Effects of Gas Inertia Forces on Dynamic Characteristics of Externally Pressurized Gas-Lubricated Thrust Bearings*
(Evaluation of Various Approximate Solutions Under Highly Unsteady Conditions)

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In this report, the modified averaging approach to the solution for the dynamic performance of an externally pressurized, gas-lubricated, circular thrust bearing in a laminar flow regime, is presented under the assumption of a small harmonic vibration. This approach can evaluate the inertia effects under highly unsteady conditions. The accuracy of the approximate solutions presented in a previous report are compared and evaluated under similar conditions and the following conclusions are obtained: (1) The conventional averaging approach,in which all the inertia terms are averaged out across the film thickness, gives a good approximation. (2) The first-order solution of the perturbation method becomes remarkably inaccurate as the unsteadiness becomes high.

Key Words: Lubrication, Theoretical Analysis, Externally Pressurized Gas Bearing, Inertia Effect, New Approximate Analysis, Dynamic Performance

1. Introduction

In a previous report[1], analytical models were introduced to predict the effects of the inertia forces of a gas film flow on the dynamic properties of an externally pressurized, gas-lubricated, circular thrust bearing with a single central supply hole. Those models enabled the analysis of the dynamic performance of such a bearing over a wide range of design conditions. The calculated results may, however, become inaccurate as the unsteadiness becomes high. In other previous reports[2][3], the modified averaging approach was introduced for the dynamic performance of an externally pressurized thrust bearing lubricated with an incompressible fluid, and it was found that this approach yield good results under highly unsteady conditions because of the exact treatment of the local acceleration terms in the momentum equation. In this report, the dynamic performance of an externally pressurized gas-lubricated circular thrust bearing is analyzed by means of such a modified averaging approach. The previous approximate solutions[1] are compared with this to evaluate their accuracy under highly unsteady conditions.

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2. Governing Equations and Boundary Conditions

Figure 1 shows schematically the externally pressurized circular thrust, gas bearing with a single central supply hole of inherent compensation. This bearing, subjects to analysis, has a uniform film thickness without any recess. The gas, with a constant pressure p_s, is fed into the central hole. The flow is assumed to be laminar and isothermal. With these assumptions and the usual assumptions of gas-film lubrication theory the momentum equations, the continuity equation and the equation of state are given as follows :

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\frac{\partial^2 u}{\partial z^2} \qquad (1)$$

$$0=\frac{\partial p}{\partial z} \tag{2}$$

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(r\rho u)+\frac{\partial}{\partial z}(\rho w)=0 \tag{3}$$

$$\frac{p}{\rho}=\mathcal{R}\,T_0 \tag{4}$$

where u and w are the velocity components in the r and z directions, p is the pressure, ρ is the density, t is the time, $\mathcal{R}$ is the gas constant, and T_0 is the temperature of the bearing. The boundary conditions are

$$\left.\begin{array}{l} z=0 : u=w=0 \\ z=h : u=0,\ w=\dfrac{dh}{dt} \\ r=r_0 : p=p_a \\ r=r_s : p=p_l \end{array}\right\} \tag{5}$$

where p_a is the ambient pressure and p_l is the pressure at the feeding section which is determined by the continuity condition of mass flow rate.

It is difficult to obtain exact solutions to the above governing equations. Therefore, various approximate solutions have been introduced. In the next chapter, assuming a small harmonic vibration, we will formulate the modified averaging approach, in which the local acceleration terms in the momentum equation are treated as exactly as possible.

3. Modified Averaging Approach

To apply the modified averaging approach to this problem, it should be premised that the flow is unchoked at the inlet to the clearance space, because this approach cannot be formulated for a choked condition. Since the velocity and the pressure can be divided into two-components of the quasi-statical (subscript q) and the dynamical (subscript t) ones, we can write

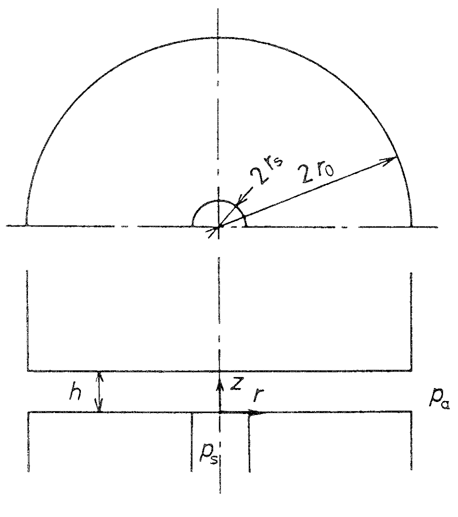

Fig. 1 Schematic view of externally pressurized thrust bearing with a single central supply hole

them as

$$(u,\ w,\ p)=(u_q,\ w_q,\ p_q)+(u_t,\ w_t,\ p_t). \tag{6}$$

Assuming a small harmonic variation in the film thickness,

$$h=h_0+h_1e^{j\omega t}. \tag{7}$$

Corresponded to this expression, the dynamical components of velocity and pressure are given :

$$(u_t,\ w_t,\ p_t)=(\tilde{u}_t,\ \tilde{w}_t,\ \tilde{p}_t)e^{j\omega t} \tag{8}$$

$$(u_q,\ w_q,\ p_q)\gg(u_t,\ w_t,\ p_t) \tag{9}$$

Substituting Eqs.(6) through (8) into Eqs.(1) through (5), we can formulate the following governing differential equations and boundary conditions:

Quasi-statical components

$$\frac{p_q}{\mathcal{R}T_0}\left(u_q\frac{\partial u_q}{\partial r}+w_q\frac{\partial u_q}{\partial z}\right)=-\frac{\partial p_q}{\partial r}+\mu\frac{\partial^2 u_q}{\partial z^2} \tag{10}$$

$$0=\frac{\partial p_q}{\partial z} \tag{11}$$

$$\frac{1}{r}\frac{\partial}{\partial z}(rp_qu_q)+\frac{\partial}{\partial z}(p_qw_q)=0 \tag{12}$$

Boundary conditions

$$\left.\begin{array}{l} z=0,\ h : u_q=w_q=0 \\ r=r_0 : p_q=p_a \\ r=r_s : p_q=p_{lq} \end{array}\right\} \tag{13}$$

Dynamical components

$$\begin{aligned}&\left[\frac{p_q}{\mathcal{R}T_0}\left(j\omega h_1\frac{\partial u_q}{\partial h}+j\omega\tilde{u}_t+u_q\frac{\partial \tilde{u}_t}{\partial r}+\tilde{u}_t\frac{\partial u_q}{\partial r}\right.\right.\\&\left.+w_q\frac{\partial \tilde{u}_t}{\partial z}+\tilde{w}_t\frac{\partial u_q}{\partial z}\right)+\frac{\tilde{p}_t}{\mathcal{R}T_0}\\&\left.\times\left(u_q\frac{\partial u_q}{\partial r}+w_q\frac{\partial u_q}{\partial z}\right)\right]_{h=h0}\\&=-\frac{\partial \tilde{p}_t}{\partial r}+\mu\frac{\partial^2\tilde{u}_t}{\partial z^2}\end{aligned} \tag{14}$$

$$0=\frac{\partial \tilde{p}_t}{\partial z} \tag{15}$$

$$\begin{aligned}&\left[j\omega\left(h_1\frac{\partial p_q}{\partial h}+\tilde{p}_t\right)+\frac{1}{r}\frac{\partial}{\partial r}\{r(p_q\tilde{u}_t+\tilde{p}_tu_q)\}\right.\\&\left.+\frac{\partial}{\partial z}(q_p\tilde{w}_t+\tilde{p}_tw_q)\right]_{h=h_0}=0\end{aligned} \tag{16}$$

Boundary conditions

$$\left.\begin{array}{l} z=0 : \tilde{u}_t=\tilde{w}_t=0 \\ z=h_0 : \tilde{u}_t=0,\ \tilde{w}_t=j\omega h_1 \\ r=r_0 : \tilde{p}_t=0 \\ r=r_s : \tilde{p}_t=\tilde{p}_{lt} \end{array}\right\} \tag{17}$$

where $j=\sqrt{-1}$, and ω is the angular frequency of the squeeze motion. From Eq.(7), it should be noted that the real parts of the complex quantities in the solutions, u_t, w_t and p_t, have physical meaning. Since it is difficult to obtain exact solutions of the above governing equations, we use the averaging approach, in which the inertia terms, except the term of $j\omega p_q\tilde{u}_t/(\mathcal{R}T_0)$, are averaged out across the film thickness. f_q and $\tilde{f}_t$ are defined as:

$$\begin{aligned}f_q\equiv&\frac{1}{\mu}\frac{\partial p_q}{\partial r}+\frac{p_q}{\mu\mathcal{R}T_0h}\int_0^h\left(u_q\frac{\partial u_q}{\partial r}\right.\\&\left.+w_q\frac{\partial u_q}{\partial z}\right)dz\end{aligned} \tag{18}$$

$$\hat{f}_t \equiv \frac{1}{\mu}\frac{\partial \hat{p}_t}{\partial r} + \frac{1}{\mu \mathfrak{R} T_0 h_0}\int_0^{h_0}\Big[p_q\Big(j\omega h_1 \frac{\partial u_q}{\partial h} + u_q\frac{\partial \hat{u}_t}{\partial r} + \hat{u}_t\frac{\partial u_q}{\partial r} + w_q\frac{\partial \hat{u}_t}{\partial z} + \hat{w}_t\frac{\partial \hat{u}_q}{\partial z}\Big) + \hat{p}_t\Big(u_q\frac{\partial u_q}{\partial r} + w_q\frac{\partial u_q}{\partial z}\Big)\Big]_{h=h_0} dz. \quad (19)$$

The velocity components are obtained as follows:

$$u_q = \frac{f_q}{2} z(z-h) \quad (20)$$

$$w_q = \frac{-1}{12 r p_q}\frac{d}{dr}(r p_q f_q) z^2(2z-3h) \quad (21)$$

$$\hat{u}_t = \frac{-\hat{f}_t}{4 r^2 p_{q0}}\left\{1 - \frac{\cosh(2\gamma\sqrt{p_{q0}}\,z - \gamma\sqrt{p_{q0}}\,h_0)}{\cosh(\gamma\sqrt{p_{q0}}\,h_0)}\right\} \quad (22)$$

$$\hat{w}_t = \frac{1}{p_{q0}}\Big[-j\omega\Big(h_1\frac{dp_q}{dh}\Big|_{h=h_0} + \hat{p}_t\Big)z + \frac{1}{4\gamma^2 r}\frac{\partial}{\partial r}\Big[r\hat{f}_t\Big\{z - \frac{\sinh(2\gamma\sqrt{p_{q0}}\,z - \gamma\sqrt{p_{q0}}\,h_0)}{2\gamma\sqrt{p_{q0}}\cosh(\gamma\sqrt{p_{q0}}\,h_0)}\Big\}\Big] - \frac{1}{12 r}\frac{d(r f_{q0}\hat{p}_t)}{dr} z^2(2z-3h_0) - \frac{1}{8\gamma^3 r}\frac{d}{dr}\Big\{\frac{r\hat{f}_t}{\sqrt{p_{q0}}}\tanh(\gamma\sqrt{p_{q0}}\,h_0)\Big\}\Big] \quad (23)$$

where $\gamma^2 = j\omega/(4\mu\mathfrak{R}T_0)$, $(p_{q0}, f_{q0}) = [p_q, f_q]_{h=h_0}$.

With the boundary conditions of Eqs. (13) and (17), Eqs. (21) and (23) result in:

$$\frac{\partial}{\partial r}(r p_q f_q) = 0 \quad (24)$$

$$\frac{1}{r}\frac{d}{dr}\Big[r\Big\{1 - \frac{\tanh(\gamma\sqrt{p_{q0}}\,h_0)}{h_0^2\gamma\sqrt{p_{q0}}}\Big\}\hat{f}_t\Big] = 4 j h_0^2\gamma^2\omega\Big(h_1\frac{dp_q}{dh}\Big|_{h=h_0} + \hat{p}_t + \frac{h_1}{h_0}p_{q0}\Big) - \frac{h_0^4\gamma^2}{3r}\frac{d(r f_{q0}\hat{p}_t)}{dr}. \quad (25)$$

On the other hand, substitution of Eqs. (20) through (23) into Eqs. (18) and (19) leads to the following equations:

$$f_q = \frac{1}{\mu}\frac{dp_q}{dr} + \frac{h^4}{120\mu\mathfrak{R}T_0}\frac{1}{r}\frac{d}{dr}(r p_q f_q^2) \quad (26)$$

$$\hat{f}_t = \frac{1}{\mu}\frac{d\hat{p}_t}{dr} + \frac{1}{\mu\mathfrak{R}T_0 h_0}\Big[\frac{-j\omega}{12}\Big\{p_{q0}\Big(h_1 h_0^3\frac{df_q}{dh}\Big|_{h=h_0} + 3h_1 h_0^3 f_{q0}\Big) + f_{q0}\Big(h_1 h_0^3\frac{dp_q}{dh}\Big|_{h=h_0} + h_0^3\hat{p}_t\Big)\Big\} + \frac{h_0^5}{240}\frac{df_{q0}^2}{dr}\hat{p}_t + \frac{h_0^5 f_{q0}}{120 r}\frac{d}{dr}(r f_{q0}\hat{p}_t) + \frac{p_{q0}f_{q0}}{48\gamma^2}\frac{d}{dr}\Big(\frac{g\hat{f}_t}{p_{q0}}\Big) + \frac{g}{48\gamma^2}\frac{df_{q0}}{dr}\hat{f}_t + \frac{f_{q0}}{48\gamma^2}\frac{1}{r}\frac{d}{dr}(r g\hat{f}_t)\Big] \quad (27)$$

where $g = h_0^3 - \dfrac{3h_0^3}{(\gamma\sqrt{p_{q0}}\,h_0)^2} + \dfrac{3h_0^3}{(\gamma\sqrt{p_{q0}}\,h_0)^3}\tanh(\gamma\sqrt{p_{q0}}\,h_0)$.

Equations (24) through (27) are the governing equations for the gas film pressure which could be derived through the modified averaging approach. From the following dimensionless quantities,

$$R = \frac{r}{r_0},\ P_q = \frac{p_q}{p_a},\ F_q = \frac{\mu r_0}{p_a}f_q,\ \varepsilon = \frac{h_1}{h_0},$$

$$\hat{P}_t = \frac{\hat{p}_t}{\varepsilon p_a},\ \hat{F}_t = \frac{\mu r_0}{\varepsilon p_a}\hat{f}_t,$$

$$G = \frac{g}{h_0^3},\ \varphi = \gamma\sqrt{p_a}\,h_0,$$

$$\sigma = \frac{12\mu\omega}{p_a}\Big(\frac{r_0}{h_0}\Big)^2,\ a = \Big(\frac{p_a h_0^2}{12\mu\sqrt{\mathfrak{R}T_0}\,r_0}\Big)^2,$$

we can obtain normalized forms of Eqs. (24), (26), (25) and (27) as follows:

$$\frac{d}{dR}(R P_q F_q) = 0 \quad (28)$$

$$F_q = \frac{dP_q}{dR} + \frac{6aH^4}{5R}\frac{d}{dR}(R P_q F_q^2) \quad (29)$$

$$\frac{1}{R}\frac{d}{dR}\Big[R\Big\{1 - \frac{1}{\varphi\sqrt{P_{q0}}}\tanh(\varphi\sqrt{P_{q0}})\Big\}\hat{F}_t\Big] = \frac{j\varphi^2\sigma}{3}\Big(\frac{dP_q}{dH}\Big|_{H=1} + \hat{P}_t + P_{q0}\Big) - \frac{\varphi^2}{3R}\frac{d}{dR}(R F_{q0}\hat{P}_t) \quad (30)$$

$$\hat{F}_t = \frac{d\hat{P}_t}{dR} + a\Big[-j\sigma\Big\{P_{q0}\Big(\frac{dF_q}{dH}\Big|_{H=1} + 3F_{q0}\Big) + F_{q0}\Big(\frac{dP_q}{dH}\Big|_{H=1} + \hat{P}_t\Big)\Big\} + \frac{6F_{q0}^2\hat{P}_t}{5R} + \frac{6}{5}\frac{d}{dR}(F_{q0}^2\hat{P}_t) + \frac{3P_{q0}F_{q0}}{\varphi^2}\frac{d}{dR}\Big(\frac{G\hat{F}_t}{P_{q0}}\Big) + \frac{3G}{\varphi^2}\frac{dF_{q0}}{dR}\hat{F}_t + \frac{3F_{q0}}{\varphi^2 R}\frac{d}{dR}(R G\hat{F}_t)\Big] \quad (31)$$

where $(P_{q0}, F_{q0}) = [P_q, F_q]_{H=1}$, a is a parameter which represents the contribution from the inertia forces, and σ is the squeeze number. The boundary conditions in non-dimensional forms are:

$$\left.\begin{aligned} &R=1:\ P_q=1,\ \hat{P}_t=0\\ &R=R_s:\ P_q=P_{lq},\ \hat{P}_t=\hat{P}_{lt}\\ &\qquad\quad F_q=F_{lq},\ \hat{F}_t=\hat{F}_{lt}\end{aligned}\right\} \quad (32)$$

where $P_{lq}, F_{lq}, \hat{P}_{lt}, \hat{F}_{lt}$ are determined through the continuity condition of the mass flow rate at the feeding section. Equations (28) through (31) are calculated numerically by means of the Runge-Kutta-Gill method.

The dimensionless dynamic stiffness (K) and the dimensionless damping coefficient (B) are defined in the same manner as those in the previous report[1]. They are determined by the following four dimensionless designing parameters:

$R_s = \dfrac{r_s}{r_0}$: (the dimensionless radius of the supply hole)

$P_s = \dfrac{p_s}{p_a}$: (the dimensionless supply pressure)

$\Gamma = \dfrac{-12\mu C_D r_s\sqrt{\mathfrak{R}T_0}}{p_a h_0^2}\ln R_s$: (the feeding parameter)

$Re^{**} = \dfrac{\rho_0 h_0^2\omega}{\mu}$: (the unsteadiness parameter)

and the discharge coefficient (C_D). In the unsteadiness parameters, ρ_0 is the density at the ambient pressure (p_a). The unsteadiness parameter (Re^{**}) represents the ratio of the local acceleration term to the viscous term. The above parameters are related to σ, a and φ

by the following equations:

$$\sigma=\frac{1}{12}\left(\frac{-\Gamma}{C_D R_s \ln R_s}\right)^2 Re^{**} \tag{33}$$

$$a=\left(\frac{-C_D R_s \ln R_s}{\Gamma}\right)^2 \tag{34}$$

$$\varphi=\frac{\sqrt{jRe^{**}}}{2}. \tag{35}$$

As mentioned already calculated results by this modified averaging approach are expected to be accurated under highly unsteady conditions because the local acceleration terms are treated exactly.

4. Comparison among Various Approximate Solutions

The approximate solutions presented in the previous paper[1] are valid under the conditions of $a \ll 1$ and $Re^{**} \ll 1$ because they are based on rough treatment of the inertia terms including the local acceleration terms. On the other hand, the modified averaging approach is expected to be valid for any value of Re^{**}. The calculated results by this approach can be accurate under highly unsteady conditions because, in this approach, the local acceleration termes in the momentum equation are treated as exactly as possible. In this chapter, the previous approximate solutions[1] are compared with the calculated results by the modified averaging approach to evaluate their accracy. This comparison is made for an unchoked conditions because the modified averaging approach cannot be applied for a choked condition.

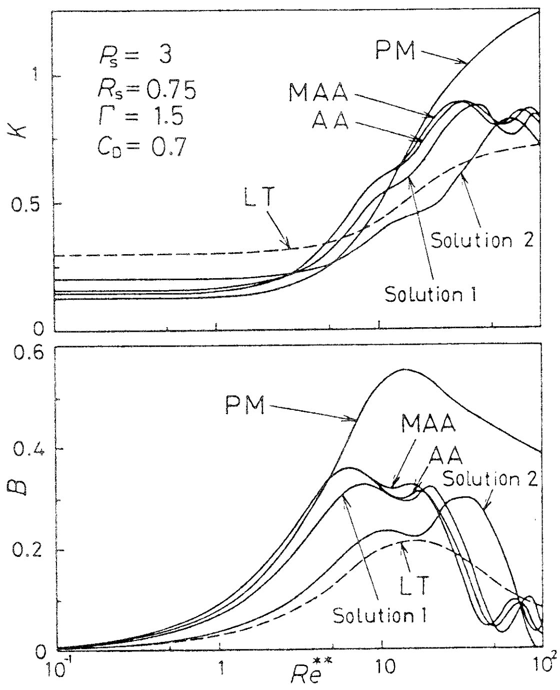

Fig. 2 Comparison of approximate solutions for dynamic stiffness and damping coefficient

Figure 2 shows the comparison of various approximate solutions for the dimensionless dynamic stiffness (K) and the dimensionless damping coefficient (B). In this figure, the symboles, MAA, AA and PM indicate the results by the modified averaging approach, the averaging approach, and the first-order perturbation method, respectively. The symbols, Solution 1 and Solution 2, indicate the results by the analytical models presented in a previous report[1]. The broken lines (LT) show the results by the classical lubrication theory. One can find in this figure that the conventional averaging approach, in which all the inertia terms are averaged out across the film thickness, gives a good approximation even under highly unsteady conditions, and that the results of Solution 1 also give a fairly good approximation. The results of Solution 2 are inaccurate except the region of small values of Re^{**}. The first-order solutions of the perturbation method become remarkably inaccurate when the value of Re^{**} is large.

5. Conclusions

The modified averaging approach to the solution for the dynamic stiffness and the damping coefficient of an externally pressurized, gas-lubricated, circular thrust bearing, was presented under the assumption of a small harmonic vibration. The previous approximate solutions was compared with the results of this approach to evaluate their accuracy under highly unsteady conditions. The following conclusions have been obtained:

(1) The averaging approach gives good approximations, and solution 1 gives fairly good approximations.

(2) The perturbation method becomes remarkably inaccurate when the value of Re^{**} is large.

(3) Solution 2 is quite inaccurate except for small values of Re^{**}.

These conclusions are obtained for an unchoked condition. Similar conclusions may be expected for a choked condition.

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