Math. Rep. Toyama Univ.
Vol. 4(1981), 179-182

CORRECTION TO : THE DISCRETE MAXIMUM PRINCIPLE FOR NONCONFORMING FINITE ELEMENT APPROXIMATIONS TO STATIONARY CONVECTIVE DIFFUSION EQUATIONS

(This Reports, Vol. 2, pp. 33-52)

Katsushi OHMORI

LEMMA 10 and THEOREM 4 in the paper " The discrete maximum principle for nonconforming finite element approximations to stationary convective diffusion equations " are not correct. We want to give here the corrections for them as follows;

LEMMA 10. *If* div $b \leq 0$ *and* h *is sufficiently small, then* $a_h(\cdot,\cdot)$ *is* V_{oh}*-elliptic, i.e., there exists a positive constant* $\tilde{\alpha}$ *such that*

$$(5.3) \qquad a_h(v_h, v_h) \geq \tilde{\alpha} \, \|v_h\|_h^2 \quad \text{for all } v_h \in V_{oh} .$$

PROOF. For any $v_h \in V_{oh}$, we have by using Green's formula over each $K \in T_h$,

$$a_h(v_h,v_h) = \nu \sum_{K\in T_h} |v_h|^2_{1,K} + \frac{1}{2}\sum_{K\in T_h}\{\int_{\partial K} v_h^2(b\cdot n)\, d\gamma - \int_K (\operatorname{div} b)v_h^2\, dx\}$$

$$\geq \nu \|v_h\|_h^2 + \frac{1}{2}\sum_{K\in T_h}\int_{\partial K} v_h^2\,(b\cdot n)\, d\gamma ,$$

where the last inequality is derived from $\operatorname{div} b \leq 0$. We note that

$$\sum_{K\in T_h}\int_{\partial K} v_h^2(b\cdot n)\, d\gamma = \sum_{K\in T_h}\sum_{K'\subset\partial K}\int_{K'} v_h^2(b - b(B))\cdot n\, d\gamma + \sum_{K\in T_h}\sum_{K'\subset\partial K}\int_{K'} v_h^2\, b(B)\cdot n\, d\gamma ,$$

where B is the barycenter of K'.

By

$$\sum_{K\in T_h}\sum_{K'\subset\partial K}\int_{K'} v_h^2\, b(B)\cdot n\, d\gamma = \sum_{K\in T_h}\sum_{K'\subset\partial K}\int_{K'} v_h(v_h - v_h(B))b(B)\cdot n\, d\gamma,$$

we have, using Lemma 9

$$\left|\sum_{K\in T_h}\sum_{K'\subset\partial K}\int_{K'} v_h^2\, b(B)\cdot n\, d\gamma\right| \leq C_1 h \|v_h\|_h^2 .$$

On the other hand, since

$$\sum_{K\in T_h}\sum_{K'\subset\partial K}\int_{K'} v_h(B)^2(b - b(B))\cdot n\, d\gamma = 0 ,$$

we may write as follows ;

$$\sum_{K\in T_h}\sum_{K'\subset\partial K}\int_{K'} v_h^2(b - b(B))\cdot n\, d\gamma$$

$$= \sum_{K\in T_h}\sum_{K'\subset\partial K}\int_{K'} (v_h - v_h(B))^2(b - b(B))\cdot n\, d\gamma$$

$$+ \sum_{K\in T_h}\sum_{K'\subset\partial K}\int_{K'} 2(v_h - v_h(B))v_h(B)(b - b(B))\cdot n\, d\gamma.$$

Using the facts that $|v_h - v_h(B)| \le h_K \|\nabla v_h\|$ on K, $\|b - b(B)\| \le C_2 h_K$ on K, $\mathrm{meas}(K') \le n\cdot\mathrm{meas}(K)/2\rho_K$ and $\sqrt{\mathrm{meas}(K)}\cdot \sum_{i=1}^{n+1} |v_h(B_i)| \le C_3\sqrt{n+1}\, |v_h|_{o,K}$ for some positive constants C_2 and C_3, we have

$$\sum_{K'\subset\partial K} \left|\int_{K'} (v_h - v_h(B))v_h(B)(b - b(B))\cdot n\, d\gamma\right| \le C_4 h_K^2 |v_h|_{o,K} |v_h|_{1,K}.$$

Similarly, we have

$$\sum_{K'\subset\partial K} \left|\int_{K'} (v_h - v_h(B))^2 (b - b(B))\cdot n\, d\gamma\right| \le C_5 h_K^3\, |v_h|_{1,K}^2 .$$

Then, summing up over all the finite elements $K\in T_h$, we have

$$\left|\sum_{K\in T_h} \sum_{K'\subset\partial K} \int_{K'} (v_h - v_h(B))v_h(B)(b - b(B))\cdot n\, d\gamma\right| \le C_6 h^2 \|v_h\|_h^2$$

and

$$\left|\sum_{K\in T_h} \sum_{K'\subset\partial K} \int_{K'} (v_h - v_h(B))^2 (b - b(B))\cdot n\, d\gamma\right| \le C_7 h^3 \|v_h\|_h^2 ,$$

using the discrete Poincaré inequality (cf. Temam [4]) in the first inequality. Hence we have

$$\left|\sum_{K\in T_h} \sum_{K'\subset\partial K} \int_{K'} v_h^2 (b - b(B))\cdot n\, d\gamma\right| \le C_8 h^2 \|v_h\|_h^2$$

for h small enough.

Therefore there exists some positive constant C_9 such that

$$a_h(v_h,v_h) \ge (\nu - C_9 h)\, \|v_h\|_h^2 \quad \text{for all} \quad v_h \in \overset{\circ}{V}_{oh}$$

and $\nu - C_9 h > 0$, provided h is sufficiently small. This proves Lemma 10 completely.

THEOREM 4. *If* $\sup_{x\in\Omega}\|b\| < \infty$, $\operatorname{div} b \le 0$ *and* h *is sufficiently small, then the discrete problem* (3.7)-(3.8) *has a unique solution.*

The paper has also some misprints, which should be corrected as follows;

p.35, ℓ↓11 : $$a(u,v) = \int_\Omega \sum_{i=1}^{n} \frac{\partial u}{\partial x_i} \left(\nu \frac{\partial v}{\partial x_i} + b_i v\right) dx \,.$$

p.37, ℓ↓4 - ℓ↓5 :
$$= \nu \sum_{i=1}^{n} \left|\frac{\partial u}{\partial x_i}\right|^2_{o,\Omega} + \sum_{i=1}^{n} \left(- \frac{1}{2} \int_\Omega u^2 \frac{\partial b_i}{\partial x_i} dx + \frac{1}{2} \int_\Gamma u^2 b_i n_i \, d\gamma \right)$$
$$= \nu \, |u|^2_{1,\Omega} - \frac{1}{2} \int_\Omega (\operatorname{div} b) u^2 \, dx + \frac{1}{2} \int_\Gamma u^2 (b\cdot n) \, d\gamma \,,$$

P.37, ℓ↑3 : *If* u *is the solution of* (2.6)-(2.7) *and* $u_o = 0$, *then*

P.37, ℓ↑1 : $$|u|_{1,\Omega} \le c \, |f|_{o,\Omega}$$

p.42, ℓ↓7 : $$\sum_{j=1}^{N+M} a_{ij} \ge 0 \quad \textit{for} \ 1 \le i \le N.$$

p.48, ℓ↑7 : *for all* $u \in H^1(K)$ *and all* $v \in H^1(K), \ldots$

(Received March 20, 1981)