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# Evaluation of Various Approximate Solutions for Effects of Fluid Inertia Forces on Performance of a Plane Inclined Slider Pad *

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*Based on the Navier-Stokes equations in which the pressure is assumed to be constant across the film thickness, various approximate solutions for the static and dynamic performances of an infinitely wide, plane inclined slider pad in a laminar flow regime are presented under the assumption of a small harmonic vibration.*

*From comparison of the approximate solutions with the numerical one, it is concluded that one kind of averaging approach in which some of the time dependent terms are treated exactly while the convective inertia terms are averaged out across the film thickness gives close approximations in a wide range of designing conditions, and that an other kind of averaging approach in which all the inertia terms including the time dependent terms are averaged out across the film thickness is a fairly good approximation.*

Key Words : Lubrication, Bearing, Inertia Effect, Approximate Solution, Plane Inclined Slider Pad

## 1. Introduction

A significant influence of the inertia forces on the dynamic characteristics has been reported about externally pressurized bearings in which the speed of fluid induced by the pressure gradients is relatively high and also about self-acting bearings which are operated at high speed [1 ~ 3]. It is difficult to solve the problem exactly for dynamic performance of bearings including the fluid inertial effects because the inertia terms in the momentum equations are nonlinear. Then, various approximate solutions have been introduced. However, their accuracies are seldom discussed. The conventional approximate methods may be classified roughly into the perturbation method [2] and the averaging approach [1].

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In this paper, based on the Navier-Stokes equations in which the pressure is assumed to be constant across the film thickness, a modified perturbation method and a modified averaging approach for the dynamic performance of an infinitely wide, plane inclined slider pad in a laminar flow regime are presented under the assumption of a small harmonic vibration. These methods may evaluate adequately the contribution of the time-dependent term. Subsequently, the conventional approximate methods and these ones will be compared with the numerical one.

It should be noted that the static and dynamic performances of this kind of bearing including the fluid inertial effects have been investigated theoretically by Mori, et al. by means of a kind of averaging approach [6]. The boundary value of the film pressure at the leading edge is set as the ambient pressure, under the assumption of a negligibly small ram-pressure there .

## 2. Nomenclature

- $B$ : dimensionless damping coefficient
- $D$ : dimensionless drag coefficient
- $K$ : dimensionless dynamic stiffness
- $Q$ : dimensionless flow rate
- $Re = \rho V h_{2e}/\mu$ : Reynolds number
- $Re^* = (\rho V h_{2e}/\mu)(h_{2e}/l)$ : inertia parameter
- $Re^{**} = \rho\omega h_{2e}^2/\mu$ : unsteadiness parameter
- $\bar{W}$ : dimensionless load capacity
- $\alpha = H_1 - H_2$ : gradient of slider
- $\Lambda = 6\mu Vl/(p_a h_{2e}^2)$ : bearing modulus
- $\sigma = (12\mu\omega/p_a)(l/h_{2e})^2$ : squeeze number

Subscripts

$e$ : equilibrium
$q$ : quasi-statical component
$t$ : dynamical component
$\hat{}$ : amplitude

### 3. Governing Equations and Boundary Conditions

The plane inclined slider pad with infinite width analyzed here is shown schematically in Fig.1, in which some symbols used are also noted. With the usual assumptions of fluid film lubrication theory the momentum equations and the continuity equation for an incompressible Newtonian fluid are given by;

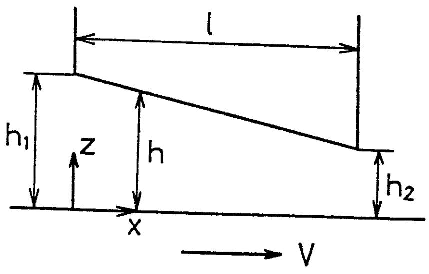


Fig.1 Configuration and coordinates

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial z^2} \tag{1}$$

$$0=\frac{\partial p}{\partial z} \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{3}$$

where $u$ and $w$ are the velocity component in the $x$ direction and the one in the $z$ direction, respectively, $p$ is the pressure, $\rho$ the density, $\mu$ the viscosity and $t$ the time.

The boundary conditions are set as follows;

$$z=0\ :\ u=V,\ w=0,\ z=h\ :\ u=0,\ w=\frac{\partial h}{\partial t},\ x=0,\ l\ :\ p=p_a \tag{4}$$

where $p_a$ is the ambient pressure and $V$ the velocity of the moving surface.

It is difficult to obtain general exact solutions to the above equations. Then, various approximate solutions have been introduced.

### 4. Various Approximate Solutions

The conventional approximate methods may be classified roughly into the perturbation method and the averaging approach. In the former, the velocity and the pressure are perturbed with respect to $Re^*$ as follows;

$$(u,\ w,\ p)=(u_0,\ w_0,\ p_0)+Re^*(u_1,\ w_1,\ p_1)+\cdots\cdots \tag{5}$$

Substituting Eq.(5) into Eqns.(1) to (4), we obtain the fundamental equations and the boundary conditions. In general, the solutions for second and higher orders of $Re^*$ are omitted because these are very complicated. In the latter, the solutions are analyzed by averaging out all the inertia terms in the momentum equations across the film thickness. Namely, Eq.(1) is replaced with

$$\frac{\rho}{h}\int_0^h\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)dz=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial z^2} \tag{6}$$

#### 4.1 Modified perturbation method

Since velocity and the pressure are regarded as the sums of quasi-statical components (subscript $q$) and dynamical ones (subscript $t$), we can write them as;

$$(u,\ w,\ p)=(u_q,\ w_q,\ p_q)+(u_t,\ w_t,\ p_t) \tag{7}$$

Next, we assume a small harmonic variation in the film thickness as follows;

$$h=h_e+h_3e^{j\omega t} \tag{8}$$

Correspondingly to this, the dynamical components of velocity and pressure are given by;

$$(u_t,\ w_t,\ p_t)=(\hat{u}_t,\ \hat{w}_t,\ \hat{p}_t)e^{j\omega t} \tag{9}$$

Introducing the following dimensionless quantities;

$$X=x/l,\ Z=z/h_{2e},\ H=h/h_{2e},\ U_q=u_q/V,\ W_q=w_q l/Vh_{2e},\ P_q=p_q/p_a$$
$$\varepsilon=h_3/h_{2e},\ \hat{U}_t=\hat{u}_t/(\varepsilon\omega l),\ \hat{W}_t=\hat{w}_t/(\varepsilon\omega h_{2e}),\ \hat{P}_t=\hat{p}_t/(\varepsilon p_a),\ R_e^*=\rho V h_{2e}^2/(\mu l)$$
$$R_e^{**}=\rho\omega h_{2e}^2/\mu,\ \Lambda=6\mu Vl/(p_a h_{2e}^2),\ \Lambda^*=6\mu\omega l^2/(p_a h_{2e}^2)$$

we can formulate the following governing differential equations and boundary conditions

( For quasi-statical components )

$$R_e^*\left(U_q\frac{\partial U_q}{\partial X}+W_q\frac{\partial U_q}{\partial Z}\right)=-\frac{6}{\Lambda}\frac{\partial P_q}{\partial X}+\frac{\partial^2 U_q}{\partial Z^2} \tag{10}$$

$$0=\frac{\partial P_q}{\partial Z} \tag{11}$$

$$\frac{\partial U_q}{\partial X}+\frac{\partial W_q}{\partial Z}=0 \tag{12}$$

$$\begin{aligned}&Z=0\ :\ U_q=1,\ W_q=0, \qquad Z=H\ :\ U_q=W_q=0\\ &X=0,\ 1\ :\ P_q=1\end{aligned} \tag{13}$$

( For dynamical components )

$$jR_e^{**}\hat{U}_t+R_e^*\left(j\frac{\partial U_q}{\partial H}+U_q\frac{\partial \hat{U}_t}{\partial X}+\hat{U}_t\frac{\partial U_q}{\partial X}+W_q\frac{\partial \hat{U}_t}{\partial Z}+\hat{W}_t\frac{\partial U_q}{\partial Z}\right)_{H=H_e}=-\frac{6}{\Lambda^*}\frac{\partial \hat{P}_t}{\partial X}+\frac{\partial^2\hat{U}_t}{\partial Z^2} \tag{14}$$

$$0=\frac{\partial \hat{P}_t}{\partial Z} \tag{15}$$

$$\frac{\partial \hat{U}_t}{\partial X}+\frac{\partial \hat{W}_t}{\partial Z}=0 \tag{16}$$

$$\begin{aligned}&Z=0\ :\ \hat{U}_t=\hat{W}_t=0, \qquad Z=H_e\ :\ \hat{U}_t=0,\ \hat{W}_t=j\\ &X=0,\ 1\ :\ \hat{P}_t=0\end{aligned} \tag{17}$$

From Eq.(8) the real parts of the complex quantities in solutions $u_t$, $w_t$ and $p_t$ have physical meaning.

Since it is difficult to obtain general exact solutions to the above governing equations, we analyze by the perturbation method. Namely, the velocity and the pressure are perturbed with respect to $Re^*$ as;

$$\begin{aligned}(U_q,\ W_q,\ P_q,\ \hat{U}_t,\ \hat{W}_t,\ \hat{P}_t)&=(U_{q0},\ W_{q0},\ P_{q0},\ \hat{U}_{t0},\ \hat{W}_{t0},\ \hat{P}_{t0})\\ &+R_e^*(U_{q1},\ W_{q1},\ P_{q1},\ \hat{U}_{t1},\ \hat{W}_{t1},\ \hat{P}_{t1})+\cdots\end{aligned} \tag{18}$$

Substituting Eq.(18) into Eqns.(10) ~ (17), we obtain the fundamental equations and the boundary conditions. The first-order solutions are obtained as follows;

$$\hat{U}_{t0}=\frac{-jX+C_1}{\Phi}\left\{1-\frac{\cosh(2\varphi Z-\varphi H_e)}{\cosh(\varphi H_e)}\right\} \tag{19}$$

$$\begin{aligned}\hat{W}_{t0}=&\frac{\alpha_e}{2}\frac{-jX+C_1}{\Phi}\frac{\cosh(2\varphi Z)-1}{\cosh^2(\varphi H_e)}\\ &+\frac{j\Phi+\alpha_e(jX-C_1)\{1-\mathrm{sech}^2(\varphi H_e)\}}{\Phi^2}\left\{Z-\frac{\sinh(2\varphi Z-\varphi H_e)+\sinh(\varphi H_e)}{2\varphi\cosh(\varphi H_e)}\right\}\end{aligned} \tag{20}$$

$$\frac{d\hat{P}_{t0}}{dX}=\frac{jR_e^{**}\Lambda^*}{6}\frac{jX-C_1}{\Phi} \tag{21}$$

$$\hat{U}_{t1}=\frac{6j}{R_e^*\Lambda^*}\frac{d\hat{P}_{t1}}{dX}\left\{1-\frac{\cosh(2\varphi Z-\varphi H_e)}{\cosh(\varphi H_e)}\right\}+F \tag{22}$$

$$\frac{d\hat{P}_{t1}}{dX}=\frac{jR_e^{**}\Lambda^*}{6\Phi}\left(\int_0^{H_e}F\,dZ+C_2\right) \tag{23}$$

where $j=\sqrt{-1}$, $\varphi\equiv\sqrt{jR_e^{**}}/2$, $\Phi\equiv H_e-\varphi^{-1}\tanh(\varphi H_e)$ and $C_1$, $C_2$ are integrations constants. $F$ is a function of $X$, $Z$ which can be determined by the following partial differential equation and the boundary condition;

$$\frac{\partial^2 F}{\partial Z^2}-4\varphi^2 F=j\frac{\partial U_{q0}}{\partial H}+U_{q0}\frac{\partial \hat{U}_{t0}}{\partial X}+\hat{U}_{t0}\frac{\partial U_{q0}}{\partial X}+W_{q0}\frac{\partial \hat{U}_{t0}}{\partial Z}+\hat{W}_{t0}\frac{\partial U_{q0}}{\partial Z} \tag{24}$$

$$Z=0,\ H_e\ :\ F=0 \tag{25}$$

The solution can be obtained in the same manner as in the previous report [4,5]. The pressure components $\hat{p}_{t0}$, $\hat{p}_{t1}$ are obtained by Runge-Kutta-Gill method, because these are are insoluble analytically.

This solution will be named modified perturbation method in this paper.

### 4.2 Modified averaging approach

In this approach, the solutions are obtained by averaging out the terms containing the inertia parameter, *Re**, in the governing equations (10) and (14) across the film thickness. Namely, Eqns.(10) and (14) are replaced with

$$\frac{R_e^*}{H}\int_0^H\left(U_q\frac{\partial U_q}{\partial X}+W_q\frac{\partial U_q}{\partial Z}\right)dZ=-\frac{6}{\Lambda}\frac{\partial P_q}{\partial X}+\frac{\partial^2 U_q}{\partial Z^2} \tag{26}$$

$$jR_e^{**}\hat{U}_t+\frac{R_e^*}{H_e}\int_0^{H_e}\left(j\frac{\partial U_q}{\partial H}+U_q\frac{\partial \hat{U}_t}{\partial X}+\hat{U}_t\frac{\partial U_q}{\partial X}+W_q\frac{\partial \hat{U}_t}{\partial Z}+\hat{W}_t\frac{\partial U_q}{\partial Z}\right)_{H=H_e}dZ$$
$$=-\frac{6}{\Lambda^*}\frac{\partial \hat{P}_t}{\partial X}+\frac{\partial^2 \hat{U}_t}{\partial Z^2} \tag{27}$$

The solutions are given by;

$$\hat{U}_t=\frac{-jX+C_3}{\Phi}\left\{1-\frac{\cosh(2\varphi Z-\varphi H_e)}{\cosh(\varphi H_e)}\right\} \tag{28}$$

$$\hat{W}_t=\frac{\alpha_e(-jX+C_3)}{2\Phi}\frac{\cosh(2\varphi Z)-1}{\cosh^2(\varphi H_e)}+\frac{j\Phi-\alpha_e(-jX+C_3)\{1-\mathrm{sech}^2(\varphi H_e)\}}{\Phi^2}$$
$$\times\left\{Z-\frac{\sinh(2\varphi Z-\varphi H_e)+\sinh(\varphi H_e)}{2\varphi\cosh(\varphi H_e)}\right\} \tag{29}$$

$$\frac{d\hat{P}_t}{dX}=\frac{jR_e^{**}\Lambda^*(jX-C_3)}{6\Phi}-\frac{R_e^*\Lambda^*}{6H_e}\int_0^{H_e}\left\{j\frac{\partial U_q}{\partial H}+U_q\frac{\partial \hat{U}_t}{\partial X}+\hat{U}_t\frac{\partial U_q}{\partial X}+W_q\frac{\partial \hat{U}_t}{\partial Z}+\hat{W}_t\frac{\partial U_q}{\partial Z}\right\}_{H=H_e}dZ \tag{30}$$

where $C_3$ is an integration constant. The pressure component $\hat{p}_t$ is obtained by Runge-Kutta-Gill method. These solutions are simple compared with these by the modified perturbation method.

This solution will be named modified averaging approach in this paper.

## 5. Bearing Performance

### 5.1 Load capacity

The dimensionless load capacity per unit width normalized by $p_a l$ is defined by;

$$\bar{W}=\int_0^1(P_q-1)_{H=H_e}dX \tag{31}$$

### 5.2 Dynamic stiffness and damping coefficient

The dynamic stiffness and the damping coefficient are respectively defined by the component of the bearing reaction force in the same phase as the displacement and the component in the same phase as the velocity of the displacement. The dimensionless dynamic stiffness and the dimensionless damping coefficient normalized by $p_a l/h_{2e}$ and $p_a l/(h_{2e}\omega)$ are respectively given by;

$$K=-\mathrm{Re}[\hat{F}_{0s}] \tag{32}$$

$$B=-\mathrm{Im}[\hat{F}_{0s}] \tag{33}$$

where $\hat{f}_{0s}=\varepsilon p_a l\hat{F}_{0s}$, and the real part of $\hat{f}_{0s}e^{j\omega t}$ represents the bearing reaction force per unit width.

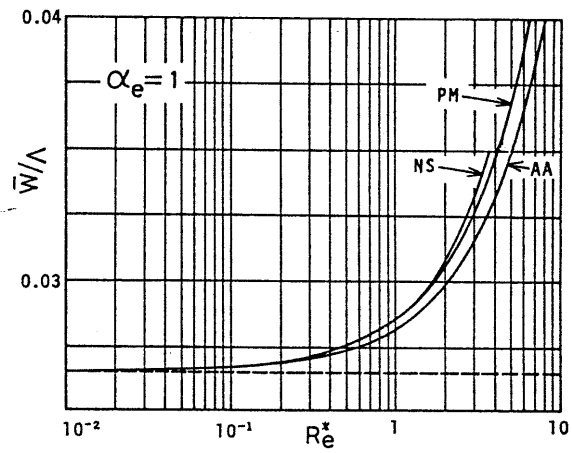


Fig.2 Comparison of approximate solutions for load capacity

### 5.3 Flow rate

The dimensionless steady flow rate per unit width normalized by $Vh_{2e}$ is defined by;

$$Q_q=\int_0^{H_e} U_q|_{H=H_e}dZ \qquad (34)$$

and the dimensionless dynamical flow rate per unit normalized by $\varepsilon\omega lh_{2e}$ is defined by;

$$\hat{Q}_t=\int_0^{H_e} \hat{U}_t dZ \qquad (35)$$

### 5.4 Drag coefficient

The dimensionless steady drag coefficient per unit width normalized by $\mu Vl/h_{2e}$ is defined by;

$$D_q=-\int_0^1 \frac{\partial U_q}{\partial Z}\Big|_{\substack{Z=0\\H=H_e}} dX \qquad (36)$$

and the dimensionless dynamical drag coefficient per unit width normalized by $\varepsilon\mu\omega l^2/h_{2e}$ is defined by;

$$\hat{D}_t=-\int_0^1 \frac{\partial \hat{U}_t}{\partial Z}\Big|_{Z=0} dX \qquad (37)$$

The real part and the imaginary part of $\hat{Q}_t$ are respectively equivalent to the component of the unsteady flow rate in the same phase as the displacement and the component in the same phase as the velocity of the displacement. The relationship between $\hat{D}_t$ and the unsteady drag coefficient is given in the same manner as above.

It can easily be understood that these characteristic quantities are governed by four parameters, i.e. $\alpha_e$, $\Lambda$ , $\sigma$ and $Re^*$. The unsteadiness parameter, $Re^{**}$, is related to these parameters by the following equations;

$$R_e^{**}=\frac{R_e^*\sigma}{2\Lambda} \qquad (38)$$

## 6. Comparison of Various Approximate Solutions

The 1st order perturbation method and the averaging approach are valid for $R_e^*\ll 1$ and also $R_e^{**}\ll 1$ in principle. The modified averaging approach are valid for $R_e^*\ll 1$ and for any value of $R_e^{**}$ .

In this chapter, these approximate solutions will be compared with the numerical one to evaluate their accuracy.

The bearing performance for $\Lambda=10$ is shown in Figs.2 to 16. In these figuers, the symbols, PM , AA , MPM , MAA and NS indicate the 1st order perturbation method, the averaging approach, the modified perturbation method, the modified averaging approach and the numerical solution respectively. The broken lines indicate the results by the classical lubrication theory.

The static performances, $\bar{W}$ , $Q_q$ and $D_q$ are shown in Figs.2 to 7. As for $\bar{W}$ , there is no significant difference among the various solutions. As for $Q_q$ and $D_q$ , the result by PM becomes inaccurate as the values of $Re^*$ and $\alpha_e$ increase and

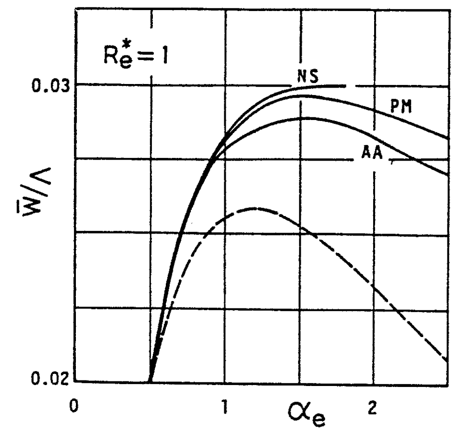


Fig.3 Comparison of approximate solutions for load capacity

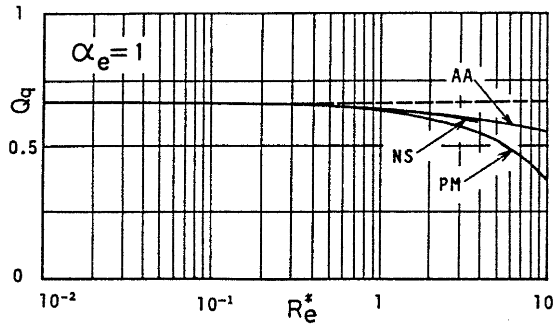


Fig.4 Comparison of approximate solutions steady flow rate

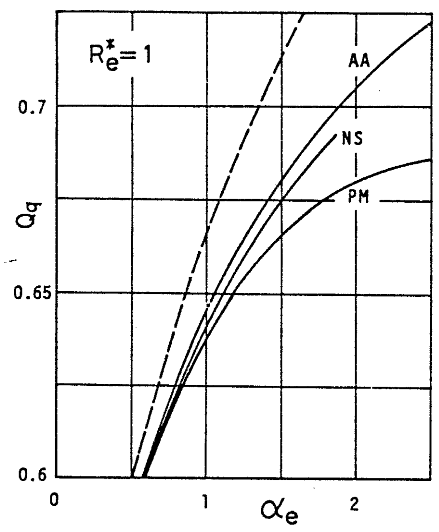


Fig.5 Comparison of approximate solutions for steady flow rate

one by AA is relatively accurate.

The dynamic stiffness, $K$, and the damping coefficient, $B$, are shown in Figs.8 to 10. As for $K$, there is no significant difference among the various solutions. As for $B$, there is either no significant difference between MPM, MAA and NS. However, the results by AA and PM become inaccurate as the value of $Re^{**}$ increases. The result by AA is relatively accurate when the value of $Re^{**}$ is comparatively small, for example smaller than 10.

The unsteady flow rate, $\widehat{Q}_t$, is shown in Figs.11 to 13. The result by PM becomes remarkably inaccurate as the value of $Re^{**}$ increases and those by MPM, MAA and AA are fairly accurate.

The unsteady drag coefficient, $\widehat{D}_t$, is shown in Figs.14 to 16. The result by PM becomes remarkably inaccurate as the value of $Re^{**}$ increases. The results by MPM and MAA are fairly accurate. However, one by AA gradually becomes inaccurate with an increase in the value of $Re^{**}$. The analysis of AA is based on

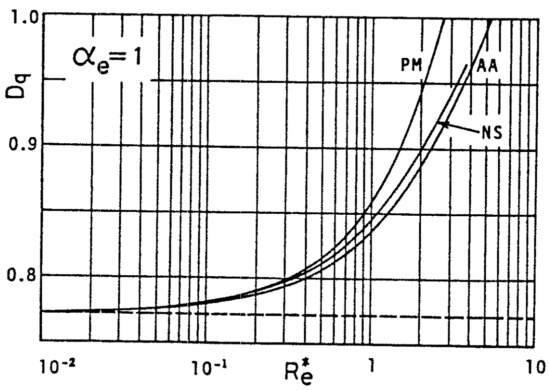


Fig.6 Comparison of approximate solutions for steady drag coefficient

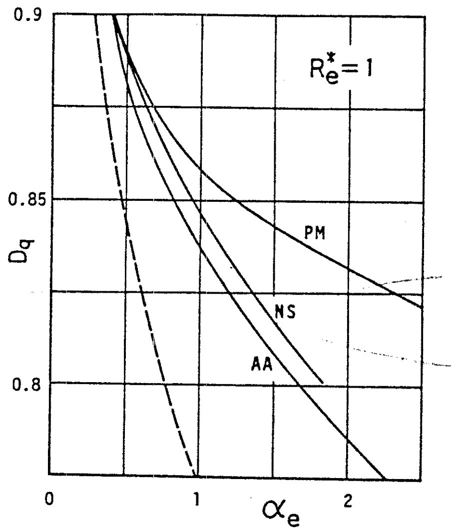


Fig.7 Comparison of approximate solutions for steady drag coefficient

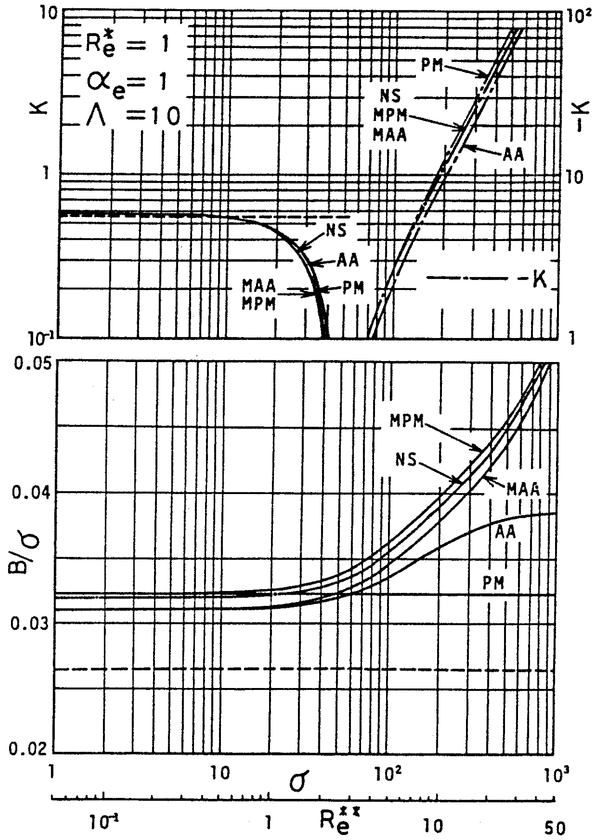


Fig.8 Comparison of approximate solutions for dynamic stiffness and damping coefficient

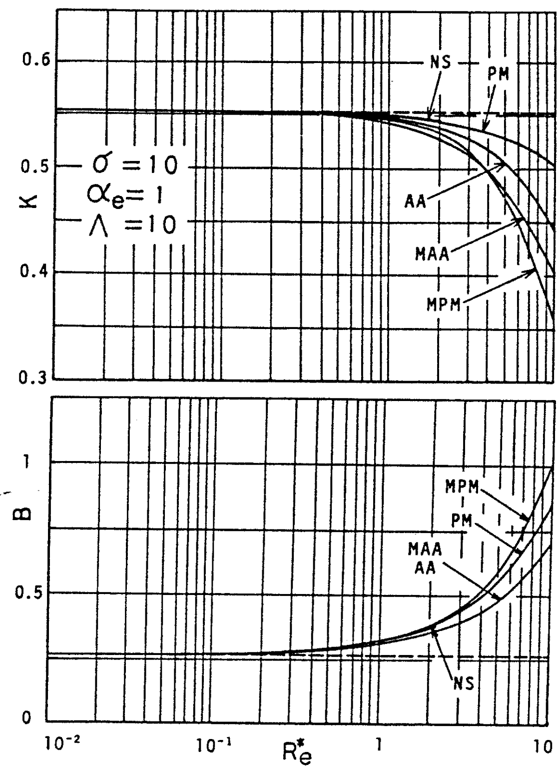


Fig.9 Comparison of approximate solutions for dynamic stiffness and damping coefficient

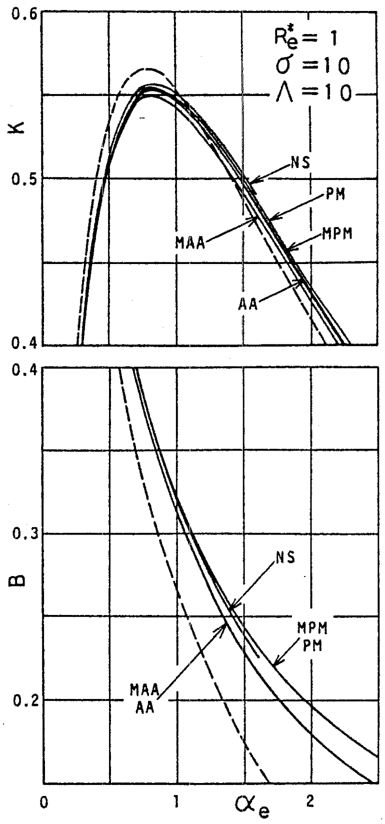


Fig.10 Comparison of approximate solutions for dynamic stiffness and damping coefficient

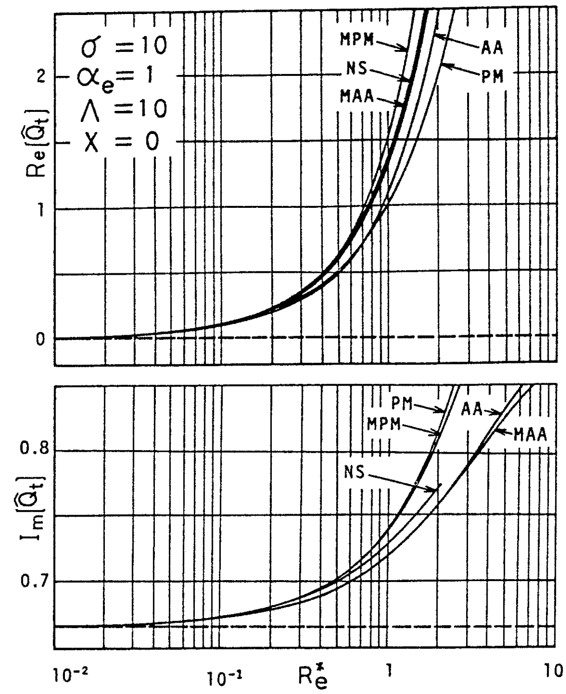


Fig.12 Comparison of approximate solutions for dynamical flow rate

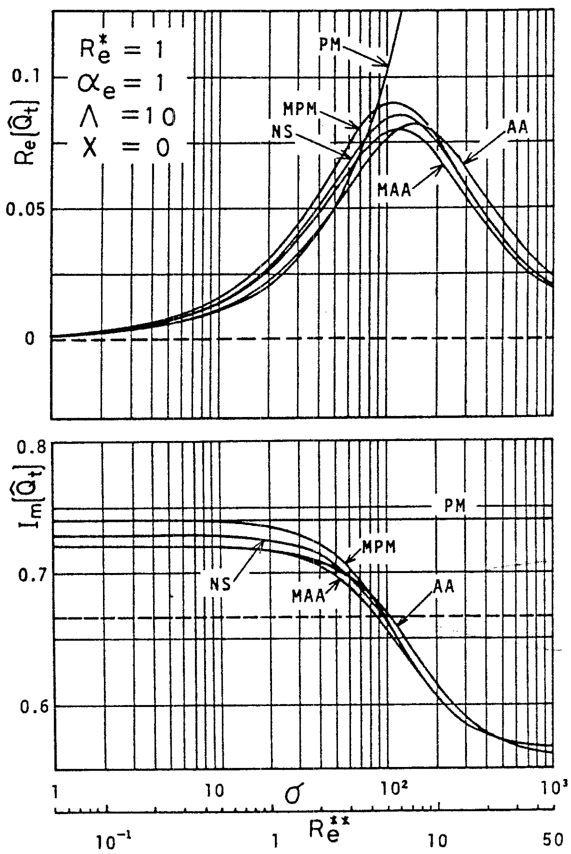


Fig.11 Comparison of approximate solutions for dynamical flow rate

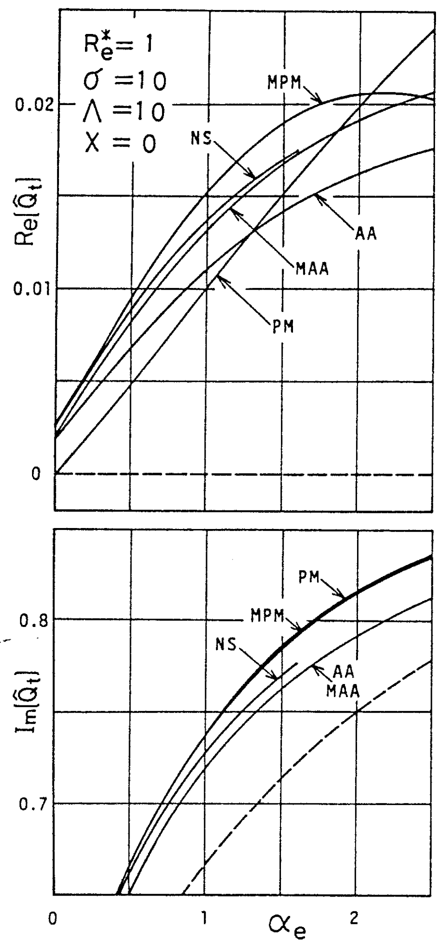


Fig.13 Comparison of approximate solutions for dynamical flow rate

the assumption of a parabolic velocity profile, and this would, therefore, be invalid when the value of $Re^{**}$ becomes large.

The above discussions show that the averaging approach is useful for analyses of the dynamic stiffness and damping ability except under the condition of $Re^{**} > 10$, though it is faulty in the analysis of the velocity field. The modified averaging approach is found much more useful. This holds for any value of $Re^{**}$.

## 7. Conclusions

A modified perturbation method and a modified averaging approach were presented for investigation of the dynamic performance of an infinitely wide, plane inclined slider pad including the fluid inertial effects. The solution was obtained under the assumption of a small harmonic vibration. From comparison of various approximate solutions with the numerical solutions, the following conclusions have been obtained:

(1) As for the static performance of load capacity, flow rate and drag coefficient, there is no significant difference between

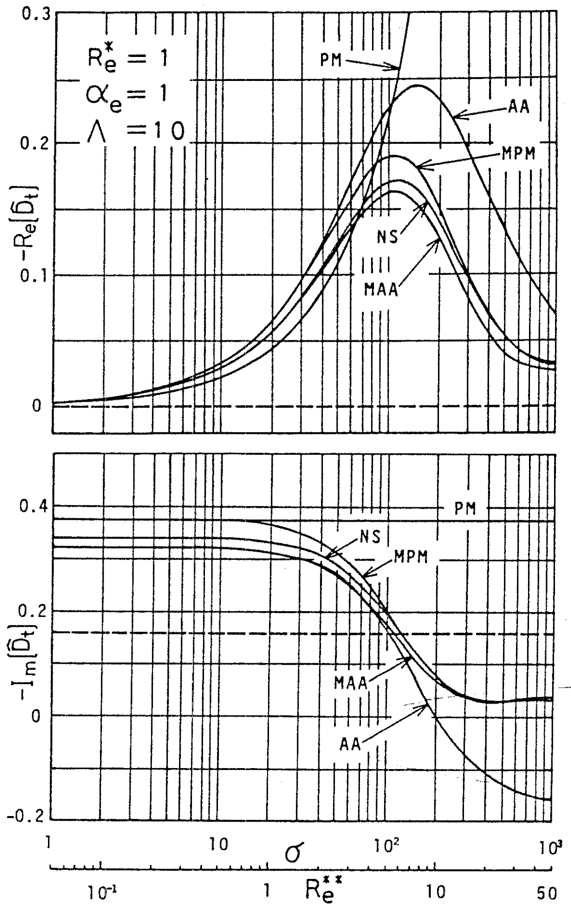


Fig.14 Comparison of approximate solutions for dynamical drag coefficient

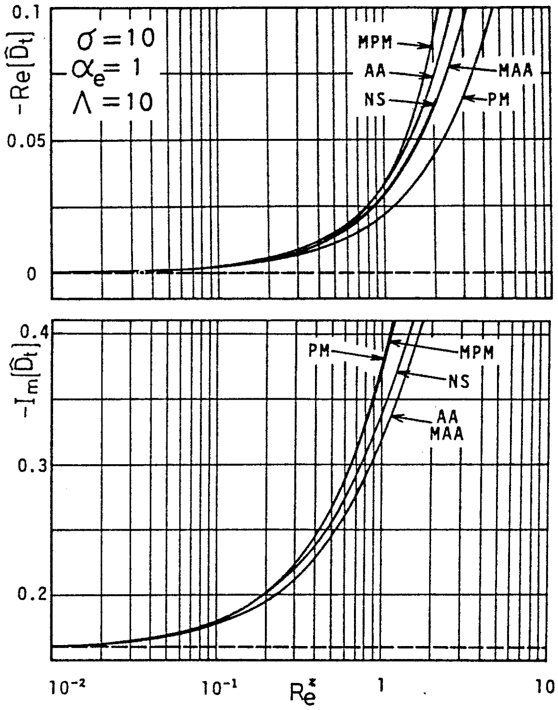


Fig.15 Comparison of approximate solutions for dynamical drag coefficient

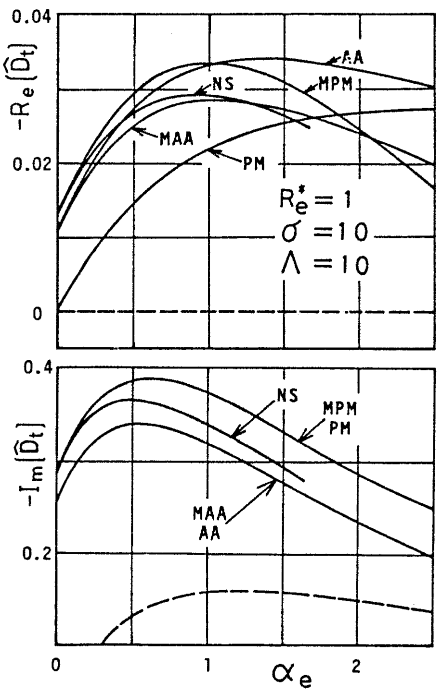


Fig.16 Comparison of approximate solutions for dynamical drag coefficient

the perturbation method and the averaging approach.

(2) The modified averaging approach, which is proposed in this paper, is simple and accurate.

(3) The averaging approach is relatively accurate except for high unsteadiness.

(4) The 1st order solution by the perturbation method becomes remarkably inaccurate as the unsteadiness becomes high.

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