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Estimation of Corrosion Fatigue Lives Based on the Variations of the Crack Lengths Distributions During Stress Cycling+

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Many small distributed cracks have been observed on the specimen during corrosion fatigue process, and the damage of corrosion fatigue is related to the behaviour of these distributed cracks. The distribution of crack lengths during corrosion fatigue was approximated well by the three parameter Weibull distribution under plane-bending fatigue tests of carbon steel in salt water.

A method of estimation of corrosion fatigue lives was proposed. The crack initiation, crack growth behaviour and the variation of the distribution of crack lengths during corrosion fatigue process were taken into consideration in the method. Comparing the estimated results with experimental results, it was concluded that this method could estimate the corrosion fatigue lives with good accuracy.

Key Words : Fatigue, Corrosion Fatigue, Distribution of crack lengths, Estimation of corrosion fatigue lives, Weibull Distribution

1. Introduction

Fracture process characterized by the initiation and propagation of distributed cracks has been frequently observed in corrosion fatigue,thermal fatigue and low-cycle fatigue. The evaluation method of the fracture process mentioned above is now an urgent problem to be solved because safety maintenance of the machines and structures which operate in corrosive environments is strongly demanded.

It was pointed out by the recent studies [1-4] that fatigue damage is associated with the decay of the surface layer of the specimen during fatigue process, that is, the initiation and growth behaviours of the micro cracks on the specimen surface, and also reported that [4] Manson-Coffin's law is a different representation of the growth law of the micro crack. Especially in corrosion fatigue, many distributed cracks are initiated in the early stage of fatigue life , and the greater part of the fatigue life is occupied by the crack growth period of these distributed cracks. Therefore,it may be important to estimate fatigue lives under corrosive environment by considering the fatigue damage substantially as the initiation and growth behaviours of the distributed cracks.

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The authors have studied [5-7] the corrosion fatigue behaviours of a smooth specimen by setting eyes to the initiation and growth behaviours of the distributed cracks on the specimen surface. From these studies, it was made clear that corrosion fatigue lives are strongly affected by the initiation and growth of the distributed cracks, and that interaction and coalescence behaviours of these distributed cracks must be taken into consideration. Crack lengths which appeared on the smooth specimen surface showed a tendency of statistical variation. The distribution of crack lengths for certain stress cycles and stress amplitudes could be obtained by the statistical calculation which took into account both initiation and growth behaviours of the distributed cracks.

In this study, S-N curves and distribution of fatigue lives will be obtained by regarding the distributed cracks substantially as corrosion fatigue damage,and further the physical methods will be discussed from the probabilistic and statistical viewpoint. The distribution of crack lengths stated in the previous paper [6][7] will be approximated by a three-parameter Weibull distributions in order to consider the variations of these distributions during fatigue process quantitatively. Using the dependence of three parameters of these distributions on stress cycles and stress amplitudes, it is possible to represent S-N curve quantitatively and to obtain the distribution of fatigue lives. Experimental examination was conducted and S-N curves and distribution of fatigue lives obtained theoretically were found in good agreement with those obtained experimentally. To clarify the probabilistic or statistical methods for the S-N curves and distribution of fatigue lives may be important to accomplish a reliable design method.

2. Main Symbols

Symbols used in this study are the same ones used in the previous studies.[6][7] New symbols used in this study are given as follows:

m_0 : Shape parameter of the three-parameter Weibull distribution
α_0 : Scale parameter of the three-parameter Weibull distribution
γ_0 : Location parameter of the three-parameter Weibull distribution
n_t : Total number of cracks
Φ : The area of the specimen surface
K_{fc} : The value of the fatigue fracture toughness
$2\ell_c$: Critical crack length
$\upsilon(2\ell \geqq 2\ell_c)$: The number of cracks which are longer than $2\ell_c$
$\Phi n(2\ell_{max})$: The finite distribution of maximum crack lengths
$\Psi(2\ell_{max})$: The asymptotic distribution of maximum crack lengths
$P_n(N_f)$: The finite distribution of fatigue lives
$P(N_f)$: The asymptotic distribution of fatigue lives

3. Estimation of S-N Curves Using the Distribution of Crack Lengths

3.1 Approximation of the distribution of crack lengths by the three-parameter Weibull distribution

First, statistical calculation method [6][7] for the distribution of crack lengths will be described briefly below:
Crack groups(F) initiated on the specimen surface were divided into two types of cracks: One(designated F_1) propagates as a single crack without interaction with others. Another(designated F_2) shows interaction and connection of closely located surface cracks. The distribution of crack lengths(F) is represented by the mixed-type distribution of $F_1(2\ell)$ and $F_2(2\ell)$ as follows.[6][7]

$$F=p_1F_1+p_2F_2 \quad \cdots\cdots(1)$$

where p_1 and p_2 are probabilities of $F_1(2\ell)$ and $F_2(2\ell)$, respectively. The sum of p_1 and p_2 is unity, that is the following equation holds,

$$p_1+p_2=1 \quad \cdots\cdots(2)$$

By considering the initiation and growth behaviours of the distributed cracks, the distributions of crack lengths F1 and F2 in equation (1) are represented by the following equations [6][7]:

$$F_1(l,\sigma,N)=\frac{\beta(\sigma)}{\sqrt{2\pi}S_k\{1-e^{-\beta(\sigma)(N-N_c)}\}}\times\int_{l_0}^{l}\int_{N_c}^{N}\exp\left[-\frac{\{\log(l^{m_1}-l_0^{m_1})-g-\log(N-N_0)\}}{2S_k^2}-\beta(N_0-N_c)\right]\left|\frac{m_1 l^{m_1-1}}{l^{m_1}-l_0^{m_1}}\right|dN_0\,dl \quad \cdots\cdots(3)$$

where $g=\bar{k}+\log\{m_1(\sigma^2{}_\pi)^{1-m_1}\}$,

$$F_2(2l)=\int_0^{2l}\int_2^{\infty}\frac{1}{\rho}f_1(\rho)f_2\left(\frac{2l}{\rho}\right)d\rho\, d(2l) \quad \cdots\cdots(4)$$

The detailed expressions for $f_1(\rho)$ and $f_2(2\ell/\rho)$ in equation (4) are given in the previous paper. [7]

As equations (1), (3) and (4) which represent the distributions of crack lengths are complicated expressions, it is not easy to examine how the distributions of crack lengths change with the number of stress cycles and stress amplitude. Accordingly, let's approximate the numerically calculated results of equation (1) by the following three-parameter Weibull distribution,

$$F(2l)\fallingdotseq 1-\exp\{-(2l-\gamma_0)^{m_0}/\alpha_0\} \quad \cdots\cdots(5)$$

where m_0, α_0 and γ_0 denote shape parameter, scale parameter and location parameter, respectively. For example, numerically calculated results of equation (1) for σ=127 MPa are shown on the Weibull probability paper of Fig. 1 by the solid lines. The dotted lines represent the ones which are obtained by approximating the numerical results of equation (1) by the three-parameter Weibull distribution of Eq. (5). Approximation by the three-parameter Weibull distribution was conducted by using Sakai and Tanaka's method [8] in which the location parameter is decided such as to make the correlation factor maximal. As seen from this figure,both solid and dotted lines nearly coincide with each other, and so, the distribution of crack lengths can be sufficiently approximated by the three-parameter Weibull distribution.

3.2 Derivation of S-N curves

From the examination in clause 3. 1, it has been made clear that the distribution of crack lengths $F(2\ell)$ represented by Eq.(1) can be approximated by the three-parameter Weibull distribution. That is, the distribution of crack lengths $F(2\ell)$ can be represented by Eq. (5). m_0, α_0 and γ_0 in Eq.(5) are functions of stress amplitudes and number of stress cycles, respectively.

It can be considered a failure of the specimen when maximum crack length ($2\ell_{max}$) among many cracks becomes equal to the critical crack length ($2\ell_c$). Number of cracks, $\upsilon(2\ell \geqq 2\ell_c)$, which are longer than $2\ell_c$ is represented by the following equation,

$$\nu(2l\geqq 2l_c)=n_t[1-F(2l_c,\sigma,N)]=n_t\exp[-\{2l-\gamma_0(\sigma,N)\}^{m_0(\sigma,N)}/\alpha_0(\sigma,N)] \quad \cdots\cdots(6)$$

Only one crack whose length is longer than $2\ell_c$ is enough to cause a failure of the specimen,and so substituting $\upsilon(2\ell \geqq 2\ell_c)=1$ into Eq. (6) and transforming the equation, a final expression for failure condition of the specimen is given by the following equation,

$$\ln\{n_t(\sigma)\}=\frac{\{2l_c-\gamma_0(\sigma,N_f)\}^{m_0(\sigma,N_f)}}{\alpha_0(\sigma,N_f)} \quad \cdots\cdots(7)$$

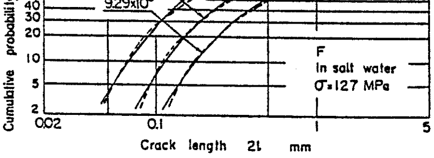

Fig. 1 Approximation of the distribution of crack lengths by the three-parameter Weibull distribution

For the shape parameter, $m_0(\sigma,N)$, scale parameter, $\alpha_0(\sigma,N)$ and location parameter, $\gamma_0(\sigma,N)$ in Eq. (7), the following equations are employed, respectively.

$$\left.\begin{aligned} m_0(\sigma,N) &\fallingdotseq m_0(\sigma)=\zeta\sigma^{\eta} \\ \alpha_0(\sigma,N) &= \zeta\sigma^{\varepsilon}N^{k_3} \\ \gamma_0(\sigma,N) &= \tau\sigma^{\xi}N^{k_4} \end{aligned}\right\} \quad \cdots\cdots(8)$$

(ζ, η, δ. ε, k3, τ, ξ, k4 : constants)
Total number of cracks, n_t, in Eq. (7) is given by the following equation,

$$n_t(\sigma)=\Phi n \quad \cdots\cdots(9)$$

where Φ and n denote the area of the specimen and number of cracks per area, respectively. The value of n in Eq. (9) is given by the following equation [6],

$$n=n_0[1-\exp\{-\beta(N-N_c)\}] \quad \cdots\cdots(10)$$

n_0 and β in above equation have the following relations [6] to stress amplitude, σ,

$$\left.\begin{aligned} \beta &= C_1\exp(C_2\sigma) \\ n_0 &= C_3\exp(C_4\sigma) \end{aligned}\right\} \quad \cdots\cdots(11)$$

(C_1, C_2, C_3, C_4 are constants which are independent of stress amplitude)
Accordingly, substituting Eq. (10) and Eq. (11) into Eq. (9), and further, performing approximation, total number of cracks (n_t) is given by the following equation,

$$n_t(\sigma)\fallingdotseq \Phi n_0=\Phi C_3\exp(C_4\sigma) \quad \cdots\cdots(12)$$

Next, fatigue fracture toughness, K_{fc}, for surface crack is given by the following equation [9],

$$K_{fc}=M_B\sigma\sqrt{\pi A_s\times(2l_c)/Q} \quad \cdots\cdots(13)$$

$$Q=E^2-0.212(\sigma/\sigma_y)^2 \quad \cdots\cdots(14)$$

where A_s, M_B and σ_y denote crack aspect ratio (=a/2ℓ, a: crack depth), correction factor and yield strength of the specimen, respectively. And, E denotes a complete elliptical integral of the second kind and is given by the following equation,

$$E=\int_0^{\pi/2}\{1-(1-4A_s^2)\sin^2\phi\}^{1/2}d\phi \quad \cdots\cdots(15)$$

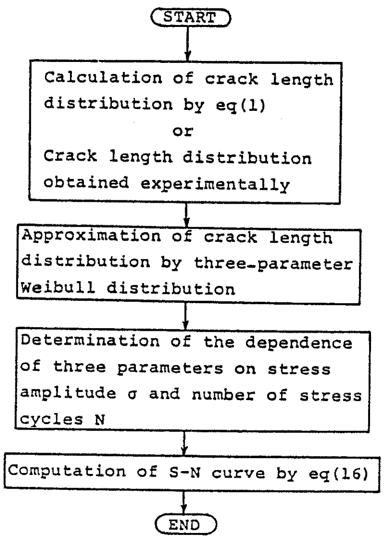

Fig. 2 Flow chart for the estimation of S-N curve

Substituting Eqs. (8), (12) and (13) into Eq.(7), the following expression for S-N curves is obtained.

$$k_3\ln N_f=\zeta\sigma^{\eta}\ln(2l_c-\gamma_0)-\ln(\delta\sigma^{\varepsilon}) -\ln\ln\{\Phi C_3\exp(C_4\sigma)\} \quad \cdots\cdots(16)$$

As seen from the above equation, S-N curve can be represented by the three parameters, m_0, α_0 and γ_0 of the Weibull distribution which describe the variation of the distribution of crack lengths during fatigue process. That is, S-N curve can be obtained by investigating the values of the constants, ζ, η, δ, k3, τ, ξ, k4 and the change of crack density n with stress cycling experimentally. Fig. 2 shows the procedure for the calculation.

4. A Comparison between Estimated S-N Curve and Experimentally Obtained Ones

4.1 Specimens and experimental procedures

The material tested was a low-carbon steel, JIS SS41, whose chemical composition and mechanical properties were reported elsewhere [6], and so omitted in this paper. The shape and dimension of the specimen are shown in Fig. 3. The surface of the specimen was coated with silicone rubber except for the central area of 16 by 16mm. Fatigue tests were carried out using a Schenk-type bending machine whose cyclic speed was 60 Hz, and a completely reversed plane bending was made. The environment in the present tests was salt water (3% NaCl) kept at 298K which was attained at a rate of 20∿30 ml per minute on the specimen surface. If specimen's stiffness decreased during fatigue, the loading condition was adjusted to secure constant bending moment at any time.
Removing the specimen surface steadily by

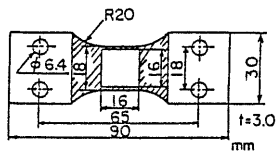

Fig. 3 Shape and dimension of the specimen used

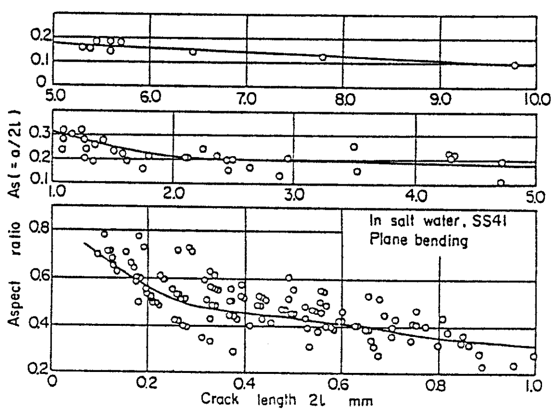

Fig. 4 Relation between crack aspect ratio and crack length

electro-polishing or grinding, and thereafter, taking a replica of the specimen surface, depths of the surface cracks were measured from the depth at which all cracks disappeared.

4.2 Crack aspect ratio

Fig. 4 shows the relation between crack length (2ℓ) and crack aspect ratio ($A_s=a/2\ell$, a: crack depth) during corrosion fatigue process. The figure is arranged by taking A_s on the vertical axis and 2ℓ on the horizontal axis. Experimental points in the figure were obtained by observing the specimens which were broken or whose test was interrupted during corrosion fatigue of σ= 98MPa or 118MPa. As seen from this figure, aspect ratio (A_s) at constant crack length shows a large amount of scatter, but it decreases monotonously with an increasing 2ℓ in the range of 0.7~0.1. The solid line in this figure was obtained by linking together the mean values of A_s in some range of crack lengths in a smoothed curve, and used to find the values of A_s at any crack length for the estimation of fatigue lives.

4.3 Determination of fatigue fracture toughness K_{fc}

Corrosion fatigue tests were conducted at stress amplitudes of 118MPa and 147MPa, and crack growth behaviours of the main crack which caused the final failure of the specimen were investigated by successive observation of the specimen surface during fatigue process. The results plotted on the semi-logarithmic paper are shown in Fig. 5. It seems that the final unstable failure of the specimen occurred at the crack length indicated by arrows in the figure which is designated as the critical crack length $2\ell_c$. After the values of $2\ell_c$ are determined for each of stress amplitudes, fatigue fracture toughness K_{fc} was calculated by using both Eq. (13) and A_s-2ℓ relation of Fig. 4. The results are summarized in Table 1. As seen from Table 1, the values of fatigue fracture toughness for σ=118MPa and 147MPa are somewhat different from each other. Though the one for 147MPa is larger than the one for 118MPa, the average of both values will be employed in this study.

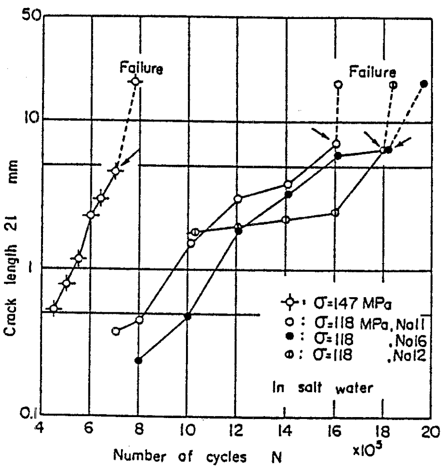

Fig. 5 Crack growth behaviours of the main crack which caused the final failure of the specimen

4.4 Variation of the three parameters of Weibull distribution with stress amplitude and number of stress cycles

Using Eqs. (1), (3) and (4), the distribution of crack lengths was calculated numerically [6][7], and thereafter the distributions were approximated by the three-parameter Weibull distribution. Variations of the obtained shape, scale and location parameters with stress amplitudes and stress cycling are plotted on the logarithmic paper of Fig. 6. As seen from the upper figure of Fig. 6(a), it may be acceptable to regard shape parameter m_0 as a constant though it changes a little during fatigue process. This tendency is independent of stress am-

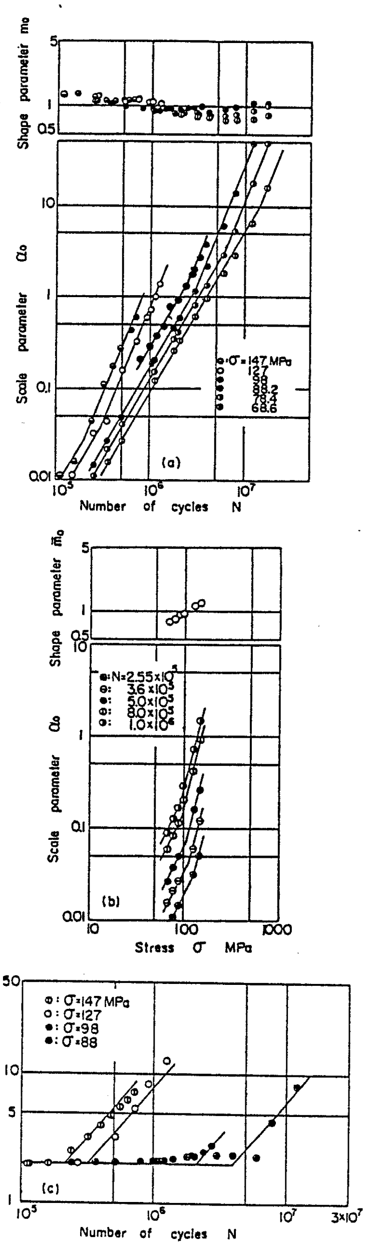

Fig. 6 Variations of the three-parameters of Weibull distribution with stress amplitude and number of stress cycles

plitudes. So, the relation between the average value of m_0, $\bar{m}_0$, during fatigue and stress amplitudes was investigated, and the results are shown in the upper figure of Fig. 6(b). As seen from this figure, a linear relation holds between $\bar{m}_0$ and σ on the logarithmic paper and $\bar{m}_0$ increases monotonously with an increase of σ. Seeing the dependency of the scale parameter, α_0, upon stress amplitude and number of stress cycles, from the lower figures of both Fig. 6 (a) and 6(b), both α_0-N and α_0-σ relations are composed of two sections represented by the different straight lines on the logarithmic paper. Similar tendency is observed in Fig. 6(c) which represents the dependency of location parameter, γ_0, upon stress cycles. The reasons will be discussed later in this study. Expressions of Eq. (8) for the three-parameters of Weibull distribution which describe the distribution of crack lengths stated in the clause 3. 2 were decided by taking the character of the three-parameters shown in Fig. 6 into consideration.

4.5 Comparison between an estimated S-N curve and one obtained experimentally

The variation of the three parameters of Weibull distribution which represent the distribution of crack lengths with stress amplitude and stress cycles shown in Fig. 6 was approximated by Eq. (8), and the values of constants, ζ, η,$\cdots$, k_4, could be decided.

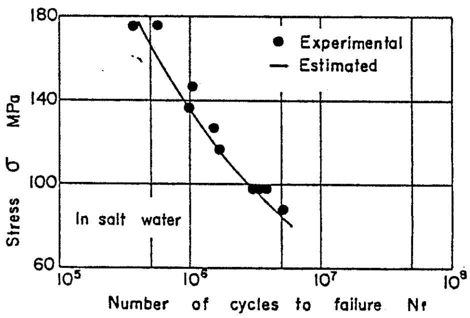

Fig. 7 A comparison between estimated S-N curve and experimentally obtained one

Thereafter, substituting the values of constants into Eq. (16), S-N curve could be estimated. The results are shown in Fig. 7. Data points represented by the mark "•" in this figure are ones obtained experimentally, and they are in good agreement with the estimated S-N curve. From the above consideration, it was made clear that S-N curve for the smooth specimen in the corrosive environment could be obtained theoretically through statistical calculation which took the initiation, the growth and the coalescence behaviours of the distributed surface cracks into consideration.

Though the dependencies of scale and location parameters of the Weibull distribution on the stress amplitude and number of stress cycles are represented by the two different straight lines on the logarithmic paper, the latter straight line will be adopted for the reasons stated in the clause 5. 1. The values of the constants used for the estimation of fatigue lives are listed in Table 2.

5. Considerations

5.1 The dependencies of scale parameter α_0 and location parameter γ_0 on the stress amplitude and number of stress cycles

As stated in clause 4. 4, the dependencies of α_0 and γ_0 on the stress amplitude and number of stress cycles are represented by the two different straight lines on the logarithmic paper. The reasons will be considered in this clause.

Now, let's denote the probability of crack initiation in unit stress cycles (N, N+ΔN) by q(N). If the variation of the crack density during corrosion fatigue is given by Eq. (10), q(N) can be expressed by the following equation,

$$q(N)=\beta\exp\{-\beta(N-N_c)\} \quad\cdots\cdots(17)$$

Denoting the mean value of number of cycles to crack initiation, N_i, by $\bar{N}_i$, $\bar{N}_i$ is expressed as follows:

$$\bar{N}_i=\int_{N_c}^{\infty}N_i q(N_i)dN_i=\frac{1}{\beta}+N_c \quad\cdots\cdots(18)$$

The values of $\bar{N}_i$ for each stress amplitude can be calculated by substituting the values

Table 1 The values of fatigue fracture toughness

σ MPa	$2l_c$ mm			A_s (=a/2l)	K_{fc} MPam$^{1/2}$	
118	TP.11	TP.12	TP.16	0.15	6.38	Average 6.50
	7.1	6.6	6.5			
	Average 6.7					
147	4.5			0.18	6.61	

Table 2 The values of the constants used for the estimation of S-N curve

ζ	η	δ	ε	k_3	Φ mm^2
0.062	0.601	6.67×10^{-30}	6.43	·2.58	256
τ	ξ	k_4	c_3	c_4	K_{fc} MPam$^{1/2}$
9.21×10^{-23}	7.41	1.20	6.22×10^{-4}	0.06	6.50

of β and N_c into Eq. (18). In the α_0-N relation of Fig. 6(a), number of cycles at the point where two different straight lines intersect corresponds to nearly 1∿2 times the value of $\bar{N}_i$. And also, in the α_0-σ relation of Fig. 6(b), the value of the stress amplitude at the point where two different straight lines intersect corresponds to the one calculated in the case when the values of N for each line just correspond to the values of $\bar{N}_i$. The same tendency as those stated above can be observed in the γ_0-N relation of Fig. 6(c). Thereafter, if the former and the latter regions of two different straight lines can be considered the crack initiation and the crack growth periods, respectively, the dependencies of scale and location parameters on stress amplitude and number of stress cycles for both periods are found to be different from each other. In this experiment, as final failure of the specimen occurs in the crack growth period, the dependencies of α_0 and γ_0 on the stress amplitude and number of stress cycles used for the estimation of fatigue lives in the clause 4. 5 are decided by using the character of the latter region of two different straight lines.

5. 2 Estimation of the distribution of maximum crack lengths and the distribution of fatigue lives

In chapter 3. supposing that the weakest link theory can be applied for estimation of fatigue lives in the case where many distributed cracks are initiated during fatigue, that is, considering that the specimen is broken when maximum crack length in the specimen obtained from the distribution of crack lengths becomes equal to the critical crack length $2\ell_c$, an expression for failure condition of Eq. (7) was introduced. From the viewpoint stated above, the scatter of fatigue lives among many specimens, that is, the distribution of fatigue lives may be closely related to the variation of the distribution of maximum crack lengths with stress cycling. In this clause, the relation between the distribution of crack lengths and the distribution of maximum crack lengths or the distribution of fatigue lives will be discussed.

It is supposed that there exists an infinite population of the distributions of crack lengths at a certain stress amplitude and number of stress cycles. Let's represent the distribution function and the probabilistic density function of it by $F(2\ell, \sigma, N)$ and $f(2\ell, \sigma, N)$, respectively. When fatigue tests are performed by using each test specimen, let's denote the number of cracks initiated during fatigue by n_t. In this case, the distribution of crack lengths observed on the specimen surface may be equivalent to that for n_t cracks extracted from the infinite population of crack lengths. In corrosion fatigue, all of the n_t cracks initiated during fatigue are supposed to be able to grow steadily with stress cycling. According to statistics of extremes [10], the distribution of the maximum crack lengths which are observed among n_t cracks is given by the following equation,

$$\Phi_n(2l_{max})=[F(2l, \sigma, N)]^{n_t} \quad \cdots\cdots(19)$$

When the value of n_t is large, Eq. (19) is transformed into the following asymptotic distribution [11] of the largest value for exponential type,

$$\psi(2l_{max})=\exp[-\exp\{-\hat{a}(2l_{max}-W)\}] \quad \cdots\cdots(20)$$

where, $\hat{a}$ and W are scale parameter and characteristic maximum value of the asymptotic distribution of the largest values for exponential type, respectively. The specimen is supposed to be broken when one of the n_t cracks initiated in the specimen becomes equal to the critical crack length $2\ell_c$. Accordingly, taking Eq. (19) into consideration, the probability for failure of the specimens, $P_n(N_f)$, at stress σ and stress cycles N_f is given by the following equation,

$$P_n(N_f)=1-\Phi_n(2l_c)$$
$$=1-[F(2l_c, \sigma, N)]^{n_t} \quad \cdots\cdots(21)$$

When the value of n_t is large, Eq. (21) is transformed into the following equation,

$$P(N_f)=1-\psi(2l_c)$$
$$=1-\exp[-\exp\{-\hat{a}(2l_c-W)\}] \quad \cdots\cdots(22)$$

$\hat{a}$ and W in Eqs.(20) and (22) have the following relationships [12] with the functions, $F(2\ell, \sigma, N)$, $f(2\ell, \sigma, N)$ and n_t,

$$\left.\begin{array}{l} F(W, \sigma, N)=1-1/n_t \\ \hat{a}=n_t f(W, \sigma, N) \end{array}\right\} \quad \cdots\cdots(23)$$

When a three-parameter Weibull distribution of Eq. (5) is adopted for $F(2\ell, \sigma, N)$, $\hat{a}$ and W are given finally by the following equations,

$$\left.\begin{array}{l} \hat{a}=(\ln n_t)^{1-(1/m_0)} m_0 \alpha_0^{-1/m_0} \\ W=\gamma_0+(\alpha_0 \ln n_t)^{1/m_0} \end{array}\right\} \quad \cdots\cdots(24)$$

Though an infinite population of crack lengths is needed to obtain the distribution of fatigue lives actually, the distribution of crack lengths obtained from a single specimen was used in place of the infinite population of crack lengths in this paper. That is, the distribution of fatigue lives was calculated by substituting Eq. (8) in chapter 3 into Eqs. (21) and (22). The values of n_t and $2\ell_c$ were decided by using Eq. (12) and Eq. (13), respectively. The distribution of fatigue lives calculated as stated above is shown in Fig. 8 plotted on

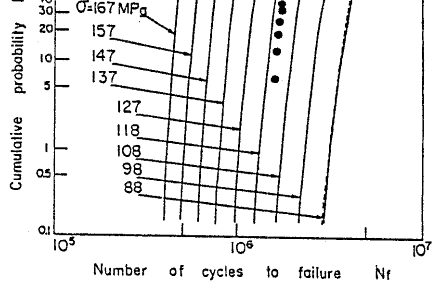

Fig. 8 Numerical results for distribution of corrosion fatigue lives

the Weibull probability paper. The solid and dotted lines in the figure indicate the asymptotic distribution of Eq. (22) and the finite distribution of Eq. (21), respectively. As seen from this figure, the scatter of fatigue lives tends to become larger with a decrease of stress amplitude. This tendency is the same as that [13] observed during fatigue in air. And also, it is seen that there is no difference between the finite distribution and the asymptotic distribution because the solid and dotted lines in this figure correspond to each other. This result stated above may be due to the fact of numerous crack initiations during corrosion fatigue process. Accordingly, it is considered that the distribution of corrosion fatigue lives is fully represented by the asymptotic distribution of Eq. (22).

Experimental points in this figure show the distribution [14] of corrosion fatigue lives which was obtained by using 15 specimens at σ=118MPa. Comparing the experimental distribution of corrosion fatigue lives with the estimated one, both distributions nearly coincide with each other, though the estimated distribution is somewhat shorter than the experimental one.

As stated above, it is clear that fatigue lives of the materials which contain many small distributed cracks can be explained by considering the situation of crack distribution. It is also shown that the distribution of fatigue lives depends on the distribution of crack lengths and that both distributions correspond with each other within the range of the present experiments.

Many factors can be listed as the causes of the scatter of fatigue lives :For example, (i)variations of experimental conditions, that is, the differences in the characters of fatigue machines and the variations in the set values of stress amplitude, (ii) the differences in both the initial conditions and microstructures of the material, and (iii) the scatter in the progress of fatigue damage. But it has not been clarified sufficiently which is the most influential one of these factors. In the estimation method of S-N curve and distribution of fatigue lives until now, based on the P-S-N curves which are obtained by performing fatigue tests with many specimens, the safety factor in design is decided. But in the method, these factors stated above have not been discriminated sufficiently from one another, and therefore,the probabilistic and statistical ground can't be given in the estimation method of safety factor. Recently, many studies have been made in the field of reliability and have pointed out that it is important to clarify the physical factors for the advance of fatigue damage and to investigate the probabilistic features of fatigue process. Considering that fatigue process is composed of crack initiation period and crack growth period, if the probabilistic features of each period can be made clear, the features of the distribution of fatigue lives will be clarified and the probabilistic and statistical ground may be given to the method of design. From the result of this study, it may be appropriate to consider the features of the distributed cracks substantially as fatigue damage,because fatigue lives obtained by using the character of the distributed cracks coincide with experimental results. The method stated in this study may be able to contribute to the advances of machine design by increasing the reliability in decision of safety factor, and can be applied as a quantitative evaluation method for the remaining fatigue life. Though above discussions on the scatter of fatigue lives have been made from the standpoint of (iii) stated before, it may be possible to investigate the scatter of fatigue lives from the standpoint of (ii) by applying probabilistic and statistical method for the distributed cracks to the distribution of the defects at initial state of the material. If fatigue cracks are initiated from the defects contained in the material, by investigating the distribution of these defects it will be possible to evaluate fatigue lives of the material by the same method as the one stated in this study. Especially in the case of corrosion fatigue, 80% of the crack initiation parts are occupied with corrosion pits [15]. Therefore, if the features of the distribution of corrosion pits can be known, it is likely that the estimation of corrosion fatigue lives can be performed. Detailed examination will be left to the future.

As the cause of the scatter of fatigue lives (i) is considered to be an artificial one and to have a different character from the causes (ii) and (iii) which are the material factors, it must be investigated separately. In the other study [16], the authors performed two-level fatigue tests which involved the variance of environments and investigated the physical means of Miner's rule by setting eyes to the distribution of crack lengths. And it has been made clear that Miner's rule corresponds to the change in the state of the distributed cracks with the change of stress amplitude or environment.

6. Conclusions

Probabilistic method of determining fatigue lives and their distributions where many cracks were initiated during fatigue process is studied, and the results obtained are summarized as follows:

(1) Distribution of crack lengths calculated by taking the initiation and growth behaviours of the distributed cracks into consideration could be approximated sufficiently by the three-parameter Weibull distribution. In the present experiment, the change in the three parameters with stress amplitude and number of stress cycles can be given as follows: Shape parameter m_0 was assumed constant though it changes a little during fatigue process, and so the mean value of m_0, $\bar{m}_0$, during fatigue was adopted as the value of shape parameter. There exists a linear relation between $\bar{m}_0$ and stress amplitude σ on the logarithmic paper and $\bar{m}_0$ increases monotonously with an increase of σ. On the other hand, the dependencies of the scale parameter α_0 and location parameter γ_0 on stress amplitude and number of stress cycles were represented by straight lines

which were composed of two sections on the logarithmic paper. Both parameters increased monotonously with an increase of σ and N.

(2) Assuming that the failure of the specimen will occur when maximum crack length among many distributed cracks becomes equal to the critical crack length, the estimation of S-N curve was conducted by taking the variation of the three parameters of the Weibull distribution which represent the distribution of crack lengths with stress cycles into consideration. Estimated results and experimental ones coincide well with each other.

(3) If the weakest link theory holds, the scatter of fatigue lives may be closely connected with the scatter of maximum crack lengths. Based on statistics of extremes, the expressions for the distributions of maximum crack lengths and fatigue lives could be obtained from the distribution of crack lengths. From the numerical examination, it was shown that the scatter of corrosion fatigue lives tended to become wider with a decrease of stress amplitude and this tendency was the same as that observed during fatigue in air. And also, it was shown that the distribution of fatigue lives obtained experimentally under stress amplitude of 118 MPa coincided well with calculated one.

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