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Influence of the Fluid Inertia Forces on the Dynamic Characteristics of Externally Pressurized Thrust Bearings*

(2nd Report, Evaluation of Various Approximate Solutions for the Influence of Film Inertia Forces on the Dynamic Performance of Externally Pressurized Infinitely Long Thrust Bearings)

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Based on the Navier-Stokes equations in which the pressure is assumed to be constant across the film thickness, various approximate solutions and an exact one for the dynamic performance of externally pressurized infiniteley long thrust bearings in a laminar flow regime are presented under the assumption of a small harmonic vibration.

From comparison of the approximate solutions with the exact one, it is concluded that some kind of averaging approach in which a part of the time dependent term is treated exactly while the other inertia terms are averaged out across the film thickness gives close approximations in a wide range of designing conditions, and that the other kind of averaging approach in which all the inertia terms including the time dependent term are averaged out across the film thickness gives fairly good approximations.

Key Words : Lubrication, Bearing, Externally Pressurized Thrust Bearing, Inertia Effect, Dynamic Performance

1. Introduction

A significant influence of the fluid inertia forces on the dynamic performance has been noted with the externally pressurized bearings[1],and there are written many papers discussing the inertia effects in hydrodynamic lubrication[2~4,7]. In the first report[7], we investigated the influence of the fluid inertia forces generated within the supply capillary restrictors on the dynamic characteristics of externally pressurized thrust bearings and pointed out that the influence must be considerable when the low kinematic viscosity of the lubricant is used. The present report is concerned with the influence of the film inertia forces on the dynamic performance of externally pressurized infinitely long thrust bearings.

It is difficult to solve exactly the problem of the dynamic performance of bearings including the fluid inertial effects because the inertia terms in the momentum equations are nonlinear. Then, various approximate solutions have been introduced.

In this paper, an exact solution for the dynamic performance of externally pressurized infinitely long thrust bearings is presented under the assumption of a small harmonic vibration. Subsequently, various approximate solutions are compared with the above one to evaluate their accuracy.

2. Nomenclature

$A = \mu \bar{u} l / (p_a h_0^2)$

$A^* = \mu \omega l^2 / (p_a h_0^2)$

b, B : damping coefficient

C_D : discharge coefficient

d_s : diameter of supply hole

f_{0s}, F_{0s} : dynamical bearing reaction force per unit width

$g \equiv \tilde{n} \pi d_s / (2\Psi)$

h, H : bearing clearance

h_0 : equilibrium clearance

k, K : stiffnesses

$2l$: bearing width

$\tilde{n}$: number of supply holes per unit width

p, P : pressures

p_a : ambient pressure

p_l, P_l : pressures at feeding section

p_n, P_n : supply pressures

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$Re = \bar{u}h_0^2/(\nu l)$: inertia parameter
$Re^* = \omega h_0^2/\nu$: unsteadiness parameter
t : time
u, U : velocity components in x direction
$\bar{u}$: characteristic velocity
w, W : velocity components in z direction
x, X : coordinates along the film
z, Z : coordinates across the film
$\Gamma = 12\,gC_D\sqrt{2/\rho}\ \mu l/(\sqrt{p_a}\,h_0^2)$: feeding parameter
μ : viscosity
ν : kinematic viscosity
ρ : density
$\sigma = 12\mu\omega/p_a (l/h_0)^2$: squeeze number
$\tau = \omega t$
$\phi = \sqrt{jRe^*}\,/\,2$
$\Phi = 1 - \phi^{-1}\tanh\phi$
Ψ : correction factor for dispersion effect
ω : circular frequency of squeeze motion

Subscripts

q : quasi-statical component
s : statical component
t : dynamical component
$\frown$: amplitude

In the case of two variables, the former is a dimensional quantity and the latter is a dimensionless one.

3. Governing Equations and Boundary Conditions

An inherently compensated externally pressurized infinitely long thrust bearing with multiple holes admission analyzed here is shown schematically in Fig.1, in which some symbols used are also noted. The thrust plate is assumed to be parallel to the bearing surface. The number of supply holes is assumed to be large enough for the line source concept to be applied. With these assumptions and the usual assumptions of fluid-film lubrication theory the momentum equations and the continuity equation for an incompressible fluid are

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial z^2} \qquad (1)$$

$$0=\frac{\partial p}{\partial z} \qquad (2)$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \qquad (3)$$

where u and w are the velocity components in the x and z directions, p is the pressure, ρ the density, μ the viscosity and t the time.

The boundary conditions are

$$z=0:\ \ u=w=0,\ \ z=h: u=0,\ \ w=\frac{dh}{dt}$$
$$x=0:\ \ p=p_l\ ,\quad x=l:\ \ p=p_a \qquad (4)$$

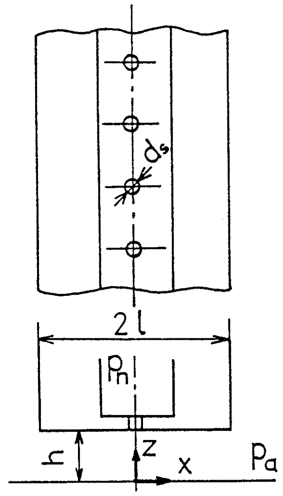

Fig.1 Configuration and coordinates

where p_a is the ambient pressure and p_l is the pressure at feeding section which is determined from the continuity condition of flow rate at feeding section as

$$\int_0^h u|_{x=0}\,dz = gC_D h\sqrt{\frac{2}{\rho}(p_n-p_l)} \qquad (5)$$

where

$$g\equiv\frac{\tilde{n}\pi d_s}{2\Psi} \qquad (6)$$

and $\tilde{n}$ is the number of supply holes per unit width, d_s the diameter of supply hole and Ψ the correction factor for the dispersion effect of a discrete source. In the case of complete line source, $g = 1$.

It is difficult to obtain general exact solutions to the above equations. Then, various approximate solutions have been introduced.

4. Various Approximate Solutions

4.1 Solution 1 In this solution, the velocity and the pressure are perturbed with respect to $Re = \rho h_0^2\,\bar{u}\,/(\mu l)$ as

$$(u, w, p) = (u_0, w_0, p_0) + Re\,(u_1, w_1, p_1) + \qquad (7)$$

Substituting Eq.(7) into Eqs.(1) to (4), we obtain the fundamental equations and the boundary conditions. Where $\bar{u}$ is a characteristic velocity which is obtained theoretically from the mean steady velocity in the x direction. In general, the solutions for second and higher orders of Re are omitted because these are very complicated. Mori, et al.[4] pointed out that the high order solutions diverge when the unsteadiness parameter, Re^*, exceeds a certain value.

4.2 Solution 2 In this approach, the solutions are analyzed by averaging out all the inertia terms in the momentum equations across the film thickness. Namely, Eq.(1)

is replaced with

$$\frac{\rho}{h}\int_0^h\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)dz=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial z^2} \tag{8}$$

4.3 Solution 3 This solution has been proposed by Murata, et al.[3] for the dynamic performance of externally pressurized circular thrust bearings. In this solution, a part of the time dependent term, $\partial u_t/\partial t$, is treated exactly.

Since the velocity and the pressure are regarded as the sums of statical components (subscript s) and dynamical ones (subscript t), we can write them as

$$(u, w, p)=(u_s, w_s, p_s)+(u_t, w_t, p_t) \tag{9}$$

Next, we assume a small harmonic variation in the film thickness as follows;

$$h=h_0+h_1e^{j\omega t} \tag{10}$$

Corresponding to this, the dynamical components of velocity and pressure are given by

$$(u_t, w_t, p_t)=(\hat{u}_t, \hat{w}_t, \hat{p}_t)e^{j\omega t} \tag{11}$$

Introducing the following dimensionless quantities

$$X=\frac{x}{l},\ Z=\frac{z}{h_0},\ U_s=\frac{u_s}{\bar{u}},\quad W_s=\frac{w_s}{\bar{u}}\left(\frac{l}{h_0}\right),$$

$$P_s=\frac{p_s}{p_a},\ H=\frac{h}{h_0},\ A=\frac{\mu\bar{u}l}{p_a h_0^2},\ R_e=\frac{\bar{u}h_0}{\nu}\left(\frac{h_0}{l}\right),$$

$$\hat{U}_t=\frac{\hat{u}_t}{\varepsilon\omega l},\ W_t=\frac{\hat{w}_t}{\varepsilon\omega h_0},\ \hat{P}_t=\frac{\hat{p}_t}{\varepsilon p_a},$$

$$A^*=\frac{\mu\omega l^2}{p_a h_0^2},\ R_e^*=\frac{\omega h_0^2}{\nu}\ ,\ \varepsilon=\frac{h_1}{h_0}$$

we can formulate the following governing differential equations and boundary conditions

(For statical components)

$$R_e\left(U_s\frac{\partial U_s}{\partial X}+W_s\frac{\partial U_s}{\partial Z}\right)=-\frac{1}{A}\frac{\partial P_s}{\partial X}+\frac{\partial^2 U_s}{\partial Z^2} \tag{12}$$

$$0=\frac{\partial P_s}{\partial Z} \tag{13}$$

$$\frac{\partial U_s}{\partial X}+\frac{\partial W_s}{\partial Z}=0 \tag{14}$$

Boundary conditions

$$\begin{aligned} Z&=0, H &:&\ U_s=W_s=0\\ X&=0 &:&\ P_s=P_{ls}\\ X&=1 &:&\ P_s=1 \end{aligned} \tag{15}$$

where $P_{ls}=P|_{H=1}$

(For dynamical components)

$$jR_e^*\hat{U}_t+R_e\left[U_s\frac{\partial \hat{U}_t}{\partial X}+\hat{U}_t\frac{\partial U_s}{\partial X}+W_s\frac{\partial \hat{U}_t}{\partial Z}+\hat{W}_t\frac{\partial U_s}{\partial Z}\right]_{H=1}=-\frac{1}{A^*}\frac{\partial \hat{P}_t}{\partial X}+\frac{\partial^2 \hat{U}_t}{\partial Z^2} \tag{16}$$

$$0=\frac{\partial \hat{P}_t}{\partial Z} \tag{17}$$

$$\frac{\partial \hat{U}_t}{\partial X}+\frac{\partial \hat{W}_t}{\partial Z}=0 \tag{18}$$

Boundary conditions

$$\begin{aligned} &Z=0\ :\ \hat{U}_t=\hat{W}_t=0\ ,\quad Z=1:\ \hat{U}_t=0,\ \ \hat{W}_t=j\\ &X=0\ :\ \hat{P}_t=\hat{P}_{lt}\ ,\quad X=1\ :\ \hat{P}_t=0 \end{aligned} \tag{19}$$

The analytical solutions of the above equations are obtained as follows;

$$U_s=\frac{P_{ls}-1}{2A}(HZ-Z^2) \tag{20}$$

$$W_s=0 \tag{21}$$

$$P_s=P_{ls}-(P_{ls}-1)X \tag{22}$$

$$\hat{U}_t=\left\{\frac{j}{2\Phi}(1-2X)-\frac{j\hat{P}_{lt}}{A^*R_e^*}\right\}\times\left\{1-\frac{\cosh(2\varphi Z-\varphi)}{\cosh\varphi}\right\}-\frac{6jR_e}{\Phi}\{F_1(Z)+F_2(Z)\} \tag{23}$$

$$\hat{W}_t=\frac{j}{\Phi}\left\{Z-\frac{\sinh(2\varphi Z-\varphi)+\sinh\varphi}{2\varphi\cosh\varphi}\right\} \tag{24}$$

$$\hat{P}_t=\hat{P}_{lt}(1-X)+\frac{A^*R_e^*}{2\Phi}(X-X^2) \tag{25}$$

where $\varphi=\sqrt{jR_e^*}/2$, $\Phi=1-\varphi^{-1}\tanh\varphi$ and $F_1(Z), F_2(Z)$ are functions of Z which can be determined by the following ordinary differential equations and the boundary conditions

$$\frac{d^2F_1}{dZ^2}-4\varphi^2F_1=(Z-Z^2)\times\left\{1-\frac{\cosh(2\varphi Z-\varphi)}{\cosh\varphi}\right\} \tag{26}$$

$$\frac{d^2F_2}{dZ^2}-4\varphi^2F_2=(2Z-1)\times\left\{Z-\frac{\sinh(2\varphi Z-\varphi)+\sinh\varphi}{2\varphi\cosh\varphi}\right\} \tag{27}$$

$$Z=0, 1\ :\ F_1=F_2=0 \tag{28}$$

They are solved as

$$\begin{aligned} F_1=&-\frac{1}{16\varphi}\left(\frac{1}{4\varphi^2}-\frac{1}{3}\right)\frac{\sinh(2\varphi Z)}{\cosh^2\varphi}\\ &+\frac{1}{4\varphi}\left(\frac{Z^3}{3}-\frac{Z^2}{2}+\frac{Z}{8\varphi^2}\right)\frac{\sinh(2\varphi Z-\varphi)}{\cosh\varphi}\\ &-\frac{1}{16\varphi^2}\left(Z^2-Z+\frac{2}{\varphi^2}\right)\frac{\cosh(2\varphi Z-\varphi)}{\cosh\varphi}\\ &+\frac{1}{4\varphi^2}\left(Z^2-Z+\frac{1}{2\varphi^2}\right) \end{aligned} \tag{29}$$

$$F_2=-\frac{3\sinh(2\varphi Z)+2\sinh(2\varphi Z-\varphi)}{32\varphi^3\cosh^2\varphi}+\frac{Z\sinh(2\varphi Z-\varphi)}{16\varphi^3\cosh\varphi}-\frac{1}{8\varphi^2}\left(Z^2-Z-\frac{2}{\varphi^2}\right)\frac{\cosh(2\varphi Z-\varphi)}{\cosh\varphi}+\frac{\sinh(2\varphi Z)}{4\varphi^2\sinh(2\varphi)}+\frac{1}{8\varphi^3}(2Z-1)\tanh\varphi-\frac{1}{4\varphi^2}\left(2Z^2-Z+\frac{1}{\varphi^2}\right) \quad (30)$$

4.4 Solution 4 In the solution 3, a part of the time dependent term, $\partial u_S/\partial t$, is neglected, but we have $\partial u_S/\partial t \neq 0$. In the solution 4, an exact solution is obtained including the term, $\partial u_S/\partial t$.

Since the velocity and the pressure are regarded as the sums of quasi-statical components (subscript q) and of dynamical ones (subscript t), we can write them as

$$(u, w, p)=(u_q, w_q, p_q)+(u_t, w_t, p_t) \quad (31)$$

The following fundamental equations and the boundary conditions are obtained under the assumption of a small harmonic vibration.

(For quasi-statical components)

$$R_e\left(U_q\frac{\partial U_q}{\partial X}+W_q\frac{\partial U_q}{\partial Z}\right)=-\frac{1}{A}\frac{\partial P_q}{\partial X}+\frac{\partial^2 U_q}{\partial Z^2} \quad (32)$$

$$0=\frac{\partial P_q}{\partial Z} \quad (33)$$

$$\frac{\partial U_q}{\partial X}+\frac{\partial W_q}{\partial Z}=0 \quad (34)$$

Boundary conditions

$$Z=0, H \;:\; U_q=W_q=0$$
$$X=0 \;:\; P_q=P_{lq} \quad (35)$$
$$X=1 \;:\; P_q=1$$

where P_{lq} is the quasi-statical pressure at feeding section.

(For dynamical components)

$$jR_e\frac{\partial U_q}{\partial H}\bigg|_{H=1}+jR_e^*\hat{U}_t+R_e\left[U_q\frac{\partial \hat{U}_t}{\partial X}+\hat{U}_t\frac{\partial U_q}{\partial X}+W_q\frac{\partial \hat{U}_t}{\partial Z}+\hat{W}_t\frac{\partial U_q}{\partial Z}\right]_{H=1}=-\frac{1}{A^*}\frac{\partial \hat{P}_t}{\partial X}+\frac{\partial^2\hat{U}_t}{\partial Z^2} \quad (36)$$

$$0=\frac{\partial \hat{P}_t}{\partial Z} \quad (37)$$

$$\frac{\partial \hat{U}_t}{\partial X}+\frac{\partial \hat{W}_t}{\partial Z}=0 \quad (38)$$

Boundary conditions

$$Z=0 \;:\; \hat{U}_t=\hat{W}_t=0\;,\quad Z=1 \;:\; \hat{U}_t=0,\quad \hat{W}_t=j$$
$$X=0 \;:\; \hat{P}_t=\hat{P}_{lt}\;,\quad X=1 \;:\; \hat{P}_t=0 \quad (39)$$

The first term of the left-hand side of Eq. (36) is obtained from $\partial u_q/\partial t$.

The exact solutions to the above equations are obtained as follows;

$$U_q=\frac{P_{lq}-1}{2A}(HZ-Z^2) \quad (40)$$

$$W_q=0 \quad (41)$$

$$P_q=P_{lq}-(P_{lq}-1)X \quad (42)$$

$$\hat{U}_t=\left\{\frac{j}{2\Phi}(1-2X)-\frac{j\hat{P}_{lt}}{A^*R_e^*}\right\}\times\left\{1-\frac{\cosh(2\varphi Z-\varphi)}{\cosh\varphi}\right\}-\frac{6jR_e}{\Phi}\{F_1(Z)+F_2(Z)\}+6jR_eF_3(Z)-\frac{jR_e}{2A}\frac{dP_{lq}}{dH}\bigg|_{H=1}F_4(Z) \quad (43)$$

$$\hat{W}_t=\frac{j}{\Phi}\left\{Z-\frac{\sinh(2\varphi Z-\varphi)+\sinh\varphi}{2\varphi\cosh\varphi}\right\} \quad (44)$$

$$\hat{P}_t=\hat{P}_{lt}(1-X)+\frac{A^*R_e^*}{2\Phi}(X-X^2) \quad (45)$$

where $F_3(Z)$ and $F_4(Z)$ are functions of Z which can be determined by the following differential equations and the boundary conditions

$$\frac{d^2F_3}{dZ^2}-4\varphi^2F_3=Z \quad (46)$$

$$\frac{d^2F_4}{dZ^2}-4\varphi^2F_4=Z^2-Z \quad (47)$$

$$Z=0,1 \;:\; F_3=F_4=0 \quad (48)$$

They are solved as

$$F_3=\frac{1}{4\varphi^2}\left\{\frac{\sinh(2\varphi Z)}{\sinh(2\varphi)}-Z\right\} \quad (49)$$

$$F_4=\frac{\cosh(2\varphi Z-\varphi)}{8\varphi^4\cosh\varphi}-\frac{1}{4\varphi^2}\left(Z^2-Z+\frac{1}{2\varphi^2}\right) \quad (50)$$

4.5 Solution 5 In this approach, the solutions are obtained by averaging out the terms containing the inertia parameter, Re, in the governing equations (32) and (36) across the film thickness. Namely, Eqs. (32) and (36) are replaced with

$$\frac{R_e}{H}\int_0^H\left(U_q\frac{\partial U_q}{\partial X}+W_q\frac{\partial U_q}{\partial Z}\right)dZ=-\frac{1}{A}\frac{\partial P_q}{\partial X}+\frac{\partial^2 U_q}{\partial Z^2} \quad (51)$$

$$jR_e^*\hat{U}_t+R_e\int_0^1\left[j\frac{\partial U_q}{\partial H}+U_q\frac{\partial \hat{U}_t}{\partial X}+\hat{U}_t\frac{\partial U_q}{\partial X}+W_q\frac{\partial \hat{U}_t}{\partial Z}+\hat{W}_t\frac{\partial U_q}{\partial Z}\right]_{H=1}dZ$$

$$= -\frac{1}{A^*}\frac{\partial \hat{P}_t}{\partial X} + \frac{\partial^2 \hat{U}_t}{\partial Z^2} \qquad (52)$$

The solutions are given by

$$U_q = \frac{P_{tq}-1}{2A}(HZ - Z^2) \qquad (53)$$

$$W_q = 0 \qquad (54)$$

$$P_q = P_{tq} - (P_{tq}-1)X \qquad (55)$$

$$\hat{U}_t = \left\{\frac{j}{2\Phi}(1-2X) - \frac{j\hat{P}_{lt}}{A^* R_e^*} - C\right\} \times \left\{1 - \frac{\cosh(2\varphi Z - \varphi)}{\cosh \varphi}\right\} \qquad (56)$$

$$\hat{W}_t = \frac{j}{\Phi}\left\{Z - \frac{\sinh(2\varphi Z - \varphi) + \sinh \varphi}{2\varphi \cosh \varphi}\right\} \qquad (57)$$

$$\hat{P}_t = \hat{P}_{lt}(1-X) + \frac{A^* R_e^*}{2\Phi}(X - X^2) \qquad (58)$$

where

$$C = \frac{R_e}{R_e^*}\left\{\frac{6}{\Phi}\left(\frac{1}{\varphi^2} - \frac{1}{\varphi^3}\tanh \varphi - \frac{1}{3}\right) + 3 + \frac{1}{12A}\left.\frac{dP_{tq}}{dH}\right|_{H=1}\right\} \qquad (59)$$

These solutions are simple compared with the solution 4.

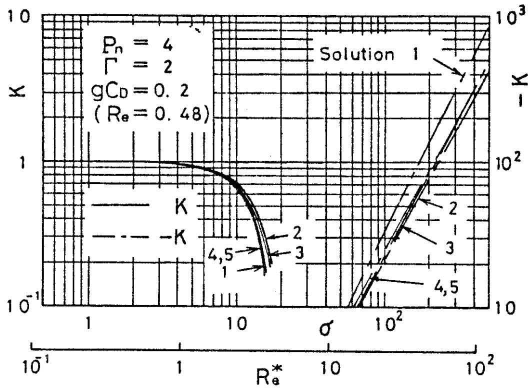

Fig.2 Comparison of approximate solutions, dynamic stiffness

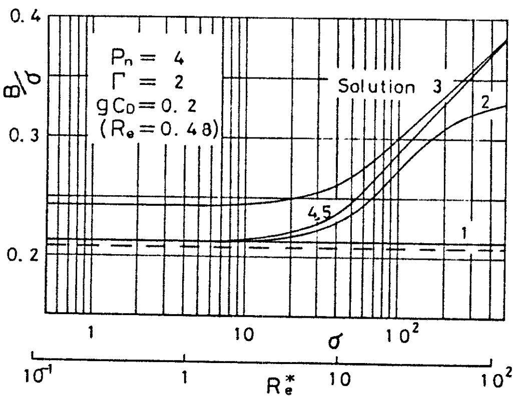

Fig.3 Comparison of approximate solutions, damping coefficient

5. Dynamic Stiffness and Damping Coefficient

The dynamic stiffness and the damping coefficient are respectively defined as the component of the bearing reaction force in the same phase as the displacement and the component in the same phase as the velocity of the displacement. The dimensionless dynamic stiffness and the dimensionless damping coefficient normalized by $2p_a l / h_0$ and $2p_a l / (h_0 \omega)$ are respectively given by

$$K = -\mathrm{Re}[\hat{F}_{0s}] \qquad (60)$$

$$B = -\mathrm{Im}[\hat{F}_{0s}] \qquad (61)$$

where $\hat{F}_{0s} = \hat{f}_{0s} / (2\varepsilon p_a l)$, and the real part of $\hat{f}_{0s} e^{j\omega t}$ represents the bearing reaction force per unit width.

K and B can be derived using the three conventional dimensionless designing parameters: $P_n = p_n / p_a$ (the dimensionless supply pressure), $\Gamma = 12 g C_D \sqrt{2/\rho}\, \mu l / (\sqrt{p_a} h_0^2)$ (the feeding parameter) and $\sigma = 12 \mu \omega l^2 / (p_a h_0^2)$ (the squeeze number) and a new dimensionless parameter: $g C_D$.

The inertia parameter, Re, and the unsteadiness parameter, Re^*, are related to those parameters by the following equations

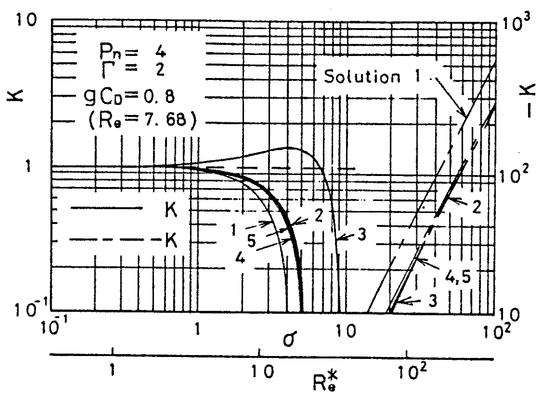

Fig.4 Comparison of approximate solutions, dynamic stiffness

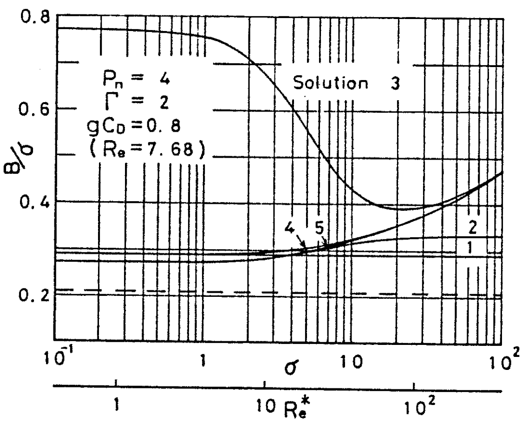

Fig.5 Comparison of approximate solutions, damping coefficient

$$R_e=\frac{12}{\Gamma}(\sqrt{\Gamma^2+4(P_n-1)}-\Gamma)(gC_D)^2 \qquad (62)$$

$$R_e^*=\frac{24}{\Gamma^2}(gC_D)^2\sigma \qquad (63)$$

6. Comparison of Various Approximate Solutions With Exact One

The soltion 4 is analyzed exactly under the assumption of a small harmonic vibration, therefore it is valid fore any values of *Re* and *Re**. This shall represent an exact solution in the present work. The solution 3 also is valid for any values of *Re* and *Re**. In this solution, however, the term, $\partial u/\partial t$, is not treated exactly. The solution 1 and the the solution 2 are valid for $Re \ll 1$ and for $Re^* \ll 1$ in principle. The solution 5 is valid for $Re \ll 1$ and for any value of *Re**. In this chapter, those approximate solutions will be compared with the solution 4, an exact one, to evaluate their accuracy.

The bearing performances for $P_n = 4$, $gC_D = 0.2$ and 0.8 are shown in Figs.2 to 9. In this figuers, the broken lines represent the results obtained by neglecting the inertia effect.

The results by the solution 3 and the solution 4 must approach the result by the solution 1 when $Re^* \to 0$ ($\sigma \to 0$). In Figs.3 and 5, the result of *B* by the solution 4 approaches the result of *B* by the solution 1, but the result by the solution 3 does not. This may be owing to an incomplete treatment of the time dependent term, $\partial u/\partial t$, in the solution 3. When the value of *Re** tends to zero, there is some difference between the results by the perturbation method (the solution 1 and the solution 4) and the averaging approach (the solution 2 and the solution 5), but the difference is insignificant. Figs.6 to 9 show the results for the feeding parameter, Γ. In these figures, the solution 5 gives close approximations in a wide range of designing conditions and the solution 2 gives fairly good approximations except for high unsteadiness. The solution 1 becomes remarkably inaccurate as the unsteadiness becomes high.

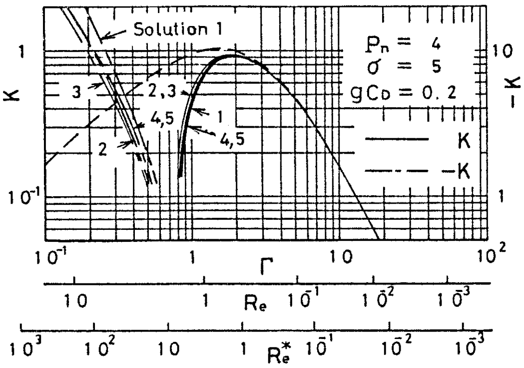

Fig.6 Comparison of approximate solutions, dynamic stiffness

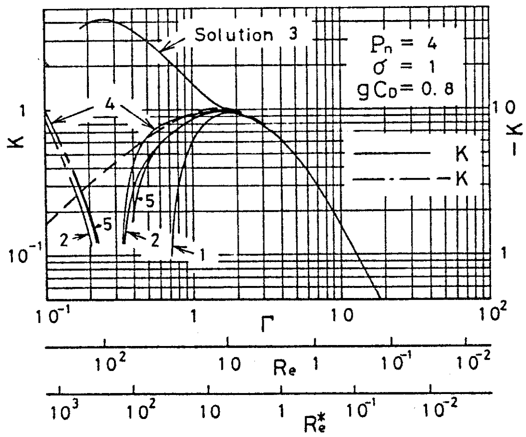

Fig.8 Comparison of approximate solutions, dynamic stiffness

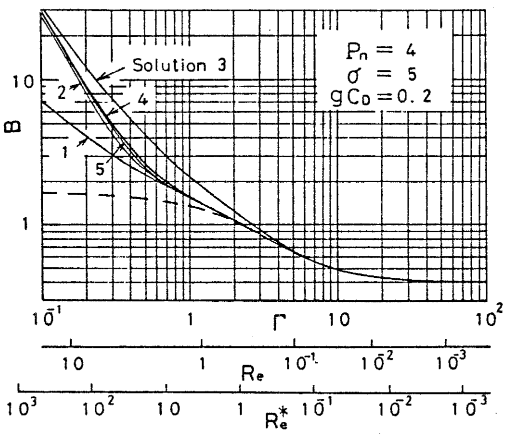

Fig.7 Comparison of approximate solutions, damping coefficient

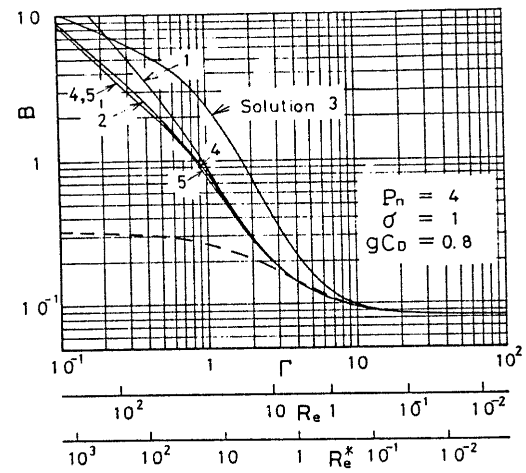

Fig.9 Comparison of approximate solutions, damping coefficient

The above discussions show that the solution 2 is useful for analyses of the dynamic stiffness and the damping coefficient if the values of Re and σ are not too large. The solution 5 is much more useful. This is accurate for any value of Re^*.

7. Conclusions

An exact solution was presented for the dynamic performance of externally pressurized infinitely long thrust bearings including the fluid inertia effects. The solution was obtained under the assumption of a small harmonic vibration. From comparison of various approximate solutions with the above one, the following conclusions have been obtained:

(1) The solution 5 in which a part of the time dependent term is treated exactly while the other inertia terms are averaged out across the film thickness gives close approximations in a wide range of designing conditions.

(2) The solution 2 in which all the inertia terms including the time dependent term are averaged out across the film thickness gives fairly good approximations except high unsteadiness.

(3) The solution 1 which is the 1st-order solution of the perturbation method becomes remarkably inaccurate as the unsteadiness becomes high.

(4) The solution 3 does not give good approximations except high unsteadiness because the time dependent term, $\partial u/\partial t$, is treated incompletely.

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