the frequency is low. However, it is obvious from Eq.(26) that the influence on the damping coefficient of the inertia parameter, α, is not small even when the squeeze number, σ, is very small.

4. Examples of calculated results

Examples of calculated results are given in Figs.2 to 7. The influence of the inertia forces has a tendency to become remarkable when the feeding parameter, Γ, increases in value.

Fig.2 shows the influence of the inertia parameter, α, on the dimensionless stiffness, K. As a value of α increases, the influence of the inertia forces becomes significant and the value of Γ which makes a maximum value of K increases. Fig.3 shows the influence of the supply pressure, P_s, on the stiffness, K. The influence of P_s is not so remarkable. Fig.4 shows the influence of the squeeze number, σ. The influence of the inertia forces becomes significant as σ increases in value.

Fig.5 shows the influence of α on the dimensionless damping coefficient, B. As a value of α increases, the influence of the inertia forces becomes significant and the value of Γ which makes a maximum value of B increases. Fig.6 shows the influence of P_s on B. The influence of the inertia forces becomes significant as P_s increases in value. Fig.7

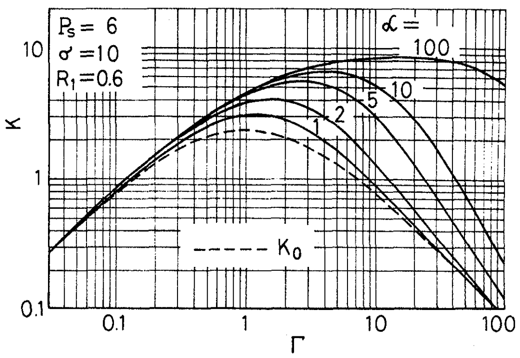


Fig.2 K for different α

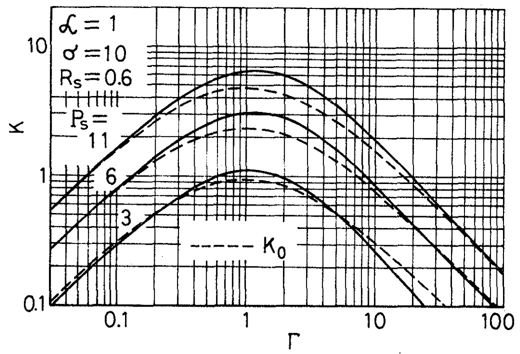


Fig.3 K for different P_s

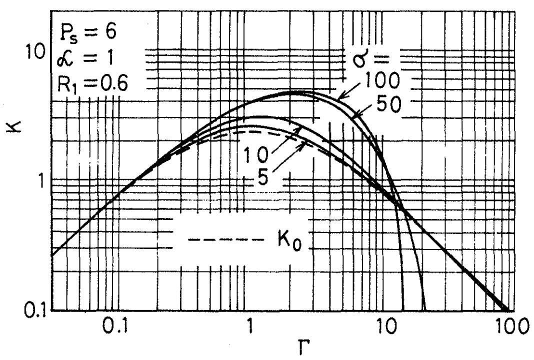


Fig.4 K for different σ

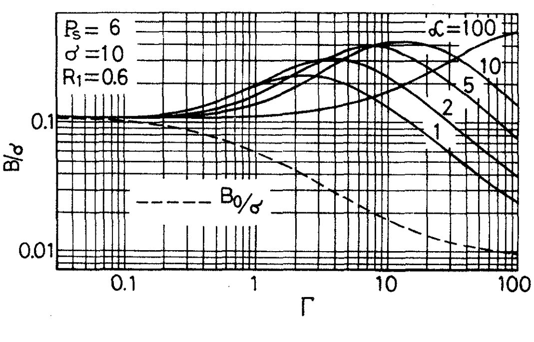


Fig.5 B for different α

shows the influence of σ on B. The influence of the inertia forces becomes significant as σ decreases in value. In this figure, the values of $B/\sigma|_{\sigma\to 0}$ which are given by Eq.(26) are indicated with an alternately long and short dash line. It can be known from this figure that the values of B/σ approach this line as σ decreases in value. The values of B are proportional to the squeeze number, σ, when the influence of the inertia forces is neglected, and so the ordinate is given in the form of B/σ in Figs.5 to 7.

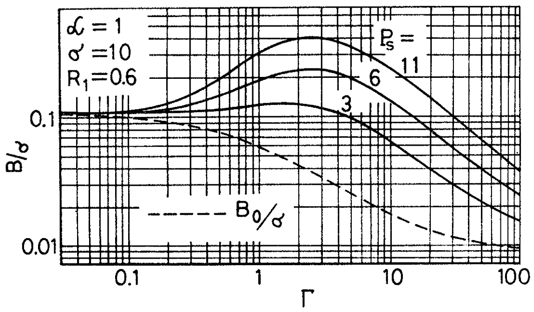


Fig.6 B for different P_s

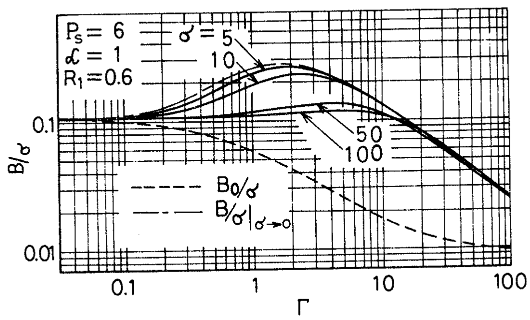


Fig.7 B for different σ

5. Comparison with the experimental results

In this chapter, comparison with the experimental results will be made to confirm the validity of the presented analysis.

The experimental apparatus is schematically illustrated in Fig.8 . The rotor ② is supported by externally pressurized thrust bearing ① , externally pressurized gas journal bearing ③ and a back pressure ④ of lower volume. The vibrator which is connected to the end of the rotor causes the rotor to vibrate with a small sinusoidal vibration in the axial direction. The bearing clearance is set by adjusting the back pressure. The displacement of the thrust plate is measured by the electric capacitance micrometer ⑥ . The lubricant which is pressurized by the compressed air is supplied into the bearing recess through the capillary restriction ⑤ . In this experiment, the capillary is set long enough so that the influence of the oil column within the capillary on the supply pressure can be considered.

Experimental procedure is as follows: (1) Control the back pressure to get a specified value of the bearing clearance, (2) make the rotor vibrate with a steady sinusoidal vibration of which the frequency is 60 Hz , (3) measure the displacement of the thrust plate and the oscillatory recess pressure by the electric capacitance micrometer ⑦ and the pressure transducer ⑧ respectively and (4) analyze the frequency, the amplitude and the phase difference by means of an oscilloscope.

The temperature of the lubricant was measured at both the entrance of the restriction and the exit of the bearing by a thermocouple. Since the difference is 1°C at the most, it may be assumed that the temperature of the lubricant is equal to the average of these temperatures.

The dimensions of the test bearings and the conditions of the experiments are shown in Fig.9 and Table 1 respectively. Since the ambient pressure p_a is almost equal to the atmospheric pressure, it may be assumed that the values of p_a are equal

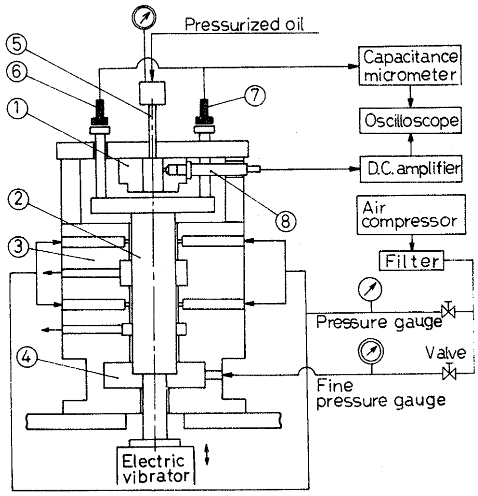
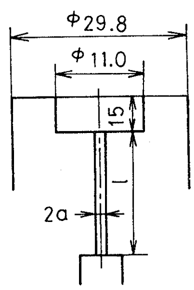


Fig.8 Schematic of experimental apparatus



	a mm	l mm
No.1	0.95	655
2	1.50	2 000

Fig.9 Dimensions of test bearings

Table 1 Conditions of experiments

Bearing No.	P_s	ρ kgs²/cm⁴	μ kgs/cm²	P_a kg/cm²
1	2	$0.880^{\times 10^{-6}}$	$0.174^{\times 10^{-6}}$	1.04
1	3	0.880	0.172	1.04
2	2	0.881	0.184	1.03
2	3	0.880	0.173	1.03

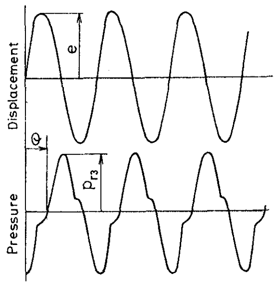


Fig.10 Examples of measurements

to the atmospheric pressure which is measured by a mercurial barometer. The examples of the measurements for the displacement and the oscillatory pressure are shown in Fig.10 . As seen from this example, the wave profile of pressure is not always sinusoidal. The bearing clearance is given by

$$H=1+\varepsilon\sin\omega t \quad\cdots\cdots(27)$$

and then the recess pressure can be expressed as follows

$$P_r \fallingdotseq P_{r0}+\varepsilon(P_{r1}\sin\omega t+P_{r2}\cos\omega t)$$
$$=P_{r0}+\varepsilon P_{r3}\sin(\omega t-\theta) \quad\cdots\cdots(28)$$

and so the values of P_{r3} and θ may be obtained by the amplitude of the oscillatory pressure and the difference in the center of the wave respectively. For the waves of P_{r3} and θ are adopted the arithmetic mean values, because the values differ depending on whether the bearing clearance increases or decreases in value.

In Figs.11 to 14, the theoretical estimations of P_{r3} and θ are compared with the experimental results. The frequency is 60 Hz . In these figures, the theoretical results in which the influence of inertia forces is neglected are indicated with a broken line. From these figures, it is seen that good agreement can be found between the estimations by the present theory and the experimental data and that the experimental data of θ tend to diviate from the theoretical estimations depending on whether Γ is small or large.

In this experiment, the diameter of the capillary is considerably large so that the influence of the inertia forces generated at the restricted part appears too large to be neglected.

6. Conclusions

The influence of the fluid inertia forces generated at the restricted part on the dynamic characteristics of externally

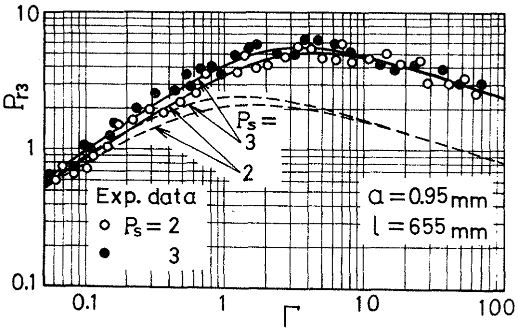


Fig.11 Comparison with experiment

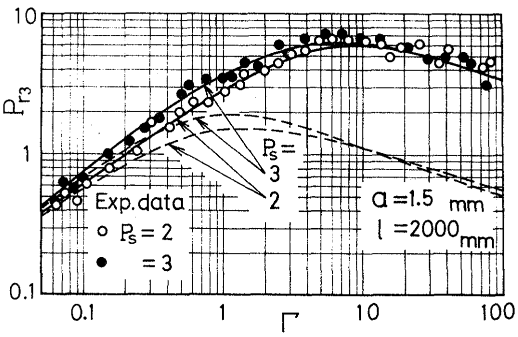


Fig.12 Comparison with experiment

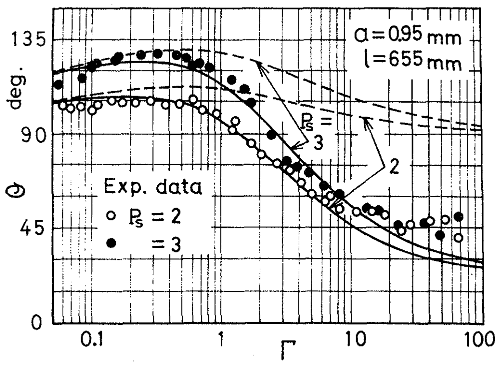


Fig.13 Comparison with experiment

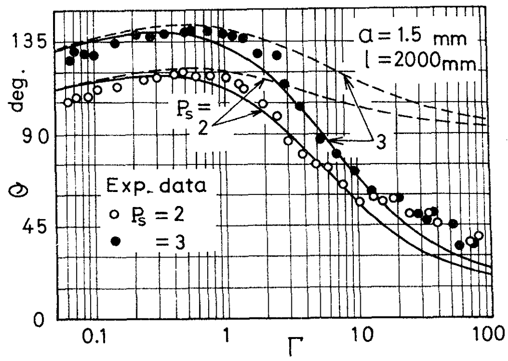


Fig.14 Comparison with experiment

pressurized circular thrust bearings with capillary restriction was examined theoretically and experimentally. In consequence, the following conclusions have been obtained:

(1) The influence of the fluid inertia forces generated at the restricted part on the dynamic characteristics is considerable when the kinematic viscosity of the lubricant is low.

(2) The inertia effect becomes remarkable as the feeding parameter, Γ , increases in value.

(3) The inertia effect on the bearing stiffness becomes remarkable as both the inertia parameter, α , and the squeeze number, σ , increase in value. In general, however, the inertia effect hardly depend on the supply pressure, P_s .

(4) In general, the damping coefficient increases in value as the fluid inertia forces do. The influence becomes significant as both the inertia parameter, α , and the supply pressure, P_s , increase in value and as the values of the squeeze number, σ , decrease.

These results imply that the dynamic characteristics are influenced by the inertia parameter, even if the bearings are similarly designed in the conventional sense. In any practical bearings, the influence of the fluid inertia forces generated at both the restricted part and the bearing clearance may be considerable when the kinematic viscosity of the lubricant is low.

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