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Regional Disparities of Income, Environmental Quality and Medical Care in China : A Multidimensional Majorization Approach

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Abstract

How serious is the trade-off between economic growth and other social welfare indicators? In this paper, we investigate whether the recent extreme economic growth in China has improved social welfare when considering not only income but also the environment and medical care. We construct an analytical framework based on multidimensional majorization which allows us to evaluate welfare orderings based on multivariate attributes. For the period from 2006 to 2010, we investigate the welfare orderings of multivariate distributions of attributes compiled from city statistics in China. The result shows that social well-being in 2010 is better than 2006. However, pairs other than {2006, 2010} are not rankable. In particular, we find that pollution emissions are a bottleneck that hinders improving social welfare. On the other hand, recent changes in the distributions of GDP and of medical resources work toward improving social well-being.

Keywords: regional inequality in China; multidimensional majorization; welfare ordering

JEL classification codes: D3; D63; R12.

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1 Introduction

In the process of economic growth, various distortions appear while national income as a whole expands. Firstly, per capita income among regions does not necessarily grow at the same rate. As a result of economic growth, regional income disparity may increase. Secondly, in an economic growth with rapid industrialization, we sometimes face serious environmental degradation. Indeed, developed countries have experienced various pollution problems in the process of economic growth. Thirdly, economic growth accompanying a transition from planned to a market economy brings a lack of social security services such as medical care and social insurance.

In order to evaluate the performance achieved by economic growth, it is not sufficient to focus on the income or the growth rate as a whole. Not only monetary income but also the factors noted above play key roles as determinants of welfare. In this paper, we investigate whether the recent extreme economic growth in China has improved social welfare when considering not only income but also the environment and medical care. That is, taking per capita GDP, environmental quality, and the endowment of medical resources as attributes affecting individuals' utility, we compare social welfare based on data from Chinese cities.

To this end, we compare the distribution of attributes by using an extended version of the generalized Lorenz (GL) dominance criterion proposed by Shorrocks (1983) and Kakwani (1984).[1] GL dominance is an appropriate criterion for analyzing the Chinese economy which shows both rapid economic growth and increased inequality since it takes into account not only inequality but also changes in the mean. Extending GL to multidimensional attributes, we evaluate recent trends in Chinese regional inequalities from a social welfare point of view.

Needless to say, reducing regional inequality is an important policy issue for the Chinese government.[2] Figure 1 shows the trend of regional inequality in income measured by per capita GDP by province. After the mid-70's, the regional inequality tended to decline for the 10 years that included the early stages of the open door policy. However, regional inequality increased from 1990 to 2003. The Gini index in 2003 in particular has indicated the highest value except for period of confusion caused by the Great Leap Forward and the famine period. Although inequality has been declining since 2003, it still remains high compared with the conditions in the late-80's.

The Chinese government has promoted the development policy in inland areas such as Great Western Development in order to reduce economic disparity. Also, substantial amount of income has been transferred to reduce income inequality between urban and rural areas. However, the basic structure of Chinese regional inequality has not changed. Table 1 summarizes the 20 cities with highest per capita GDP and those with lowest per capita GDP. Although the statistics are based on cities rather than provinces, they have similar tendencies. The cities and provinces with low GDP are mainly located in inland areas. On the other hand, the cities with high per capita GDP are in the

1 . For a detail discussion, see also Lambert (1993, ch.3). For a detailed discussion of the Lorenz-type multidimensional dominance criteria, see Savaglio (2006) and Trannoy (2006).

2. For a comprehensive study on China's inequality, see Gustafsson et al.(2008).

northeastern and coastal areas. At the same time, there are is a province, namely Gansu, where there are cities from both the lowest GDP and highest GDP lists. This fact suggests that regional disparities in economic activity in China should be investigated in more detail than at the provincial level.

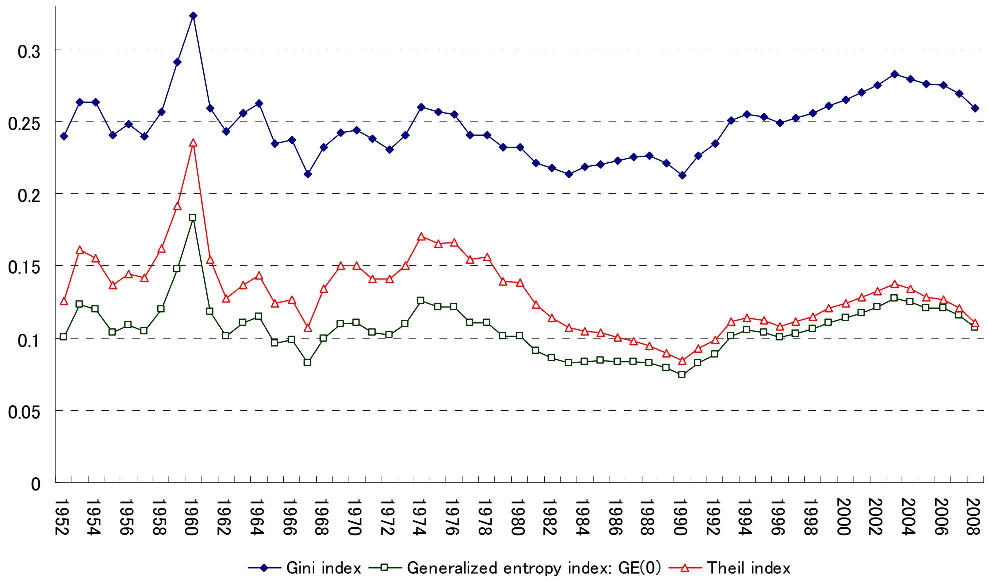

Figure 1 Interprovincial income inequality trend

Source: *China Compendium of Statistics 1949-2008*.

Notes: Hainan province is excluded from the sample. Generalized entropy index, $GE(0)$, is defined as $GE(0) = \sum_i f_i \ln(\mu/y_i)$, where f_i, y_i, and μ denote the population share in province *i*, per capita GDP in province *i*, and average per capita GDP, respectively.

Reflecting concern about regional inequality, there is much extant literature even if we restrict our attention to just income inequality. In addition, several studies focus on inequalities other than income such as the environment, health status, education and social security. In what follows, we summarize the related literature to clarify our motivation.

Tsui (1991, 1993, 1996, and 1998) contributed pioneering work on regional inequality in China.[3] For example, using the Gini index, the coefficient of variation and the entropy index, Tsui (1996) investigated the interprovincial GDP inequality from 1978 to 1989. He revealed that the growth-oriented policy toward coastal areas in the late 80s caused an expansion of income inequality, even though in the early stages of the open door policy the inequality has been shrank. In addition, decomposing the overall inequality into sectors, he found that reform in the industrial sector implemented by the open door policy resulted in increased interprovincial inequality.[4]

3. In this paper, we focus on the regional income inequality based on the difference in per capita GDP. On the other hand, Gustafson et al. (2008) investigates the inequality using household survey data set of Chinese household income (CHIP) conducted in 1988, 1995, and 2002.

4. Kanbur and Zhang (2005) conducted a long term analysis that includes the period before the open door policy. They considered interprovincial-income-inequality based on GDP from 1952 to 2000 using Gini and generalized entropy indices. This calculated inequality is decomposed into rural-urban inequality and coastal-inland

Table 1 Per capita GDP in Chinese cities (2010)

Lowest 20 cities				Highest 20 cities			
	City	Province	Per capita GDP a)		City	Province	Per capita GDP a)
1	Dingxi	Gansu	0.578	1	Karamay	Xinjiang	18.193
2	Longnan	Gansu	0.660	2	Ordos	Inner Mongolia	13.620
3	Zhaotong	Yunnan	0.728	3	Dongying	Shandong	11.595
4	Guyuan	Ningxia	0.847	4	Daqing	Heilongjiang	9.985
5	Bazhong	Sichuan	0.855	5	Baotou	Inner Mongolia	9.285
6	Lincang	Yunnan	0.893	6	Shenzhen	Guangdong	9.250
7	Tianshui	Gansu	0.920	7	Wuxi	Jiangsu	9.091
8	Fuyang	Anhui	0.950	8	Suzhou	Jiangsu	8.818
9	Pu'er	Yunnan	0.976	9	Guangzhou	Guangdong	8.463
10	Anshun	Guizhou	1.014	10	Jiayuguan	Gansu	7.950
11	Shaoyang	Hunan	1.028	11	Foshan	Guangdong	7.856
12	Baoshan	Yunnan	1.041	12	Zhuhai	Guangdong	7.746
13	Bozhou	Anhui	1.057	13	Dalian	Liaoning	7.710
14	Pingliang	Gansu	1.121	14	Shanghai		7.457
15	Lijiang	Yunnan	1.153	15	Wuhai	Inner Mongolia	7.340
16	Lu'an	Anhui	1.205	16	Beijing		7.196
17	Suzhou	Anhui	1.215	17	Tianjin		7.130
18	Shangluo	Shaanxi	1.221	18	Weihai	Shandong	6.933
19	Ankang	Shaanxi	1.244	19	Hangzhou	Zhejiang	6.838
20	Wuwei	Gansu	1.260	20	Ningbo	Zhejiang	6.788

Source: *China City Statistical Yearbook 2011* and *2010 Population Census*.
Note: a) Ten thousand CNY (current price).

Based on Tsui (1983), Lee (2000) investigated regional inequality by using data on consumption and production in the industrial and agricultural sectors and decomposed overall inequality. As a result, Lee (2000) found that sources of inequality can shift from interprovincial to intra-provincial and from rural-urban to intra-rural areas. Fan and Sun (2008) investigated interprovincial income inequality as measured by the coefficient of variation, the Gini index, and the Theil index and decomposed inequality at a sub-national level (the eastern, central, and western regions). They found that interregional inequality has increased while intraregional inequality has dropped. They also pointed out that regional inequality tends to increase and that intraregional inequality in the eastern region significantly affects overall inequality.[5]

Inequality exacerbated by rapid growth affects another aspect of individuals' well-being. Environmental quality has deteriorated grievously in industrial areas.[6] Figure 2 represents sulfur dioxide (SO_2) emissions per land area and per capita GDP in the prefecture-level cities and direct-controlled municipalities. Although there are large deviations, cities with higher income have

inequality. They also argued that the inequality can be explained by the share of heavy industry, the degree of decentralization, and market openness.

5. While much of the literature focusing on decomposing of overall inequality used the Theil index, which allows inter-group inequality to be distinguished from intra-group inequality, Wan (2001) investigated regional inequality by making use of the Gini index, decomposing it using the concentration index.
6. For example, see World Bank (2010).

substantially more SO_2 emissions which may indicate tradeoff between the environment and economic growth. Vennemo et al. (2009) argued that air pollution in China, especially in the industrial areas of the northern and central regions, causes serious environmental problems. In the 11th five-year plan, China's government, facing serious environmental damage, has set compulsory targets for environmental protection (e.g. Cao et al., 2009).

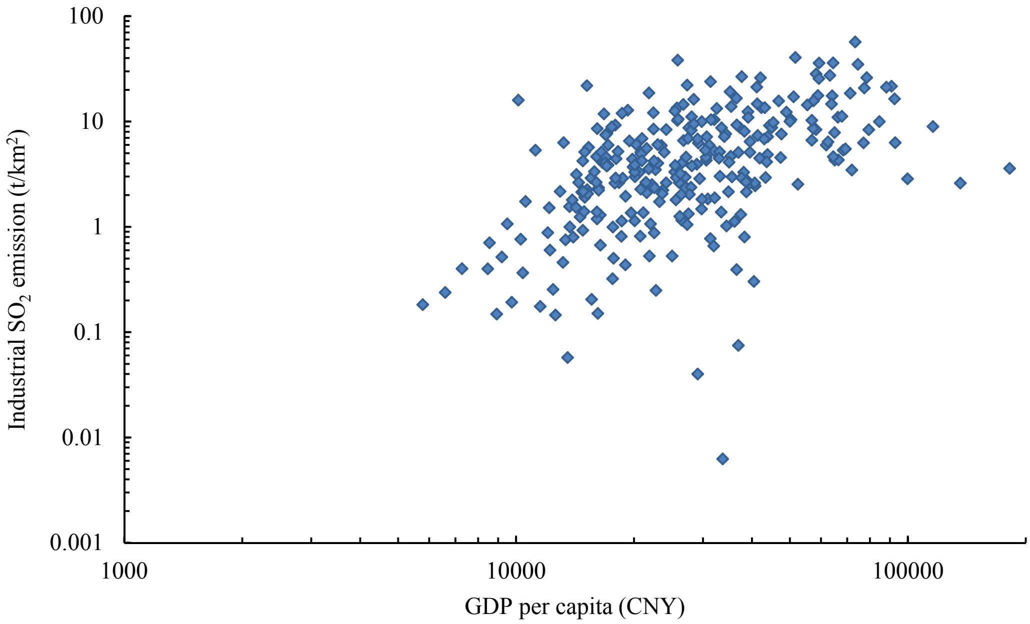

Figure 2 Per capita GDP and industrial SO_2 emissions in Chinese cities (2010)

Note: Observations are prefecture-level cities except for Lhasa in Tibet, and direct-controlled municipalities.
Sources: *China City Statistical Yearbook 2011* and *2010 Population Census*.

Brajer et al. (2010) explicitly related regional income inequality with environmental quality in China. By using city data from 1995 and 2004, they estimated income adjusted by environmental quality, which takes into account health damage due to SO_2 and NO_x. Comparing adjusted income and unadjusted income, they found that the difference in environmental pollution between cities negatively affects inequality. Clarke-Sather et al. (2011) investigated inequalities in carbon dioxide (CO_2) emissions in China at provincial level and compared its pattern with that of income. They found that inter-provincial CO_2 emissions inequality is primarily intra-regional (Eastern, Western, and Central regions), while inter-provincial income inequality is inter-regional.

Accessibility to medical care services has caused confusion during the transition period from a planned economy to a market economy. According to Liu et al. (1999), the number of workers in medical and health care sectors decreased by 35.9% from 1980 to 1989. In addition, Liu et al. (1999) and Bloom (2001) argued that medical professionals tend to be concentrated in urban areas.[7] Figure 3 shows the relationship between the number of doctors per thousand population and per capita GDP in the prefecture-level cities and direct-controlled municipalities. From Figure 3, we can confirm that the number of doctors tends to increase as per capita GDP increases. This suggests that the income inequality accelerates the concentration of medical care resources and negatively affects social welfare.

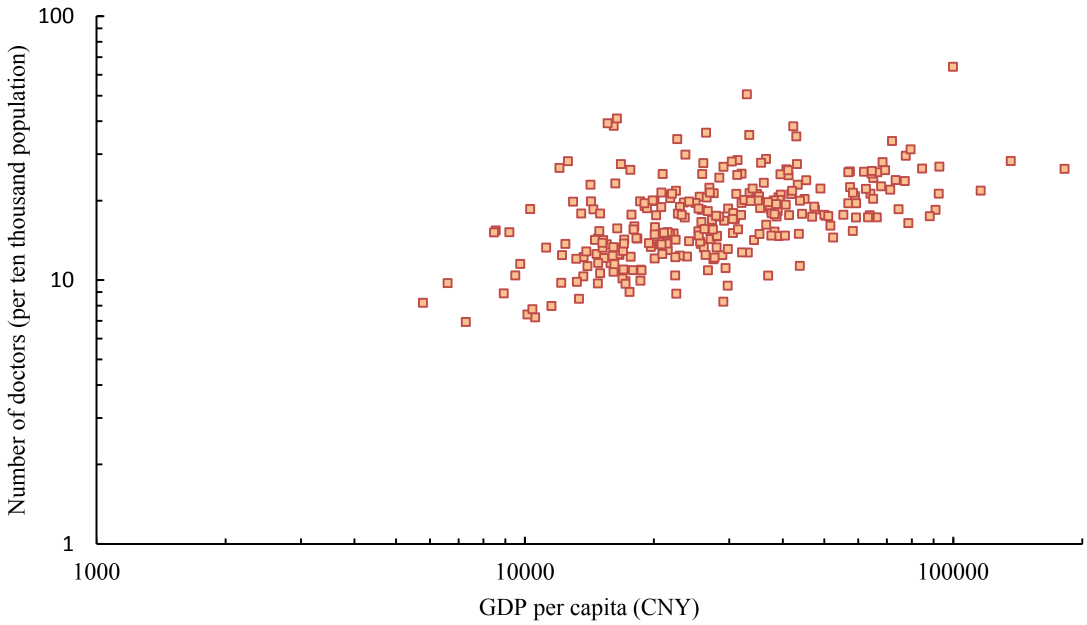

Figure 3 Per capita GDP and the number of doctors in Chinese cities (2010)

Note: Observations are the same as in Figure 2.
Sources: *China City Statistical Yearbook 2011* and *2010 Population Census*.

There are several studies that focus on the relationship between medical care and income inequality. Zhang and Kambur (2005) analyzed the spatial inequality of medical care and education in China using the Gini index and an entropy measure. According to their analysis, at the provincial level, inequality measured by the number of beds and professionals has shrunk during the 50s and 70s and was flat through the 80s. On the other hand, inequality between urban and rural areas based on the number of beds and doctors has continuously increased. Zhang and Kambur (2005) argued that the disparity in infant mortality rate between urban and rural has grown due to inequality of medical resources.[8] They suggested that tight fiscal constraints caused by the decentralization lie in the background of this trend.

Although the intensive studies described above reveal various aspects of regional income inequality, the environment, and medical care, some issues still need to be clarified. Firstly, further

7. Also, Zhao (2006) argued, using data from the National Health Services Survey of 2003, that there are substantial disparities in accessibility medical care service between urban and rural areas, and between large and small cities.
8. Fang et al. (2010) pointed out that regional health inequality in China is associated with health resources. They also argued that primary health care rather than hospital services is important in reducing health inequality.

investigation should be done on the indices and procedures used to analyze regional inequality. Much literature on regional inequality utilizes the aggregate measures such as the Gini and Theil indices, which concentrate relative inequality. However, if we take into account the substantial increase in mean income in the Chinese economy, it is not sufficient to focus on only relative inequality. We should analyze regional inequality giving consideration to increased absolute levels of income. In addition, to obtain robust welfare implications from changes in the regional inequality, rather than aggregative indices, non-aggregative dominance criteria should be employed.[9]

Secondly, there is room for expansion in the treatment of multidimensional attributes. In the related literature, the regional inequality of individual items such as income, health and education has been considered. However, no comprehensive investigation that contains multiple attributes has been implemented as far as we know.[10] Brajer et al. (2010) is an informative analysis in the sense that the effects of environmental deterioration on health status are incorporated into regional income inequality. However, in their analysis, inequality is measured using Gini or Theil coefficients. In addition, although they revealed that income inequality grew from 1995 to 2004, they do not take into account any increase in mean income.

Thirdly, the policy implications obtained from the analyses are limited. Since the bulk of studies have dealt with regional inequality using aggregative means, it is not necessarily clear which attributes, and at what level, should be devoted to improving the social welfare. The existing studies on regional inequality have focused on the sources of inequality such as inter- intra-provincial or urban-rural factors. Such studies make substantial contributions by clarifying the sources of income inequality. However, in order to identify the future direction of public policy, analyses that enable us to the problems in more detail will be needed.

To address the remaining issues described above, we will employ a novel analytical procedure to evaluate regional inequality. First, employing data for 2006 to 2010 for prefecture-level cities and direct-controlled municipalities, we utilize a concept that extends GL to multidimensional attributes, in our case, income, the environment, and medical care. By using city-based data, we can capture regional inequality in more detail than with provincial data. Indeed, intra-provincial inequality is an important factor in investigating recent trends in Chinese regional inequality. For example Akita (2003) argued that intra-provincial income inequality has a significant effect on total regional inequality by decomposing the Theil index.

In addition, by extending GL to multidimensional attributes, we can comprehensively consider regional inequality, which is construed to contain income, environment, and medical care.

9. While the focus is slightly different from our analysis, Bishop et al. (1996) considers income inequality within a region using a micro data set according to the Lorenz dominance criterion. Although Gustafsson et al. (2008) provided the Lorenz function for the whole of China and for cities and rural areas based on CHIP, they analyzed only the years 1988, 1995, and 2002.

10. For countries other than China, several studies analyze social well-being using multidimensional dominance criteria. For example, Nilsson (2010) considered the inequality among Zambian households using the sequential generalized Lorenz dominance criterion. Muller and Trannoy (2011) proposed a multidimensional dominance criterion having attributes consisting of income, health and education, and investigated the change in welfare distribution among countries between 2000 and 2004.

The dominance criterion employed in this paper, which can be interpreted as an extension of uniform majorization (UM) as proposed by Kolm (1977), can produce welfare ordering for a broad class of social welfare functions similar to UM.[11] Moreover, we can frame the procedure for checking the criterion as a linear programming problem.[12] With such a formulation, we can clarify which directions will improve social welfare if the distribution of attributes cannot be seen as desirable compared with the distribution of the base year.

The rest of the paper is organized as follows. In Section 2, we provide an analytical framework. In Section 3, we construct the data set and provide preliminary results. In Section 4, we show the results and provide an interpretation. In Section 5, we conclude the analysis.

2 Analytical Framework

In this section, we provide a framework for the analysis. Consider an economy consisting of $r \geq 2$ regions and $n_J \geq 2$ individuals for period J, where $J \in \{X, Y\}$. Hereafter, X and Y are referred to as the given year and the base year, respectively. We define a set of indexed individuals as $N_J = \{1, \dots, n_J\}$ and define a set of indexed regions as $R = \{1, \dots, r\}$. Let $N_{i(J)}$ be an indexed set of individuals who lives in region $i \in R$ for period $J \in \{X, Y\}$. That is, $\bigcup_{i \in R} N_{i(J)} = N_J$. Thus, $|N_{i(J)}|$ represents the population in region i in period J, which is denoted by $n_{i(J)}$ and $n_J = \sum_{i=1}^{r} n_{i(J)}$. The distribution of the population over the region is represented by a vector,

$$\mathbf{n}_J = \left(n_{1(J)}, \dots, n_{r(J)}\right),$$

for $J \in \{X, Y\}$.

Each individual's well-being depends on m attributes. The attributes of individual j in periods X and Y are respectively denoted by

$$\mathbf{x}_j = \begin{bmatrix} x_{1j} \\ \vdots \\ x_{mj} \end{bmatrix}, \qquad \mathbf{y}_j = \begin{bmatrix} y_{1j} \\ \vdots \\ y_{mj} \end{bmatrix}.$$

Since our concern is on regional inequality, we assume that all individuals living in the same region have the same attributes. Let $\mathbf{x}_i$ and $\mathbf{y}_i$ for $i \in R$ be the vectors that represent the attributes of an individual living in region i in the given year and the base year, respectively. Thus, the distributions of attributes in the given year and the base year are respectively represented as follows:

$$\widetilde{\mathbf{X}} = [\underbrace{\mathbf{x}_1, \dots, \mathbf{x}_1}_{n_{1(X)}}, \dots, \underbrace{\mathbf{x}_r, \dots, \mathbf{x}_r}_{n_{r(X)}}],$$

11. See Marshall et al. (2011) for a detailed discussion of the theory of majorization.

12. Recently, using data envelopment analysis (DEA), some researchers have focused on the efficiency of Chinese economic activity taking undesirable outputs such as air pollution into account (e.g. Liu and Lo, 2007; Song et al., 2012). Our study differs from the DEA-based analyses in the sense that our main concern is welfare ordering, although both employ linear programming techniques.

$$\widetilde{\mathbf{Y}} = [\underbrace{\mathbf{y}_1, \dots, \mathbf{y}_1}_{n_{1(Y)}}, \dots, \underbrace{\mathbf{y}_r, \dots, \mathbf{y}_r}_{n_{r(Y)}}].$$

In the given year, the well-being of an individual who lives in region i is represented by the following utility index.

$$u_{i(X)} = u(\mathbf{x}_i), \tag{1}$$

for $i \in R$. Similarly, the well-being of an individual for the base year is represented by $u_{i(Y)} = u(\mathbf{y}_i)$. The utility index is not necessarily the same as the utility function that represents an individual's preferences. Rather, the utility index can be interpreted as a function that the policy maker or the researcher employs to evaluate the well-being of an individual. The utility index is assumed to be cardinal. In addition, the same functional form is applied to every individual. Furthermore, we make the following assumption on the utility index.

Assumption 1 The utility index is continuous, non-decreasing and concave in the attributes.

In what follows, we denote the class of utility indices that satisfy Assumption 1 as Ω_u.

Social welfare is evaluated using a social evaluation function which is a function of an individual's well-being. That is,

$$SW_J = \frac{1}{n_J}\sum_{i=1}^{r} n_{i(J)}u_{i(J)}, \tag{2}$$

for $J \in \{X, Y\}$. Equation (2) implies that the society's well-being can be represented as a weighted sum of its individuals' well-being.

Later, we compare the distributions of attributes in a given year with the base year. However, since the sizes of matrices $\widetilde{\mathbf{X}}$ and $\widetilde{\mathbf{Y}}$ are extremely large, it is difficult to compare $\widetilde{\mathbf{X}}$ with $\widetilde{\mathbf{Y}}$ directly. Therefore, we replace $\widetilde{\mathbf{X}}$ and $\widetilde{\mathbf{Y}}$ with $\widehat{\mathbf{X}}$ and $\widehat{\mathbf{Y}}$ defined as follows:

$$\widehat{\mathbf{X}} = [\mathbf{x}_1, \dots, \mathbf{x}_r],$$

$$\widehat{\mathbf{Y}} = [\mathbf{y}_1, \dots, \mathbf{y}_r].$$

Furthermore, in order to compare social welfare based on $\widehat{\mathbf{X}}$ and $\widehat{\mathbf{Y}}$, we define the following matrix, which can be interpreted as a compressed form of a doubly stochastic matrix.

Definition 1 (Compressed doubly stochastic matrix) An r-by-r nonnegative matrix $\widehat{\mathbf{Q}}$ is said to be a *compressed doubly stochastic matrix* if the following properties are satisfied,

(1) $\mathbf{e}_r\widehat{\mathbf{Q}} = \frac{1}{n_X}\mathbf{n}_X$,

(2) $\widehat{\mathbf{Q}}\mathbf{e}_r^T = \frac{1}{n_Y}\mathbf{n}_Y^T$.

where $\mathbf{e}_r$ denotes an r-dimensional row vector whose entries are one.

That is, $\widehat{\mathbf{Q}}$ is a nonnegative square matrix whose column sum is equal to the population share in the given year and whose row sum is equal to the population share in the base year. We denote the set of compressed doubly stochastic matrices determined by the distributions of population, $\mathbf{n}_X$ and $\mathbf{n}_Y$, as $\Psi(\mathbf{n}_X, \mathbf{n}_Y)$.[13]

By using the notation mentioned above, we can define a dominance criterion, which we then use herein.

Definition 2 (Uniform supermajorization). Let $\widehat{\mathbf{X}}$ and $\widehat{\mathbf{Y}}$ be m-by-r matrices and $\mathbf{S}$ be an r-by-r diagonal matrix defined as $\mathbf{S} \equiv \mathrm{diag}\left(\frac{n_{1(X)}}{n_X}, \dots, \frac{n_{r(X)}}{n_X}\right)$.[14] $\widehat{\mathbf{X}}$ is said to be *uniformly supermajorized* by $\widehat{\mathbf{Y}}$ (in symbols $\widehat{\mathbf{X}} \prec^w \widehat{\mathbf{Y}}$), if

$$\widehat{\mathbf{Y}}\widehat{\mathbf{Q}} \leq \widehat{\mathbf{X}}^*, \tag{3}$$

holds for some $\widehat{\mathbf{Q}} \in \Psi(\mathbf{n}_X, \mathbf{n}_Y)$, where $\widehat{\mathbf{X}}^* \equiv \widehat{\mathbf{X}}\mathbf{S}$.

Uniform supermajorization (USM) can be interpreted as an extension of UM as proposed by Kolm (1977) in the economics literature.[15] In the theory of stochastic dominance, USM is known as increasing-concave (second-order) stochastic ordering (ICV).[16] By considering ICV based on an empirical distribution, we can easily relate ICV with other multivariate majorization criteria.

USM's welfare implications are similar to those of UM.

Proposition 1 Let $\widehat{\mathbf{X}}$ and $\widehat{\mathbf{Y}}$ be m-by-r matrices. The following two conditions are equivalent.

(1) $\widehat{\mathbf{X}} \prec^w \widehat{\mathbf{Y}}$.

(2) $SW_X \geq SW_Y$ holds for all $u \in \Omega_u$.

Proof See Appendix.

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13. For $I, J \in \{X, Y\}$ and $I \neq J$, let $\mathbf{C}_J$ be an r-by-$\tilde{n}(\equiv n_X n_Y)$ block matrix defined as

$$\mathbf{C}_J \equiv \frac{1}{n_J}\begin{bmatrix} \mathbf{e}_{n_I n_{1(J)}} & & \mathbf{O} \\ & \ddots & \\ \mathbf{O} & & \mathbf{e}_{n_I n_{r(J)}} \end{bmatrix}.$$

For a given $\tilde{n}$-dimensional doubly stochastic matrix, $\mathbf{Q}$, a compressed doubly stochastic matrix $\widehat{\mathbf{Q}} \in \Psi(\mathbf{n}_X, \mathbf{n}_Y)$ is obtained as $\widehat{\mathbf{Q}} = \mathbf{C}_Y \mathbf{Q} \mathbf{C}_X^T$.

14. Hereafter, $\mathrm{diag}(z_1, \dots, z_r)$ is an r-dimensional diagonal matrix whose diagonal elements are $z_1, \dots, z_r$.

15. Since the present analysis focuses on data aggregated by region, the definition of UM is slightly modified from that in Kolm (1977). In Kolm (1977), for given r-by-n matrices, $\mathbf{X}$ and $\mathbf{Y}$, $\mathbf{X}$ is said to be uniformly supermajorized by $\mathbf{Y}$, if $\mathbf{YQ} = \mathbf{X}$ holds for some doubly stochastic matrix $\mathbf{Q}$. Within the context of the present analysis, $\widehat{\mathbf{X}}$ is said to be *uniformly majorized* by $\widehat{\mathbf{Y}}$ (in symbol $\widehat{\mathbf{X}} \prec \widehat{\mathbf{Y}}$), if $\widehat{\mathbf{Y}}\widehat{\mathbf{Q}} = \widehat{\mathbf{X}}^*$ holds for some $\widehat{\mathbf{Q}} \in \Psi(\mathbf{n}_X, \mathbf{n}_Y)$.

16. For example, see Müller and Stoyan (2002).

Proposition 1 can be regarded as an extension of Shorrocks theorem to the multidimensional attributes. From Proposition 1, if $\widehat{\mathbf{X}} \prec^{w} \widehat{\mathbf{Y}}$ holds then we can say that the social well-being in the given year is more desirable than that in the base year under the social evaluation function defined as equation (2) with all utility indices satisfying Assumption 1. In contrast, if $\widehat{\mathbf{X}} \nprec^{w} \widehat{\mathbf{Y}}$ then there exists a utility index such that $\widehat{\mathbf{Y}}$ is more desirable than $\widehat{\mathbf{X}}$. It should be noted that $\widehat{\mathbf{X}} \nprec^{w} \widehat{\mathbf{Y}}$ does not always imply $\widehat{\mathbf{Y}} \prec^{w} \widehat{\mathbf{X}}$. Since we cannot exclude the possibility that both $\widehat{\mathbf{X}} \nprec^{w} \widehat{\mathbf{Y}}$ and $\widehat{\mathbf{Y}} \nprec^{w} \widehat{\mathbf{X}}$ hold, there may be a case in which the two distributions are not rankable in the sense of social well-being.[17]

If we consider a univariate case such as m=1, USM coincides with the familiar GL-dominance criterion. In such a situation, USM can be easily checked using the GL curve. When we consider more than two attributes, we can check USM using a linear programming model instead of the GL curve. Vectorizing inequality (3), we obtain the following problem.

Problem 1 (P1). Consider the following problem.

$$\min_{\mathbf{q}} \mathbf{aq},$$

subject to

$$\mathbf{Aq} = \mathbf{b},$$

$$\mathbf{q} \geq \mathbf{0},$$

where

$$\mathbf{a} \equiv (\mathbf{0}_{r^2+mr}, \mathbf{w}_L), \qquad \mathbf{b} \equiv \begin{bmatrix} \mathrm{vec}\widehat{\mathbf{X}}^* \\ \frac{1}{n_Y}\mathbf{n}_Y^T \\ \frac{1}{n_X}\mathbf{n}_X^T \end{bmatrix}, \qquad \mathbf{A} \equiv \begin{bmatrix} \mathbf{I}_r \otimes \widehat{\mathbf{Y}} & \mathbf{I}_{mr} & -\mathbf{I}_{mr} \\ \mathbf{e}_r \otimes \mathbf{I}_r & \mathbf{O} & \mathbf{O} \\ \mathbf{I}_r \otimes \mathbf{e}_r & \mathbf{O} & \mathbf{O} \end{bmatrix},$$

$\mathbf{I}_s$ is a s-dimensional identity matrix, and $\mathbf{w}_L$ is an mr-dimensional row vector with positive constants. In addition, vec and $\otimes$ denote the column stacking operator and Kronecker product, respectively.[18]

Since (P1) is a standard linear programming model, we can easily check USM by investigating the optimal solution of (P1).

Proposition 2 Let $\widehat{\mathbf{X}}$ and $\widehat{\mathbf{Y}}$ be m-by-r matrices. The following two conditions are equivalent.

17. In what follows, inequality is to be interpreted component-wise.

18. For an m-by-n matrix $\mathbf{Z} = [\mathbf{z}_1, \ldots, \mathbf{z}_n]$ consisting of m-dimensional column vectors $\mathbf{z}_j$, vec$\mathbf{Z}$ is defined as

$$\mathrm{vec}\mathbf{Z} \equiv \begin{bmatrix} \mathbf{z}_1 \\ \vdots \\ \mathbf{z}_n \end{bmatrix}.$$

(1) For a given $\mathbf{w}_L > \mathbf{0}$, the optimal value for (P1) is zero,

(2) $SW_X \geq SW_Y$ holds for all $u \in \Omega_u$.

Proof. That is clear from (P1).

Whether the optimal solution of (P1) is zero or positive does not depend on $\mathbf{w}_L$. When the optimal solution of (P1) is not zero, its value depends on the coefficient vector. By solving (P1), we obtain a recommended policy for which attributes should be improved and in which region the utility should be improved. Let $\mathbf{q}^*$ be an optimal solution vector of (P1). We decompose $\mathbf{q}^*$ into column vectors such as

$$\mathbf{q}^* = \begin{bmatrix} \mathbf{q}_{DS}^* \\ \mathbf{q}_P^* \\ \mathbf{q}_N^* \end{bmatrix}.$$

where $\mathbf{q}_{DS}^*$ is an r^2-dimensional column vector, and $\mathbf{q}_P^*$ and $\mathbf{q}_N^*$ are mr-dimensional column vectors. From the equality constraint of (P1), we obtain $(\mathbf{I}_r \otimes \widehat{\mathbf{Y}})\mathbf{q}_{DS}^* + \mathbf{q}_P^* - \mathbf{q}_N^* = \mathrm{vec}\widehat{\mathbf{X}}^*$ which can be rewritten in matrix form as follows:

$$\widehat{\mathbf{Y}}\widehat{\mathbf{Q}}^* + \widetilde{\mathbf{Q}}_P^* - \widetilde{\mathbf{Q}}_N^* = \widehat{\mathbf{X}}^*, \tag{4}$$

where $\widehat{\mathbf{Q}}^* \in \Psi(\mathbf{n}_X, \mathbf{n}_Y)$ such that $\mathrm{vec}\widehat{\mathbf{Q}}^* = \mathbf{q}_{DS}^*$ holds. Similarly, $\widetilde{\mathbf{Q}}_P^*$ and $\widetilde{\mathbf{Q}}_N^*$ are m-by-r matrices such that $\mathrm{vec}\widetilde{\mathbf{Q}}_P^* = \mathbf{q}_P^*$ and $\mathrm{vec}\widetilde{\mathbf{Q}}_N^* = \mathbf{q}_N^*$ hold, respectively. Considering equation (4), we obtain an m-by-r matrix defined as

$$\Delta\widehat{\mathbf{X}} = \widetilde{\mathbf{Q}}_N^* \mathbf{S}^{-1}, \tag{5}$$

which satisfies $\widehat{\mathbf{X}} + \Delta\widehat{\mathbf{X}} \prec^w \widehat{\mathbf{Y}}$.[19] That is, $\Delta\mathbf{x}_i$ in $\Delta\widehat{\mathbf{X}} \equiv [\Delta\mathbf{x}_1, \dots, \Delta\mathbf{x}_r]$ represents the incremental changes in the attributes in region i that must be improved to increase social well-being. Thus, the objective function of (P1) can be represented as follows:

$$E(\mathbf{w}, \widehat{\mathbf{X}}, \widehat{\mathbf{Y}}) = \min_{\mathrm{vec}\Delta\widehat{\mathbf{X}}}\{\mathbf{w} \cdot \mathrm{vec}\Delta\widehat{\mathbf{X}} : \widehat{\mathbf{X}} + \Delta\widehat{\mathbf{X}} \prec^w \widehat{\mathbf{Y}}, \Delta\widehat{\mathbf{X}} \geq \mathbf{O}\}. \tag{6}$$

In the above definition, $\mathbf{w}$ is a shadow price vector of attributes. The coefficient vector of (P1), $\mathbf{w}_L$, is related to $\mathbf{w}$ as follows:

$$\mathbf{w}_L = \mathbf{w}(\mathbf{S}^{-1} \otimes \mathbf{I}_m). \tag{7}$$

Decomposing $\mathbf{w}$ into m-dimensional row vectors $\mathbf{w} \equiv (\mathbf{w}_1, \dots, \mathbf{w}_r)$, we obtain the shadow prices of attributes applied to region i in the form of $\mathbf{w}_j = (w_{1j}, \dots, w_{mj})$.

19. Vectorizing $\Delta\widehat{\mathbf{X}}$, we can rewrite (5) as $\mathrm{vec}\Delta\widehat{\mathbf{X}} = (\mathbf{S}^{-1} \otimes \mathbf{I}_r)\mathbf{q}_N^*$.

Example Consider a society consisting of four regions and three attributes. Suppose that the distributions of residents in the base and given years are $\mathbf{n}_Y = (10, 20, 30, 40)$ and $\mathbf{n}_X = (5, 20, 45, 55)$, respectively. In addition, the distributions of attributes in the base year and the given year are assumed to be

$$\hat{\mathbf{Y}} = \begin{bmatrix} 200 & 300 & 400 & 500 \\ 50 & 100 & 300 & 20 \\ 300 & 200 & 100 & 400 \end{bmatrix}, \quad \hat{\mathbf{X}} = \begin{bmatrix} 250 & 300 & 400 & 450 \\ 30 & 120 & 260 & 40 \\ 300 & 200 & 160 & 370 \end{bmatrix}.$$

Since $\mathbf{S} = \mathrm{diag}\left(\frac{5}{125} \quad \frac{20}{125} \quad \frac{45}{125} \quad \frac{55}{125}\right)$, we have

$$\hat{\mathbf{X}}^* \equiv \hat{\mathbf{X}}\mathbf{S} = \begin{bmatrix} 10 & 48 & 144 & 198 \\ 1.2 & 19.2 & 93.6 & 17.6 \\ 12 & 32 & 57.6 & 162.8 \end{bmatrix}.$$

The shadow price vector for region *j*(=1,…,4) is set as $\mathbf{w}_j = (100 \quad 100 \quad 100)$. Hence, according to equation (7), $\mathbf{w}_L = \left(\frac{125}{5}\mathbf{w}_1 \quad \frac{125}{20}\mathbf{w}_2 \quad \frac{125}{45}\mathbf{w}_3 \quad \frac{125}{55}\mathbf{w}_4\right)$**.** Solving (P1), we obtain the optimal value as $\min_{\mathbf{q}} \mathbf{aq} = 2250$ which implies $\hat{\mathbf{X}} \not\prec^w \hat{\mathbf{Y}}$. From the solution vector of (P1), we can construct a matrix,

$$\tilde{\mathbf{Q}}_N^* = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

That is, if the second attribute in region 1 is increased by 22.5=0.9/0.04, then USM is achieved. Indeed, when we consider a distribution of attributes such as

$$\hat{\mathbf{X}}^{*\prime} \equiv \hat{\mathbf{X}}^* + \tilde{\mathbf{Q}}_N^* = \begin{bmatrix} 10 & 48 & 144 & 198 \\ 2.1 & 19.2 & 93.6 & 17.6 \\ 12 & 32 & 57.6 & 162.8 \end{bmatrix},$$

we have a compressed doubly stochastic matrix such that $\hat{\mathbf{Y}}\hat{\mathbf{Q}} \le \hat{\mathbf{X}}^{*\prime}$ holds as follows:

$$\mathbf{Q} = \begin{bmatrix} .030 & .000 & .004 & .066 \\ .005 & .160 & .035 & .000 \\ .000 & .000 & .278 & .022 \\ .005 & .000 & .043 & .352 \end{bmatrix}.$$

Thus, from Definition 1, we obtain $\hat{\mathbf{X}}' \prec^w \hat{\mathbf{Y}}$, where $\hat{\mathbf{X}}' \equiv \hat{\mathbf{X}}^{*\prime}\mathbf{S}^{-1}$.

In the subsequent sections, we consider the change in social well-being in China by solving (P1) based on statistics from Chinese cities.

3 Data and Preliminary Results

In this section, we construct a data set for the analysis and provide preliminary results. The time period for the analysis is from 2006 to 2010. This period coincides exactly with the 11th five-year plan. We analyze China's regional inequality based on statistics aggregated for prefecture-level cities and direct-controlled municipalities. In 2010, there were four direct-controlled municipalities: Beijing, Tianjin, Shanghai and Chongqing. The direct-controlled municipalities have authority similar to that of the provinces. On the other hand, the prefecture-level cities belong to the provinces. In 2010, there were 283 prefecture-level cities, which did not change during the time period of the analysis.

The prefecture-level cities differ from typical cities since most of them contain rural areas. Hence, secondary or tertiary industries are not necessarily the main activities there. Figure 4 depicts the industrial structure in each city measured by the GDP shares of primary, secondary, and tertiary industries. As seen in Figure 4, in some cities, the GDP share of primary industries is higher than 30%.

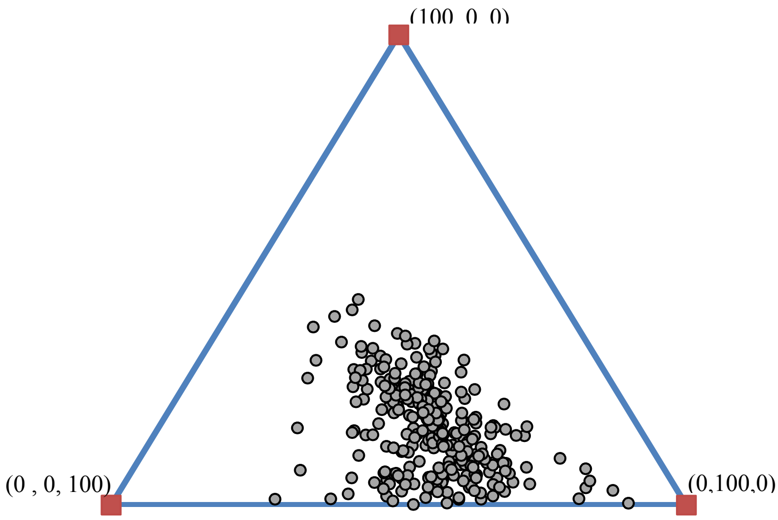

Figure 4 Cities' industrial structure (2010)
Source: *China City Statistical Yearbook 2011*.
Notes: (x, y, z) denotes the GDP share of the primary, secondary, and tertiary industries, respectively. Observations are the same as in Figure 2.

Since the prefecture-level cities and the direct-controlled municipalities are not identical with respect to their sizes, authorities etc., treating them in the same category may be controversial. However, exclusion of the direct-controlled municipalities from the data set would ignore major trends in inequality arising from disparities between metropolitan and rural areas. On the other hand, analysis based on data consisting of provinces, autonomous areas and the direct-controlled municipalities cannot capture intra-provincial disparities. Therefore, we investigated regional inequality based on data for prefecture-level cities and the direct-controlled municipalities.

When using Chinese cities as the unit for analysis, demographic statistics should be treated carefully. Demographic statistics of Chinese cities are sometimes based on residents registered by public security office (*hukou*). For analytical purposes, the actual number of residents, not the *hukou* population, is appropriate. Indeed, there are huge differences between the *hukou* and resident populations.[20] But reliable statistics on actual population of each city do not exist except for the years 2000, 2005 and 2010 when censuses or population sample surveys were conducted. In the present analysis, we use the estimated population based on the *2000 Population Census*, *2005 1% Population Sample Survey,* and *2010 Population Census*.[21]

Table 2 Summary statistics for cities

	2006	2007	2008	2009	2010
Population (ten thousand)[a)]					
Mean	418	422	426	430	434
Std. Dev.	295	301	307	315	323
Min	19	20	21	22	23
Max	2815	2832	2850	2867	2885
Total	119638	120667	121742	122865	124037
Land area (km^2)[b)]					
Mean	16398	16435	16448	16475	16484
Std. Dev.	21830	21833	21840	21962	21960
Min	1113	1113	1113	1113	1113
Max	253356	253356	253356	253356	253356
Number of cities[c)]					
	286	286	286	286	286

Notes. a) Estimation by author based on *2000 Population Census*, *2005 1% Population Sample Survey* and *2010 Population Census*. b) Based on *China City Statistical Yearbook.* c) Sum of direct-controlled municipalities and prefecture-level cities except for Lhasa in Tibet.

Table 2 presents summary statistics for the population and land area of the cities included in the analysis. Generally, prefecture-level cities have sub-divisions such as county-level cities, counties, and districts. But all county-level cities do not necessarily belong to prefecture-level cities. That is, the data set we construct does not contain the entire area of China. In addition, Lhasa in the Tibet autonomous area is excluded from the analysis due to the lack of data. As a result, through the period for which we investigate, we include 286 cities: 282 prefecture-level cities and 4 direct-

20. In Beijing, for example, the population reported in the *China City Statistical Yearbook,* which is based on *hukou*, is 12578 (in thousand persons), while its resident population based on the *2010 Population Census* is 19612. See Chan (2006, 2007) for a detailed discussion of the demographic statistics of Chinese cities.
21. The resident population of city i in year t is simply estimated as $n_{i(t)} = (1+\beta_i)^{t-t_0} n_{i(t_0)}$,where $n_{i(t_0)}$ is the resident population obtained from the *2000 Population Census*, *2005 1% Population Sample Survey*, or the *2006 Statistical Communiqué on National Economic and Social Development* for each city. We basically employ the data based on the *2005 1% Population Sample Survey* as $n_{i(t_0)}$. However, this data is not available in some cities. In such cases, the *2006 Statistical Communiqué on National Economic and Social Development* of each city is used for the analysis. Furthermore, if neither 2005 nor 2006 resident population is available, data from the *2000 population census* is employed. The average growth rate of population of city i, is calculated as $1+\beta_i = (n_{i(T)}/n_{i(t_0)})^{1/(T-t_0)}$, where $n_{i(T)}$ is the resident population of city i in the 2010 Population Census.

controlled municipalities. Since the total population in 2010 is 1.34 billion, more than 90% of the population is included in the analysis.

We compiled the attribute data from the *China City Statistical Yearbook* edited by the National Bureau of Statistics of China. However, in the *China City Statistical Yearbook*, data for specific years in some cities is lacking. In addition, some values provided in the *China City Statistical Yearbook* seem to be inaccurate. We supplemented the missing data and corrected the inaccurate data using other statistics and governmental official reports.

Five variables are adopted as proxy variables for the attributes that affect the well-being of individuals. As a proxy variable for individual income, we employ per capita GDP in each city, deflated by the GDP deflator of 2005=100. Using GDP as a proxy for individual income may be controversial since GDP is a territorial, rather than a personal concept for economic activity.

For environmental quality, we focus on air pollution. In particular, we employ industrial SO_2 and soot emissions as proxy variables for environmental quality. In the 11th five-year plan, reduction of total SO_2 emissions was set as a binding target. Recently, soot emission from energy consumption has become suspected to cause serious health damage. In the analysis, both emissions are measured by their intensity in the geographical area of each city: emission intensity is defined as the *amount of emissions* (t)/*land area* (km^2). Since increases in SO_2 and soot emissions cause the environment to deteriorate, we capture the environmental quality as *E - emission intensity*, where *E* denotes the environmental capacity per land area. The results do not change if we do not include *E* in the calculation as long as it is constant across the various regions. Since environmental quality has the property of equitable consumption, we do not transform it into per a capita level.

The distribution of medical resources is measured by the number of doctors and hospital beds in each city. The number of doctors per ten thousand population can be interpreted as an indicator of human resources for medical care. We include professional doctor and assistant doctors in the analysis. The number of hospital beds per ten thousand population reflects the physical level of medical resources. We concentrate our attention on the supply side of medical care services although the demand side, such as establishing a medical care system, is an important issue. Regional inequality in medical care due to demand will induce concentration of medical resources in particular region. Therefore, by investigating the inequality of medical resources distribution, we can also take into account part of the demand-side problem.

In summary, the variables for city j in the given year are defined as follows:

$$\mathbf{x}_j = \begin{bmatrix} x_{1j} \\ x_{2j} \\ x_{3j} \\ x_{4j} \\ x_{5j} \end{bmatrix} = \begin{bmatrix} GDP\ per\ capita\ in\ city\ j \\ -Industrial\ SO_2\ emission\ intensity\ in\ city\ j \\ -Industrial\ soot\ emission\ intensity\ in\ city\ j \\ Number\ of\ doctors\ per\ ten\ thousand\ population\ in\ city\ j \\ Number\ of\ hospital\ beds\ per\ ten\ thousand\ population\ in\ city\ j \end{bmatrix}$$

For the base year, the variables for city j, $\mathbf{y}_j$, are represented similarly.

Table 3 presents the summary statistics for the five attributes mentioned above. From Table 3 it is clear that the mean value of each attribute moves toward improving social well-being: the mean GDP, the numbers of doctors and hospital beds increase, while the mean values of SO_2 and soot emissions decline during the period. Since the utility index is assumed to be concave, it is undesirable that the minimum utility-index, hence minimum values of GDP and of medical resources, and maximum values of emissions, deteriorates. While the minimum number of hospital beds continuously increased, the minimum number of doctors declined from 2008 to 2009. Similarly, the maximum value of SO_2 emissions increased from 2009 to 2010. Furthermore, soot emissions show unsteady changes.

Table 3 Summary statistics of attributes

	2006	2007	2008	2009	2010
GDP per capita (ten thousand 2005 CNY)					
Mean	1.72	1.89	2.09	2.29	2.56
Std. Dev.	1.44	1.47	1.59	1.54	1.71
Min	0.27	0.32	0.31	0.40	0.45
Max	15.62	14.68	16.25	11.04	14.26
Industrial SO_2 emission intensity (t/km^2)					
Mean	8.59	8.42	7.74	7.19	7.11
Std. Dev.	10.61	10.03	8.82	7.72	7.64
Min	0.06	0.06	0.05	0.04	0.01
Max	67.96	62.96	56.16	56.10	57.01
Industrial soot emission intensity (t/km^2)					
Mean	2.87	2.53	2.34	2.10	2.04
Std. Dev.	3.05	2.81	2.70	2.37	2.35
Min	0.01	0.01	0.02	0.02	0.02
Max	15.81	15.11	16.88	16.08	14.85
Number of doctors (per ten thousand population)					
Mean	15.39	16.43	16.97	17.47	18.28
Std. Dev.	6.37	6.48	6.32	6.54	7.01
Min	3.78	4.26	5.85	4.80	6.93
Max	59.09	50.73	47.07	49.59	64.26
Number of hospital beds (per ten thousand population)					
Mean	27.24	28.32	29.97	32.42	34.08
Std. Dev.	10.62	9.89	9.79	10.23	9.47
Min	10.36	10.81	11.07	11.77	12.36
Max	84.25	73.42	73.62	104.27	77.82
GDP deflator (2005=100)					
	103.81	111.74	120.41	119.68	127.54

Note: Weighted by population

As discussed in the previous section, when USM does not hold, the optimal value of (P1) depends on the shadow prices of the attributes. In the present analysis, the shadow prices applied to region j, $\mathbf{w}_j = (w_{1j}, \ldots, w_{5j})$, are set as follows:

$$w_{1j} = \frac{n_{j(X)}}{n_X} \times 10^4 \times p_1 : \text{GDP} \tag{8a}$$

$$w_{2j} = \frac{l_{j(X)}}{n_X} \times p_2 : \text{SO}_2 \text{ emissions} \tag{8b}$$

$$w_{3j} = \frac{l_{j(X)}}{n_X} \times p_3 : \text{Soot emissions} \tag{8c}$$

$$w_{4j} = \frac{n_{j(X)}}{n_X} \times p_4 : \text{Doctors} \tag{8d}$$

$$w_{5j} = \frac{n_{j(X)}}{n_X} \times p_5 : \text{Hospital beds} \tag{8e}$$

where $l_{j(X)}$ denotes the land area of city j in the given year.[22] In the above expressions, p_i denotes a positive coefficient. For example, p_2 can be interpreted as the cost of reducing one ton of SO_2 emissions. As a benchmark case, we set p_1=100, p_2=4, p_3=1, p_4=135 and p_5=195 based on estimated costs for treating the pollution emissions (p_2 and p_3) and on government expenditures for the medical care (p_4 and p_5).

Prior to the USM investigation, we investigate trends in regional disparities with an item-by-item approach, where the GL dominance criterion is applied to each attribute. Table 4 presents the results.

Table 4 Item-by-item dominance relation

		Given year: *X*				
		2006	2007	2008	2009	2010
Base year: *Y*	2006	-	I, E_1, E_2, M_1, M_2	I, E_1, M_1, M_2	I, E_1, M_1, M_2	I, E_1, E_2, M_1, M_2
	2007	None	-	E_1, M_1, M_2	I, E_1, M_1, M_2	I, E_1, E_2, M_1, M_2
	2008	None	None	-	I, E_2, M_2	I, E_1, M_1, M_2
	2009	None	None	None	-	I, M_1, M_2
	2010	None	None	None	None	-

Notes: I=Per capita GDP, E_1=SO_2 emission, E_2=Soot emission, M_1=Number of doctors, M_2=Number of hospital beds. In each cell, J(=I, E_1, E_2, M_1, M_2) implies that variable J in the base year is GL-dominated by that in the given year.

From Table 4, we see the following: First, for per capita GDP, except for the pair {2007, 2008}, the distribution of the later year dominates the distribution of the previous year in terms of GL.[23] Second, in medical resources, the later year's distribution dominates the previous year's one except for the number of doctors in {2008, 2009}. Generally speaking, regional inequality in medical resources improved from 2006 to 2010 assuming that this is appropriately measured by the numbers of doctors and hospital beds. Finally, environmental quality shows somewhat unsteady changes. Except for the pairs {2006, 2007}, {2006, 2010}, and {2007, 2010}, at least one of two emissions is not rankable according to the GL dominance criterion. In this sense, we can predict that

22. In equation (8a), 10^4 is multiplied because the population is measured in 10000 persons.
23. In what follows, $\{t_1, t_2\}$ implies the comparison of $\{\mathit{base\ year}, \mathit{given\ year}\}$.

the distribution of environmental variables significantly affects welfare ordering during the time period of the analysis.

A necessary condition for USM is that each attribute can be related to GL dominance.[24] From the results described above, this condition is satisfied for the pairs {2006, 2007}, {2006, 2010}, and {2007, 2010}. For these pairs, we can say that social well-being in the given year is better than the base year if we specify the utility index as additive separable, $u_{i(X)} = v_1(x_{1i})+, \dots, +v_m(x_{mi})$. However, for the general form of utility indices, this may not be necessarily true. In the next section, we consider social well-being in the recent Chinese economy based on this data set.

4 Results and Implications

First, solving (P1) for the distribution pairs in the data from 2006 to 2010, we check the USM criterion. As mentioned in the previous section, the pairs {2006, 2007}, {2006, 2010}, and {2007, 2010} are candidates for USM. However, the optimal solution of (P1) will provide information useful for understanding the situation. So, we solve (P1) for all distribution pairs. As shown in Proposition 2, a given year's distribution of attributes is uniformly supermajorized by the base year's distribution if the optimal value of (P1) is zero. In this case, social well-being, as characterized by equation (5), has improved from the base year to the given year.

The results are shown in Table 5. We can obtain the following information from Table 5. Firstly, the distribution of 2010 dominates that of 2006 in USM. That is, social well-being in 2010 is better than 2006. However, the optimal solutions of (P1) other than {2006, 2010} are positive: these pairs are not rankable by USM. In the item-by-item approach, three pairs of distributions showed GL dominance for all attributes. Nevertheless, the fact that two of them are not robust for USM stresses the importance of simultaneous consideration of multidimensional attributes.

Table 5 Optimal value of (P1)

		Given year: *X*				
		2006	2007	2008	2009	2010
Base year: *Y*	2006	-	2.061E+00	3.260E-01	5.078E-02	**5.669E-15**
	2007	1.845E+05	-	3.478E+00	1.078E-01	1.665E-02
	2008	3.818E+05	1.976E+05	-	1.838E+00	2.723E-01
	2009	6.323E+05	4.479E+05	2.509E+05	-	1.385E+00
	2010	8.986E+05	7.141E+05	5.170E+05	2.670E+05	-

Notes: Calculations implemented using 'linprog' function of MATLAB. Coefficient vector is set according to (7), (8), p_1=100, p_2=4, p_3=1, p_4=135, and p_5=195.

Secondly, when the given year is after the base year, the optimal value of (P1) becomes smaller as the given year becomes later compared with the base year. The optimal value represents the increments of attributes needed to achieve USM when we regard $\mathbf{w}_L$ as a shadow price vector of attributes. Applying this interpretation to the results, we can say that, although not actually improved,

24. This property is sometimes referred to as row-wise majorization. See Marshall et al.(2011).

the distribution of attributes has been gradually changing in the direction of social well-being improvement.

Now, we investigate which attribute(s) may be bottlenecks that hamper improved welfare. According to the item-by-item analysis in the previous section, we can predict that the distribution of environmental quality hinders achieving welfare improvement. In order to clarify this prediction, we solve (P1) without SO_2 and soot intensities. The results are summarized in Table 6.

Table 6 Optimal value of (P1) without SO_2 and soot emissions

		Given year: *X*				
		2006	2007	2008	2009	2010
Base year: *Y*	2006		5.803E-01	**2.677E-11**	3.050E-02	**1.707E-14**
	2007	1.845E+05		3.087E+00	**2.738E-15**	**3.666E-16**
	2008	3.818E+05	1.975E+05		6.998E-01	**4.679E-15**
	2009	6.323E+05	4.479E+05	2.509E+05		**1.969E-14**
	2010	8.985E+05	7.141E+05	5.170E+05	2.670E+05	

Notes: Calculations are implemented using the 'linprog' function of MATLAB. Coefficient vector is set according to equations (7) and (8), p_1=100, p_2=4, p_3=1, p_4=135, and p_5=195.

Table 6 shows that the distribution in 2010 consisting of GDP and medical resources is superior based on USM to any base year. Therefore, considering Table 5 and Table 6 together, we can conclude that in 2010, the distribution of environmental quality is the reason some pairs of distributions are non-rankable.

Next, we focus on the most recent pair {2009, 2010}. How much of an attribute is required in what region to achieve USM? To answer this question, we consider the attribute increments according to equation (5) for the pair {2009, 2010}. For city *j*, we express the total amounts of the attributes to be increased as follows:

$$\Delta GDP_j = n_{j(X)}\Delta x_{1j}$$

$$\Delta(SO_2\ emission)_j = -\Delta x_{2j}$$

$$\Delta(Soot\ emission)_j = -\Delta x_{3j}$$

$$\Delta(Doctors)_j = n_{j(X)}\Delta x_{4j}$$

$$\Delta(Hospital\ beds)_j = n_{j(X)}\Delta x_{5j}$$

Table 7 summarizes the results. From Table 7, we see that, as discussed above, environmental quality is the main factor that should be improved to achieve the dominance relation. Also, cities with low per capita GDP do not necessarily need attributes to be improved. Indeed, comparing the cities listed in Table 7 with those of Table 1, we recognize that Anshun in Guizhou province is the only city among the 20 lowest-GDP cities where attributes need to be improved. In contrast, four of the twelve cities listed in Table 7 are from the 20 cities with the highest GDP. In this sense, only focusing on reducing the income disparity between the coastal and inland regions, which is emphasized in the literature, is insufficient to develop public policies that will improve social well-being.

As discussed in the previous section, the optimal solution depends on the coefficient vector, hence the shadow price vector. Taking into account recent economic growth trends, a drastic reduction in pollution may be difficult for practical reasons. If the results summarized in Table 7 do not change under the different shadow prices, improving environmental quality is essential for the future improvement in China's social well-being.

Table 8 summarizes the attribute increments when the shadow prices of environmental quality are high: p_2=400 and p_3=100 instead of p_2=4 and p_3=1. Most of the cities listed in Table 7 where emissions must be reduced to improve social welfare are also listed in Table 8. Moreover, in seven of the eleven cities listed in both Table 7 and Table 8, reducing the emissions is still needed under the relatively high shadow price for environmental quality. Therefore, reducing SO_2 and soot emissions in these cities is indispensable in improving social welfare. In addition, we can see that the required increments of medical resources are larger than the benchmark case. In particular, some cities located in inland areas such as Henan, Sichuan and Guizhou provinces need to increase the amount of medical resources.

Table 7 Changes in attributes to improve social well-being {2009, 2010}

City	Province	Δ Attributes									
		GDP		$-SO_2$		-Soot		Doctor		Hospital bed	
		10^4CNY	%	t	%	t	%	Person	%		%
Wuhai	Inner Mongolia	0	(0.00)	1977	(1.98)	92	(0.48)	15	(1.15)	0	(0.00)
Shanghai		0	(0.00)	12913	(5.83)	5450	(13.04)	0	(0.00)	0	(0.00)
Zhoushan	Zhejiang	0	(0.00)	0	(0.00)	2921	(16.68)	0	(0.00)	0	(0.00)
Huainan	Anhui	0	(0.00)	0	(0.00)	462	(1.20)	0	(0.00)	0	(0.00)
Puyang	Henan	0	(0.00)	0	(0.00)	229	(1.07)	0	(0.00)	0	(0.00)
Luohe	Henan	0	(0.00)	0	(0.00)	890	(9.24)	0	(0.00)	0	(0.00)
Shenzhen	Guangdong	0	(0.00)	1755	(5.38)	0	(0.00)	0	(0.00)	0	(0.00)
Shantou	Guangdong	0	(0.00)	0	(0.00)	728	(14.74)	0	(0.00)	0	(0.00)
Foshan	Guangdong	0	(0.00)	0	(0.00)	4119	(12.33)	0	(0.00)	0	(0.00)
Dongguan	Guangdong	0	(0.00)	9155	(9.16)	2146	(7.37)	0	(0.00)	0	(0.00)
Neijiang	Sichuan	0	(0.00)	0	(0.00)	5829	(24.09)	0	(0.00)	0	(0.00)
Anshun	Guizhou	0	(0.00)	1351	(0.91)	0	(0.00)	284	(16.69)	0	(0.00)
Total		0	(0.00)	27150	(0.16)	22866	(0.42)	298	(0.01)	0	(0.00)

Notes: p_1=100, p_2=4, p_3=1, p_4=135, p_5=195.

Table8 Changes in attributes to improve social well-being {2009, 2010} (relatively high shadow price of environment)

City	Province	Δ Attributes									
		GDP		$-SO_2$		-Soot		Doctor		Hospital bed	
		10^4CNY	%	t	%	t	%	Person	%		%
Wuhai	Inner Mongolia	0	(0.00)	1977	(1.98)	92	(0.48)	15	(1.15)	0	(0.00)
Qitaihe	Heilongjiang	0	(0.00)	0	(0.00)	0	(0.00)	75	(6.42)	0	(0.00)
Shanghai		0	(0.00)	10416	(4.70)	4914	(11.76)	411	(0.96)	0	(0.00)
Zhoushan	Zhejiang	0	(0.00)	0	(0.00)	2921	(16.68)	0	(0.00)	0	(0.00)
Huainan	Anhui	0	(0.00)	0	(0.00)	462	(1.20)	0	(0.00)	0	(0.00)
Zibo	Shandong	0	(0.00)	0	(0.00)	0	(0.00)	103	(1.31)	0	(0.00)
Zhengzhou	Henan	0	(0.00)	0	(0.00)	0	(0.00)	3431	(22.90)	0	(0.00)
Kaifeng	Henan	0	(0.00)	0	(0.00)	0	(0.00)	8	(0.13)	0	(0.00)
Luohe	Henan	0	(0.00)	0	(0.00)	0	(0.00)	98	(3.55)	0	(0.00)
Shenzhen	Guangdong	0	(0.00)	1755	(5.38)	0	(0.00)	0	(0.00)	0	(0.00)
Shantou	Guangdong	0	(0.00)	0	(0.00)	0	(0.00)	311	(5.54)	1316	(11.53)
Foshan	Guangdong	0	(0.00)	0	(0.00)	3137	(9.39)	199	(1.68)	0	(0.00)
Dongguan	Guangdong	0	(0.00)	8964	(8.97)	1735	(5.96)	453	(3.43)	0	(0.00)
Neijiang	Sichuan	0	(0.00)	0	(0.00)	0	(0.00)	531	(13.09)	0	(0.00)
Meishan	Sichuan	0	(0.00)	0	(0.00)	0	(0.00)	122	(3.81)	0	(0.00)
Liupanshui	Guizhou	0	(0.00)	0	(0.00)	0	(0.00)	226	(8.80)	0	(0.00)
Anshun	Guizhou	0	(0.00)	0	(0.00)	0	(0.00)	308	(18.09)	160	(2.90)
Total		0	(0.00)	23111	(0.14)	13261	(0.25)	6296	(0.28)	1477	(0.04)

Notes: p_1=100, p_2=400, p_3=100, p_4=135, p_5=195.

5 Concluding Remarks

In this paper, we evaluate multidimensional regional inequality in China from 2006 to 2010 based on data consisting of prefecture-level cities and direct-controlled municipalities. Taking into account not only income but also the environment and medical resources, we consider welfare ordering during that time period. As shown in Figure 1 in the Introduction, regional (relative) inequality based on provincial data has tended to decline. In this sense, one may claim that rapid economic growth may contribute to improved social welfare. However, the analysis presented in Section 3 reveals that social well-being is not necessarily improved year by year when we consider other aspects of welfare than individual income.

Based on these results, we find that the distribution of pollution emissions is a bottleneck that hinders improving social welfare. On the other hand, recent changes in the distributions of GDP and of medical resources work toward improving social well-being. That is, when we consider a utility index consisting of GDP and medical resources, the social well-being in 2010 is better than any previous year in the time period.

When we compare 2009 and 2010, it should be noted that the GDPs of cities where polluting emissions need to be reduced to improve social welfare are not necessarily low. Improving the environmental quality in rich cities such as Shanghai and Shenzhen is an urgent problem to be solved not only for each individual city but also for overall social welfare.

There are several points for future research. First, considering the other variables that affect well-being may reveal other rich implications. For example, environmental quality in each city is measured by SO_2 emission. Some other environmental aspects like water contaminants may be important factors determining well-being. Within the medical care services, we concentrate on the supply side. Alternatively, the demand side of medical care, such as health status, should be considered in more comprehensive studies.

Second, by expanding the period to be evaluated, we can obtain more insights on China's regional inequality. Although the present study of the five years between 2006 and 2010 is informative in that it reveals recent trends in inequality, studies of longer periods will be needed to evaluate development policies such as China’s Great Western Development and Northeast Area Revitalization.

As a theoretical consideration, we do not implement any statistical tests to evaluate our welfare orderings. A method for statistical inference need to be developed for the procedure presented here.

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Appendix

Proof of Proposition 1

First, we prove the following Lemma.

Lemma A1. Let **X** and **Y** be m-by-$\tilde{n}$ matrices consisting of m dimensional column vectors $\mathbf{x}_i$ and $\mathbf{y}_i$, respectively. The following two conditions are equivalent.

(1) $\mathbf{YQ} \leq \mathbf{X}$ holds for some doubly stochastic matrix $\mathbf{Q}$.

(2) $\sum_{i=1}^{\tilde{n}} u(\mathbf{x}_i) \geq \sum_{i=1}^{\tilde{n}} u(\mathbf{y}_i)$ holds for all $u \in \Omega_u$.

Proof. (1) ⇒ (2) follows from that u is non-decreasing concave function in attributes. Now, we consider 2)⇒1). Suppose that for all $\tilde{n}$-by-$\tilde{n}$ doubly stochastic matrices $\mathbf{Q}$,

$$\mathbf{YQ} \nleq \mathbf{X} \tag{A.1}$$

Vectrizing inequality (A.1), we can state that

$$\tilde{\mathbf{A}}\tilde{\mathbf{p}} \leq \tilde{\mathbf{b}} \tag{A.2}$$

does not have non-positive solution, $\tilde{\mathbf{p}}$, where

$$\tilde{\mathbf{A}} \equiv \begin{bmatrix} \mathbf{I}_{\tilde{n}} \otimes \mathbf{Y} \\ \mathbf{e}_{\tilde{n}} \otimes \mathbf{I}_{\tilde{n}} \\ -\mathbf{e}_{\tilde{n}} \otimes \mathbf{I}_{\tilde{n}} \\ \mathbf{I}_{\tilde{n}} \otimes \mathbf{e}_{\tilde{n}} \\ -\mathbf{I}_{\tilde{n}} \otimes \mathbf{e}_{\tilde{n}} \end{bmatrix}, \text{ and } \tilde{\mathbf{b}} \equiv \begin{bmatrix} \mathrm{vec}\mathbf{X} \\ \mathbf{e}_{\tilde{n}}^T \\ -\mathbf{e}_{\tilde{n}}^T \\ \mathbf{e}_{\tilde{n}}^T \\ -\mathbf{e}_{\tilde{n}}^T \end{bmatrix}.$$

From the theorem of alternative on non-negative solution in linear inequality (Gale 1960, Theorem 2.8), we know that the system of inequalities

$$\mathbf{v}\tilde{\mathbf{A}} \geq \mathbf{0}, \tag{A.3}$$

$$\mathbf{v}\tilde{\mathbf{b}} < 0, \tag{A.4}$$

have a non-negative solution, $\mathbf{v}$. Let $\mathbf{v}^* = [\mathbf{v}_1^*, \dots, \mathbf{v}_{\tilde{n}}^*, \check{\mathbf{z}}^*, \hat{\mathbf{z}}^*, \check{\mathbf{c}}^*, \hat{\mathbf{c}}^*]$ be a non-negative vector satisfying inequalities (A.3) and (A.4), where $\mathbf{v}_i^* \in \mathbb{R}_+^m$ for $i = 1, \dots, \tilde{n}$, and $\check{\mathbf{z}}^*, \hat{\mathbf{z}}^*, \check{\mathbf{c}}^*, \hat{\mathbf{c}}^* \in \mathbb{R}_+^{\tilde{n}}$. Inequality (A.3) implies that

$$\mathbf{v}_i^*\mathbf{y}_j + z_j^* + c_i^* \geq 0$$

holds for $i, j = 1, \dots, \tilde{n}$, where $z_j^* \equiv \check{z}_j^* - \hat{z}_j^*$ and $c_i^* \equiv \check{c}_i^* - \hat{c}_i^*$. Furthermore, from inequality (A.4), we obtain

$$\sum_{i=1}^{\tilde{n}} (\mathbf{v}_i^*\mathbf{x}_i + z_i^* + c_i^*) < 0.$$

Now, let define $V^* = \{[c_1^*, \mathbf{v}_1^*], \dots, [c_{\tilde{n}}^*, \mathbf{v}_{\tilde{n}}^*]\}$ as a set consisting of m+1 dimensional vector. We can confirm the following inequalities:

$$\textstyle\sum_{i=1}^{\tilde{n}} (\mathbf{v}_i^*\mathbf{x}_i + z_i^* + c_i^*) \geq \sum_{i=1}^{\tilde{n}} \min_{[c_j^*,\mathbf{v}_j^*]\in V^*} \left\{ [c_j^*, \mathbf{v}_j^*] \begin{bmatrix} 1 \\ \mathbf{x}_i \end{bmatrix} \right\} + \sum_{i=1}^{\tilde{n}} z_i^*, \tag{A.5}$$

$$\textstyle\sum_{i=1}^{\tilde{n}} \min_{[c_j^*,\mathbf{v}_j^*]\in V^*} \left\{ [c_j^*, \mathbf{v}_j^*] \begin{bmatrix} 1 \\ \mathbf{y}_i \end{bmatrix} \right\} + \sum_{j=1}^{\tilde{n}} z_j^* \geq 0. \tag{A.6}$$

Combining inequality (A.5) with inequality (A.6), we obtain

$$\textstyle\sum_{i=1}^{\tilde{n}} \min_{[c_j^*,\mathbf{v}_j^*]\in V^*} \left\{ [c_j^*, \mathbf{v}_j^*] \begin{bmatrix} 1 \\ \mathbf{y}_i \end{bmatrix} \right\} > \sum_{i=1}^{\tilde{n}} \min_{[c_j^*,\mathbf{v}_j^*]\in V^*} \left\{ [c_j^*, \mathbf{v}_j^*] \begin{bmatrix} 1 \\ \mathbf{x}_i \end{bmatrix} \right\}. \tag{A.7}$$

Consider a utility index such as

$$\tilde{u}(\mathbf{x}) = \textstyle\sum_{i=1}^{\tilde{n}} \min_{[c_j^*,\mathbf{v}_j^*]\in V^*} \left\{ [c_j^*, \mathbf{v}_j^*] \begin{bmatrix} 1 \\ \mathbf{x} \end{bmatrix} \right\}. \tag{A.7}$$

It is easily verified that $\tilde{u}(\mathbf{x}) \in \Omega_u$. Therefore, if $\mathbf{YQ} \nleq \mathbf{X}$, we can find a utility index such that $\sum_{i=1}^{\tilde{n}} u(\mathbf{x}_i) < \sum_{i=1}^{\tilde{n}} u(\mathbf{y}_i)$ holds.

Next, we focus on $\hat{\mathbf{X}}$ and $\hat{\mathbf{Y}}$. We define a replication matrix as follows:

$$\hat{\mathbf{G}}_J \equiv \begin{bmatrix} \mathbf{e}_{n_I n_{1(J)}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{e}_{n_I n_{2(J)}} & & \vdots \\ \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{e}_{n_I n_{r(J)}} \end{bmatrix},$$

for $I, J \in \{X, Y\}$ and $I \neq J$. Using the replication matrix, we define m-by-$\tilde{n}(= n_X n_Y)$ matrix as follows:

$$\mathbf{X}^* \equiv \hat{\mathbf{X}}\hat{\mathbf{G}}_X, \tag{A.9}$$

$$\mathbf{Y}^* \equiv \hat{\mathbf{Y}}\hat{\mathbf{G}}_Y. \tag{A.10}$$

It should be noted that

$$\widehat{\mathbf{G}}_X\widehat{\mathbf{G}}_X^T = n_Y \times \text{diag}\left[n_{1(X)}, \dots, n_{r(X)}\right] = \tilde{n} \times \mathbf{S}, \tag{A.11}$$

is invertible and $\widehat{\mathbf{G}}_X\widehat{\mathbf{G}}_X^T > \mathbf{O}$. Furthermore, we can consider the following block matrix

$$\widehat{\mathbf{G}}_J^- \equiv \begin{bmatrix} \frac{1}{n_I n_{1(J)}}\mathbf{e}_{n_I n_{1(J)}}^T & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \frac{1}{n_I n_{2(J)}}\mathbf{e}_{n_I n_{2(J)}}^T & & \vdots \\ \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \frac{1}{n_I n_{r(J)}}\mathbf{e}_{n_I n_{r(J)}}^T \end{bmatrix}.$$

Since $\widehat{\mathbf{G}}_J\widehat{\mathbf{G}}_J^- = \mathbf{I}_r$, we can verify that $\mathbf{Y}^*\widehat{\mathbf{G}}_Y^- = \widehat{\mathbf{Y}}$ holds.

Noting that the social evaluation function defined by equation (2) is replication invariant, we obtain the following result:

Lemma A2. For two distributions of attributes, $\widehat{\mathbf{X}}$ and $\widehat{\mathbf{Y}}$, the following two conditions are equivalent.

(1) $\mathbf{Y}^*\mathbf{Q} \le \mathbf{X}^*$ holds for some doubly stochastic matrix $\mathbf{Q}$.
(2) $\frac{1}{n_X}\sum_{i=1}^r n_{i(X)}u(\mathbf{x}_i) \ge \frac{1}{n_Y}\sum_{i=1}^r n_{i(Y)}u(\mathbf{y}_i)$ holds for all $u \in \Omega_u$.

Proof. It follows from Lemma A1 and replication invariance of the social evaluation function.

Now, we prove the following Lemma.

Lemma A3. For two distributions of attributes, $\widehat{\mathbf{X}}$ and $\widehat{\mathbf{Y}}$, the following two conditions are equivalent.

(1) $\mathbf{Y}^*\mathbf{Q} \le \mathbf{X}^*$ holds for some doubly stochastic matrix $\mathbf{Q}$.
(2) $\widehat{\mathbf{Y}}\widehat{\mathbf{Q}} \le \widehat{\mathbf{X}}\mathbf{S}$ holds for some compressed doubly stochastic matrix $\widehat{\mathbf{Q}}$.

Proof. First, suppose that $\mathbf{Y}^*\mathbf{Q} \le \mathbf{X}^*$ holds for some doubly stochastic matrix $\mathbf{Q}$. From equations (A.9) and (A.10), $\mathbf{Y}^*\mathbf{Q} \le \mathbf{X}^*$ implies $\widehat{\mathbf{Y}}\widehat{\mathbf{G}}_Y\mathbf{Q} \le \widehat{\mathbf{X}}\widehat{\mathbf{G}}_X$. Since $\mathbf{Q}$ is doubly stochastic, $\widehat{\mathbf{G}}_Y\mathbf{Q}\widehat{\mathbf{G}}_X^T$ is a r-by-r nonnegative matrix whose ith row sum and ith column sum are $n_X n_{i(X)}$ and $n_Y n_{i(X)}$, respectively. Hence, we can consider a compressed doubly stochastic matrix $\widehat{\mathbf{Q}}$ as follows:

$$\widehat{\mathbf{Q}} = \frac{1}{\tilde{n}}\widehat{\mathbf{G}}_Y\mathbf{Q}\widehat{\mathbf{G}}_X^T. \tag{A.12}$$

Multiplying both sides of $\mathbf{Y}^*\mathbf{Q} \le \mathbf{X}^*$ on the right by $\left(\frac{1}{\tilde{n}}\right)\widehat{\mathbf{G}}_X^T$, we obtain

$$\left(\frac{1}{\tilde{n}}\right)\mathbf{Y}^*\mathbf{Q}\widehat{\mathbf{G}}_X^T = \left(\frac{1}{\tilde{n}}\right)\mathbf{Y}^*\widehat{\mathbf{G}}_Y^-\widehat{\mathbf{G}}_Y\mathbf{Q}\widehat{\mathbf{G}}_X^T = \widehat{\mathbf{Y}}\widehat{\mathbf{Q}} \le \left(\frac{1}{\tilde{n}}\right)\mathbf{X}^*\widehat{\mathbf{G}}_X^T = \widehat{\mathbf{X}}\mathbf{S}. \tag{A.13}$$

From equation (A.13), we can confirm that (1) implies (2). Conversely, suppose that $\widehat{\mathbf{Y}}\widehat{\mathbf{Q}} \leq \widehat{\mathbf{X}}\mathbf{S}$ holds for some compressed doubly stochastic matrix $\widehat{\mathbf{Q}}$. Since $\widehat{\mathbf{X}}\mathbf{S} = \left(\frac{1}{\tilde{n}}\right)\widehat{\mathbf{X}}\widehat{\mathbf{G}}_X\widehat{\mathbf{G}}_X^T$ holds, $\widehat{\mathbf{Y}}\widehat{\mathbf{Q}} \leq \widehat{\mathbf{X}}\mathbf{S}$ implies that

$$\tilde{n}\mathbf{Y}^*\widehat{\mathbf{G}}_Y^-\widehat{\mathbf{Q}}\left(\widehat{\mathbf{G}}_X\widehat{\mathbf{G}}_X^T\right)^{-1} \leq \widehat{\mathbf{X}}, \tag{A.14}$$

where $\left(\widehat{\mathbf{G}}_X\widehat{\mathbf{G}}_X^T\right)^{-1} = \mathrm{diag}\left[\frac{1}{n_Y n_{1(X)}}, \dots, \frac{1}{n_Y n_{r(X)}}\right]$. Multiplying both sides of inequality (A.14) on the right by $\widehat{\mathbf{G}}_X^T$, we obtain

$$\tilde{n}\mathbf{Y}^*\widehat{\mathbf{G}}_Y^-\widehat{\mathbf{Q}}\left(\widehat{\mathbf{G}}_X\widehat{\mathbf{G}}_X^T\right)^{-1}\widehat{\mathbf{G}}_X \leq \mathbf{X}^*. \tag{A.15}$$

Furthermore, it can be verified that

$$\tilde{n}\widehat{\mathbf{G}}_Y^-\widehat{\mathbf{Q}}\left(\widehat{\mathbf{G}}_X\widehat{\mathbf{G}}_X^T\right)^{-1}\widehat{\mathbf{G}}_X\mathbf{e}_{\tilde{n}}^T = \tilde{n}\widehat{\mathbf{G}}_Y^-\widehat{\mathbf{Q}}\mathbf{e}_r^T = \mathbf{e}_{\tilde{n}}^T, \tag{A.16}$$

$$\mathbf{e}_{\tilde{n}}\tilde{n}\widehat{\mathbf{G}}_Y^-\widehat{\mathbf{Q}}\left(\widehat{\mathbf{G}}_X\widehat{\mathbf{G}}_X^T\right)^{-1}\widehat{\mathbf{G}}_X = \frac{\tilde{n}}{n_X}\mathbf{n}_X\left(\widehat{\mathbf{G}}_X\widehat{\mathbf{G}}_X^T\right)^{-1}\widehat{\mathbf{G}}_X = \mathbf{e}_{\tilde{n}}, \tag{A.17}$$

and $\tilde{n}\widehat{\mathbf{G}}_Y^-\widehat{\mathbf{Q}}\left(\widehat{\mathbf{G}}_X\widehat{\mathbf{G}}_X^T\right)^{-1}\widehat{\mathbf{G}}_X \geq \mathbf{O}$. Therefore, $\tilde{n}\widehat{\mathbf{G}}_Y^-\widehat{\mathbf{Q}}\left(\widehat{\mathbf{G}}_X\widehat{\mathbf{G}}_X^T\right)^{-1}\widehat{\mathbf{G}}_X$ is a doubly stochastic matrix. Thus, (2) implies (1).

Using Lemmas A1, A2, and A3, we can confirm that $\widehat{\mathbf{Y}}\widehat{\mathbf{Q}} \leq \widehat{\mathbf{X}}\mathbf{S} \Leftrightarrow \mathbf{Y}^*\mathbf{Q} \leq \mathbf{X}^* \Leftrightarrow SW_X \geq SW_Y$ holds for given $\widehat{\mathbf{X}}$ and $\widehat{\mathbf{Y}}$, and all $u \in \Omega_u$.

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