Working Paper

A Generalized User-Revenue Model of Financial Firms under Dynamic Uncertainty: Equity Capital, Risk Adjustment, and the Conjectural User-Revenue Model[1]

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Abstract

A generalized user-revenue model is proposed in which the volatility risk of quasi short-run profits and equity capital effects reflecting the risk of bearing the costs of financial distress are taken into consideration. This is achieved by extending the conjectural user-revenue model proposed by Homma and Souma (2005). Specifically, uncertainties are added to endogenous holding-revenue and holding-cost rates, and the utility function of financial firms is formulated in terms of both quasi short-run profits and equity capital. The conjectural user-revenue price is extended as a generalized user-revenue price, and the extended generalized-Lerner index is proposed to incorporate these extensions.

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1 Introduction

This paper introduces a generalized user-revenue model (GURM) that accounts for both the volatility risk of quasi short-run profits and equity capital effects reflecting the risk of bearing the costs of financial distress. This is achieved through the application of concepts in the consumption-based capital asset pricing model (CCAPM)[1] to the conjectural user-revenue model (CURM) proposed by Homma and Souma (2005).

The CURM is a general extension of Hancock's (1985, 1987, 1991) user-cost model (UCM) of financial firms.[2] The extensions included in the CURM relax the strictness of the three assumptions of the UCM: (i) that the financial firm under analysis is risk-neutral, (ii) that there exists no strategic interdependence between financial firms, and (iii) that there exists symmetry related to information in the financial asset and liability markets. In reality, these assumptions are seldom fulfilled, and if even one of these assumptions is not met, the estimation of user-cost prices (UCPs) derived under the UCM will be biased.

The UCP is defined based on holding revenues or the costs of financial goods, and the sign of the UCP is a useful criterion for unambiguous classification of financial goods as inputs or outputs. It is widely recognized that the UCP provides an objective criterion based on microeconomics, in contrast to conventional classifications based on a priori assumptions. To make the most use of the UCP under more general assumptions, the CURM derives stochastic user-revenue prices (SURPs) and conjectural user-revenue prices (CURPs) as generalizations of the UCP. The SURP extends the conventional

[1] See, for example, Cochrane (2005, pp.5-35) for a full account of the CCAPM. The important point to note is that the CCAPM is an equilibrium model, whereas the GURM does not aggregate individual bank behavior.

[2] As mentioned in Homma and Souma (2005), Homma et al. (1996) were the first to apply the UCM to the Japanese banking industry, and they estimated a stochastic profit frontier function and an X-profit function for panel data during the High Growth Era. Ōmori and Nakajima (2000) estimated total factor productivity and economies of scope in the Japanese banking industry using data from 1987 to 1995. Other papers applied this approach to measure the value of financial services in the national income accounts (Fixler and Zieschang, 1991, 1992). Nagano (2001) measured the nominal value of financial services in Japan using this approach.

UCP to be applicable to the case of financial firms that are not risk-neutral,[3] and the CURP extends applicability to the cases of financial firms that follow strategic interdependence and financial assets and liabilities-related market information that exhibits asymmetrical characteristics. The relationship between the SURP and CURP is used in the CURM to generalize the Lerner index of monopoly power to the oligopoly of financial firms under dynamic uncertainty.[4] The generalized Lerner index (GLI) is a Lerner index that reflects these extensions of the UCP to the SURP and CURP.

The CURM is thus a more general model that relaxes the criticality of the three assumptions of the UCM. However, there are several other implicit assumptions in the UCM that are not considered in the CURM. The most important of these implicit assumptions are that holding revenues and holding costs are certain and that the utility function of financial firms is independent of equity capital. The former assumption ignores the existence of uncertainties in actual holding revenues and holding costs, and the latter disregards the effects of financial distress costs. Financial distress costs are incurred when the financial firm is expected to experience difficulty in honoring its commitment, and thus include the cost of bankruptcy and the loss

[3] As mentioned in Homma and Souma (2005), Barnett and Zhou (1994) and Barnett et al. (1995) were the first to analyze the user-cost approach under dynamic uncertainty. This is likely to lead to generalizations similar to the analysis of the SURP. Unfortunately, their purpose is the pursuit of more desirable monetary aggregation, and thus not only did they not derive a generalized user-cost price, such as the SURP, but also they did not consider the case in which financial firms are strategically interdependent and in which there are informational asymmetries between buyers and sellers. Furthermore, the formulation of the dynamic-uncertainty model in their papers is less rigorous in terms of the stochastic properties of the exogenous state variables, as compared to the present paper.

[4] As reported in Homma and Souma (2005), other approaches that estimate first-order conditions for profit-maximizing oligopolies have been used to measure the degree of competition and collusion in Japanese financial industries. Souma and Tsutsui (2005) examined the change in the level of competition in the Japanese life insurance industry for the period form 1986 to 2002 using the asset approach and found that the industry was not very competitive, but that the industry became more competitive starting from 1995, when the New Insurance Industry Law came into effect. Uchida and Tsutsui (2005) applied an asset approach, similar to that of Souma and Tsutsui (2005), to the Japanese banking industry and estimated the degree of competition from 1974 to 2000. They found that the market had become more competitive in the 1970s, and judged that the Japanese banking sector faced perfect competition by the middle of the 1990s. Using the H-statistic, Tsutsui and Kamesaka (2005) found that the Japanese securities industry was in monopoly equilibrium in the 1980s and in monopolistic competition equilibrium in the 1990s.

in firm value. An increase in equity capital reduces the risk of the burden of financial distress costs. Financial distress costs and risk reduction by raising equity capital are thus important concerns for financial firms that conduct deposits with institutions such as banks.

In the present paper, the CURM is further extended by incorporating the fundamental premises of the CCAPM. The extension involves two parts: 1) the introduction of uncertainties into endogenous holding-revenue rates (EHRRs) and endogenous holding-cost rates (EHCRs) with corresponding definitions for stochastic endogenous holding-revenue rates (SEHRRs) and stochastic endogenous holding-cost rates (SEHCRs), and 2) the formulation of the utility function of financial firms in terms of both quasi short-run profits and equity capital. The introduction of uncertainty effectively adds risk-adjustment effects to the CURM as expressions of the covariance of uncertain factors in the SEHRR or the SEHCR given a stochastic discount factor. This modification makes it possible to consider the volatility risk of quasi short-run profits explicitly. The new formulation of the utility function introduces equity capital effects into the CURM by allowing the marginal rate of substitution between equity capital and quasi short-run profits to be expressed. This extension effectively incorporates indirect consideration of the risk of bearing the cost of financial distress in addition to subjective evaluation of equity capital by financial firms and the opportunity cost of equity capital. Consideration of these risks is thus incorporated into the SURP, the CURP, and the GLI determined by the extended CURM.

The extended CURM is derived here from the principles of the CCAPM as the generalized user-revenue model (GURM). Definitions for the extended SURP, the CURP, and the GLI are also derived from the GURM in consideration of more general assumptions. The extended SURP and CURP are generically called the generalized user-revenue price (GURP), and the extended GLI is referred to as the extended generalized-Lerner index (EGLI).

The remainder of the present paper is organized as follows. Section 2 deals with the introduction of uncertainties into the EHRR and the EHCR, and the definition of the SEHRR and SEHCR. The utility function of financial firms is also reformulated as a function of quasi short-run profits and equity capital.

In Section 3, the decision of financial firms is formulated as a stochastic dynamic program, and stochastic Euler equations are transformed into a formulation that expresses risk corrections clearly. The GURP is derived, capital effects and risk-adjustment effects are defined, and the relationship between the GURP and the CURP is clarified. The EGLI is then derived based on this relationship, and the relationship between the EGLI and the GLI is clarified. The paper is concluded in Section 4.

2 Introduction of Uncertainties and Inclusion of Equity Capital

In this section, the assumptions of the CURM concerning the certainty of the EHRR and the EHCR and the independence of the utility function of financial firms from equity capital are relaxed by the introduction of uncertainty into the EHRR and the EHCR and the inclusion of both quasi short-run profits and equity capital in the utility function of financial firms. These changes appropriately take into account the uncertainties in actual holding revenues or costs, which drive the volatility risk of quasi short-run profits, and the necessity of financial firms to consider an increase in equity capital as a means of reducing the risk of the burden of financial distress costs.

The derivations in this section are based on three preliminary assumptions. (i) Time is divided into discrete periods. (ii) These periods are sufficiently short, so that variations in exogenous (state) variables within the period can be neglected. That is, exogenous variables are constant within each period but can change discretely at the boundaries between periods. (iii) The process of adjustment is essentially instantaneous, allowing stock adjustment problems to be ignored. These assumptions are made in order to facilitate future empirical research, similar to Hancock (1985, 1987, 1991) and Homma and Souma (2005), in the expectation that the GURM may provide a consistent basis for such research.

2.1 Stochastic Endogenous Holding-Revenue Rates and Holding-Cost Rates

The net cash flow produced by a financial good, that is, a financial asset or liability, is defined in the same way as in the CURM. Similar to the UCM and the CURM, it is assumed that all financial transactions occur at the boundaries between given unit periods of time. Each financial firm holds stocks of financial assets and liabilities as inventory, and the costs or revenues that accrue from the holding of these inventories are components of the net cash flow of services to the firm. These costs or revenues are regarded to be of equal or higher importance than the costs of real resource inputs such as labor, materials, equipment, and facility-related inputs in kind.

The general price index, $p_{G,t}$, is adopted as a deflator for all financial goods at the beginning of period t. The real balance of the j-th financial good of the i-th firm at the beginning of period t is expressed as $q_{i,j,t}$, and the revenue obtained (or cost required) from holdings per currency unit for a single time period is expressed as the holding-revenue rate (or holding-cost rate) $h_{i,j,t+1}$ at the end of period t (and thus at the beginning of period $t+1$). In this case, it is assumed that the holding-revenue rate (or holding-cost rate) is contracted at the beginning of period t and the uncertainty therein is realized at the end of period t. Thus, $h_{i,j,t+1} \cdot q_{i,j,t}$ is the holding revenue or cost, which is received or paid at the end of period t. Financial assets and liabilities are divided into $j = 1, \cdots, N_A$ assets and $j = N_A + 1, \cdots, N_A + N_L$ liabilities.

Definition 1 *During period t, the net cash flow of the i-th firm produced by financial good j, denoted by $q^{NCF}_{i,j,t}$, is defined as*

$$q^{NCF}_{i,j,t} = b_j \cdot \left(h_{i,j,t} \cdot p_{G,t-1} \cdot q_{i,j,t-1} + p_{G,t-1} \cdot q_{i,j,t-1} - p_{G,t} \cdot q_{i,j,t} \right), \qquad (1)$$

where b_j is the parameter used to distinguish between financial assets and liabilities: $b_j = 1$ for assets (i.e., $j = 1, \cdots, N_A$), and $b_j = -1$ for liabilities (i.e., $j = N_A + 1, \cdots, N_A + N_L$).

For example, for an asset such as a loan (with the exception of cash),

$b_j = 1$, in which case the first term of the right-hand side of Eq. (1), $h_{i,j,t} \cdot p_{G,t-1} \cdot q_{i,j,t-1}$, indicates holding revenues, and the last two terms, $p_{G,t-1} \cdot q_{i,j,t-1} - p_{G,t} \cdot q_{i,j,t}$, represent the change in the nominal asset for the period. If loan repayments by the borrower exceed the total new loans for the period, the revised balance indicates a positive change, and if the repayments are lower than the total new loans for the period, the value is negative. These three terms thus express the net cash flow resulting from the acceptance of an asset. However, cash, which is an asset, generates no interest. As such, the holding revenue for cash, even if held, is zero. Similarly, in the case of a liability such as a deposit, $b_j = -1$, the first term of the right-hand side, $-h_{i,j,t} \cdot p_{G,t-1} \cdot q_{i,j,t-1}$, indicates holding costs, while the last two terms, $p_{G,t} \cdot q_{i,j,t} - p_{G,t-1} \cdot q_{i,j,t-1}$, represent the nominal liability change. Therefore, the change is positive if new deposits exceed withdrawn deposits and is negative if new deposits are less than withdrawn deposits. These three terms thus indicate the net cash flow resulting from the issuance of a liability.

To account for the strategic interdependence between financial firms and asymmetry in financial asset and liability market information, holding-revenue rates and holding-cost rates are assumed to be determined endogenously, as in the basic CURM. In the GURM, however, uncertainties in actual holding revenues and holding costs, the drivers of volatility risk for quasi short-run profits, are also considered. Such uncertainty is attributable to unpredictable factors such as uncollected or unpaid interest rates, future service charge rates, capital gains or losses, default rates, and insurance premium rates. The uncertainty in these factors gives rise to volatility in quasi short-run profits. The EHRR and the EHCR with uncertainty correspond to the stochastic representations SEHRR and SEHCR.

Let $r_{i,j,t}$ be the collected interest rate of the j-th asset of the i-th financial firm in period t. Then, $r^Q_{i,j,t}$, $h^S_{i,j,t}$, $h^C_{i,j,t}$, $h^D_{i,j,t}$, and $h^R_{i,j,t}$ are the certain or predictable components of the uncollected interest rate, the service charge rate, capital gains or losses, the default rate, and the SEHRR ($h^R_{i,j,t} = r_{i,j,t} + r^Q_{i,j,t} + h^S_{i,j,t} + h^C_{i,j,t} - h^D_{i,j,t}$). Let $Q_{j,t}$ be the total assets in the market, $\zeta_{i,j,t+1}$ be the sum of the uncertain or unpredictable components of $r^Q_{i,j,t}$, $h^S_{i,j,t}$, $h^C_{i,j,t}$, and $-h^D_{i,j,t}$. Furthermore, let $\mathbf{z}^k_{i,j,t}$ ($k = R, Q, S, D$) be the vectors of exogenous (state) variables affecting each endogenous component of the SEHRR, and let $\mathbf{z}^H_{i,j,t} = \left(\mathbf{z}^{R\prime}_{i,j,t}, \mathbf{z}^{Q\prime}_{i,j,t}, \mathbf{z}^{S\prime}_{i,j,t}, h^C_{i,j,t}, \mathbf{z}^{D\prime}_{i,j,t}\right)^\prime$ be the vectors of exogenous (state) variables of the SEHRR except $\zeta_{i,j,t+1}$, where $h^C_{i,j,t}$ is assumed to be exogenous similar to the case of the CURM. For tractability of analysis, it is also assumed that $h^R_{i,j,t}$ and $\zeta_{i,j,t+1}$ are separable. The SEHRR, $h_{i,j,t+1}$, can then be defined as follows.

Definition 2 *The stochastic endogenous holding-revenue rate of the j-th financial good of the i-th firm at the end of period t, denoted by $h_{i,j,t+1}$, is defined as*

$$\begin{aligned}
h_{i,j,t+1} &= b_C \cdot h^R_{i,j,t} + \zeta_{i,j,t+1} \\
&= b_C \cdot \left(r_{i,j,t} + r^Q_{i,j,t} + h^S_{i,j,t} + h^C_{i,j,t} - h^D_{i,j,t}\right) + \zeta_{i,j,t+1} \\
&= b_C \cdot \left[r_{i,j}\left(Q_{j,t}, \mathbf{z}^R_{i,j,t}\right) + r^Q_{i,j}\left(Q_{j,t}, \mathbf{z}^Q_{i,j,t}\right) + h^S_{i,j}\left(Q_{j,t}, \mathbf{z}^S_{i,j,t}\right) + h^C_{i,j,t}\right. \\
&\quad \left. - h^D_{i,j}\left(Q_{j,t}, \mathbf{z}^D_{i,j,t}\right)\right] + \zeta_{i,j,t+1} \\
&= b_C \cdot h^R_{i,j}\left(Q_{j,t}, \mathbf{z}^H_{i,j,t}\right) + \zeta_{i,j,t+1} \\
&= h_{i,j}\left(Q_{j,t}, \mathbf{z}^H_{i,j,t}, \zeta_{i,j,t+1}\right); \, j = 1, \cdots, N_A, \qquad (2)
\end{aligned}$$

where b_C is the parameter used to distinguish cash from other financial assets. That is, if $q_{i,j,t}$ represents cash (i.e., $j = 1$), then $b_C = 0$, whereas if the financial good is another type of financial asset (i.e., $j \neq 1$), then $b_C = 1$.

The case of $b_C = 0$ gives $h_{i,1,t+1} = \zeta_{i,1,t+1}$, where $\zeta_{i,1,t+1}$ is the rate of uncertain withdrawal claims, and other uncertain mobile payments and the component are deemed to be cash. Uncertain withdrawal claims and other uncertain mobile payments are the response risk to liquidity. Thus, $\zeta_{i,1,t+1} < 0$ if $\zeta_{i,1,t+1}$ predominantly reflects the response risk to liquidity, and $\zeta_{i,1,t+1} > 0$ if $\zeta_{i,1,t+1}$ mainly reflects the component deemed to be cash. The difference between the SEHRR and the EHRR in the CURM is the incorporation of $\zeta_{i,j,t+1}$ in the former. The certain or predictable component of the SEHRR is therefore equivalent to the EHRR, and the SEHRR is defined as EHRR plus the uncertainty $\zeta_{i,j,t+1}$ under the assumption of separability between the two components.

A similar treatment holds for the SEHCR. Let $r_{i,j,t}$ be the paid interest rate of the j-th liability of the i-th financial firm in period t. Then, $r^{Q}_{i,j,t}$, $h^{I}_{i,j,t}$, $h^{S}_{i,j,t}$, $r^{D}_{i,t}$, $\kappa_{i,j,t}$, and $h^{R}_{i,j,t}$ are the certain or predictable components of the unpaid interest rate, the insurance premium rate, the service charge rate, the subjective rate of time preference, the required reserve ratio, and the SEHCR ($h^{R}_{i,j,t} = r_{i,j,t} + r^{Q}_{i,j,t} + h^{I}_{i,j,t} + r^{D}_{i,t} \cdot \kappa_{i,j,t} - h^{S}_{i,j,t}$). Let $Q_{j,t}$ be the total liabilities in the market, and let $\zeta_{i,j,t+1}$ be the sum of the uncertain or unpredictable components of $r^{Q}_{i,j,t}$, $h^{I}_{i,j,t}$, and $-h^{S}_{i,j,t}$. Furthermore, let $\mathbf{z}^{k}_{i,j,t}$ ($k = R, Q, I, S$) be the vectors of exogenous (state) variables affecting each component of the SEHCR, and let $\mathbf{z}^{H}_{i,j,t} = \left(\mathbf{z}^{R\prime}_{i,j,t}, \mathbf{z}^{Q\prime}_{i,j,t}, \mathbf{z}^{I\prime}_{i,j,t}, \mathbf{z}^{S\prime}_{i,j,t}, r^{D}_{i,t}, \kappa_{i,j,t}\right)^{\prime}$ be the vectors of exogenous (state) variables of the SEHCR except $\zeta_{i,j,t+1}$, where $r^{D}_{i,t}$ and $\kappa_{i,j,t}$ are assumed to be exogenous similar to the definition in the CURM. To ensure tractability of analysis, it is also assumed that $h^{R}_{i,j,t}$ and $\zeta_{i,j,t+1}$ are separable. The SEHCR, $h_{i,j,t+1}$, can then be defined as follows.

Definition 3 *The stochastic endogenous holding-cost rate of the j-th financial good of the i-th firm at the end of period t, denoted by $h_{i,j,t+1}$, is defined as*

$$\begin{aligned}
h_{i,j,t+1} &= h^{R}_{i,j,t} + \zeta_{i,j,t+1} \\
&= r_{i,j,t} + r^{Q}_{i,j,t} + h^{I}_{i,j,t} + r^{D}_{i,t} \cdot \kappa_{i,j,t} - h^{S}_{i,j,t} + \zeta_{i,j,t+1} \\
&= r_{i,j}\left(Q_{j,t}, \mathbf{z}^{R}_{i,j,t}\right) + r^{Q}_{i,j}\left(Q_{j,t}, \mathbf{z}^{Q}_{i,j,t}\right) + h^{I}_{i,j}\left(Q_{j,t}, \mathbf{z}^{I}_{i,j,t}\right) + r^{D}_{i,t} \cdot \kappa_{i,j,t} \\
&\quad - h^{S}_{i,j}\left(Q_{j,t}, \mathbf{z}^{S}_{i,j,t}\right) + \zeta_{i,j,t+1} \\
&= h^{R}_{i,j}\left(Q_{j,t}, \mathbf{z}^{H}_{i,j,t}\right) + \zeta_{i,j,t+1} \\
&= h_{i,j}\left(Q_{j,t}, \mathbf{z}^{H}_{i,j,t}, \zeta_{i,j,t+1}\right); \, j = N_A + 1, \cdots, N_A + N_L, \qquad (3)
\end{aligned}$$

where $r^{D}_{i,t} \cdot \kappa_{i,j,t}$ is the implicit tax rate imposed by the reserve requirement similar to the UCM and the CURM.

The reserve requirement is a tax because it requires banks to hold deposits that do not bear interest. The tax is the foregone interest on uninvested required reserves. Similar to the difference between the SEHRR and the EHRR, the difference between the SEHCR and the EHCR in the CURM is the inclusion of $\zeta_{i,j,t+1}$ in the former. If all components of the EHCR are certain or predictable, the SEHCR is simply the EHCR with $\zeta_{i,j,t+1}$ included under the assumption of separability between the EHCR and $\zeta_{i,j,t+1}$.

2.2 Production Technology and Variable Cost Functions

To represent the production technology of financial firms as in the CURM, the vector of real balances of financial goods of the i-th financial firm in period t is defined as $\mathbf{q}_{i,t} = (q_{i,1,t}, \cdots, q_{i,N_A+N_L,t})'$, and the vector of real resource inputs, such as labor, materials, and physical capital, is given by $\mathbf{x}_{i,t} = (x_{i,1,t}, \cdots, x_{i,M,t})'$. The vector of exogenous (state) variables affecting the quality of financial goods is defined as $\mathbf{z}_{i,t}^{Q} = \left(\mathbf{z}_{i,1,t}^{Q\prime}, \cdots, \mathbf{z}_{i,N_A+N_L,t}^{Q\prime}\right)'$, and the index of (exogenous) technical change is expressed by the variable $\tau_{i,t}$. In this case, the efficient production technology can be defined as follows.[5]

Definition 4 *The efficient production technology of the i-th financial firm in period t is represented by the following transformation function:*

$$\phi_i\left(\mathbf{q}_{i,t}, \mathbf{x}_{i,t}, \mathbf{z}_{i,t}^{Q}, \tau_{i,t}\right) = 0. \tag{4}$$

As described in Section 2.1, $\mathbf{z}_{i,j,t}^{Q}$ ($j = 1, \cdots, N_A + N_L$), components of $\mathbf{z}_{i,t}^{Q}$, are exogenous (state) variables affecting the uncollected or unpaid interest rates, the amount of which is interpreted as one measure of the quality of the SEHRR or the SEHCR. This vector thus represents financing technological factors. As seen in the derivatives, financing technological factors affect not

[5]Under the assumption that the process of adjustment is essentially instantaneous, the level of assets a bank chooses on day t does not depend on the level it chose on day $t-1$. In other words, there are no portfolio adjustment costs. However, at present, several banks wish they had the opportunity to divest themselves of some mortgage securities. Thus, the introduction of portfolio adjustment costs is an important task for the future.

only the SEHRR and the SEHCR but also the real resource inputs (i.e., labor, materials, and physical capital) through $r^Q_{i,j,t}$. For this reason, $\mathbf{z}^Q_{i,t}$ is a variable of the transformation function ϕ_i.

As emphasized in the CURM, some elements of the real balance vector $\mathbf{q}_{i,t}$ may be outputs or inputs, but not all can be inputs, as the existence of outputs cannot otherwise be guaranteed. Moreover, the transformation function ϕ_i must satisfy appropriate regularity conditions. That is, ϕ_i is strictly convex in $(\mathbf{q}_{i,t}, \mathbf{x}_{i,t})$ and $\partial\phi_i / \partial q_{i,j,t} > 0$ if $q_{i,j,t}$ is an output, $\partial\phi_i / \partial q_{i,j,t} < 0$ if $q_{i,j,t}$ is an input, and $\partial\phi_i / \partial x_{i,j,t} < 0$, because $\mathbf{x}_{i,t}$ is an input vector.

As the times required for the adjustment of real resource inputs to optimized levels can vary among firms and industries, the vector of real resource inputs $\mathbf{x}_{i,t}$ is divided into vectors of real resource variable inputs $\mathbf{x}^V_{i,t} = \left(x^V_{i,1,t}, \cdots, x^V_{i,M_V,t}\right)'$, which include labor and materials, and real resource fixed inputs $\mathbf{x}^F_{i,t} = \left(x^F_{i,1,t}, \cdots, x^F_{i,M_F,t}\right)'$, which include physical and human capital. Real resource variable inputs are optimized within a single period, taking outputs (financial goods) and fixed inputs (financial goods and real resource fixed inputs) as given. The optimization of real resource fixed inputs therefore requires several periods, similar to the case for financial goods. As a consequence, the optimization of real resource variable inputs must be completed before the optimization of real resource fixed inputs. To deal with this requirement explicitly, it is assumed for a single period that the financial firm takes the vector of variable input prices $\mathbf{p}^V_{i,t} = \left(p^V_{i,1,t}, \cdots, p^V_{i,M_V,t}\right)'$ as given and minimizes real resource variable costs $\sum_{j=1}^{M_V} p^V_{i,j,t} \cdot x^V_{i,j,t}$ with respect to the vector of real resource variable inputs $\mathbf{x}^V_{i,t}$ subject to the transformation function ϕ_i given by Eq. (4). This assumption leads to the following definition of variable cost function.

Definition 5 *The variable cost function of the i-th financial firm in period t, denoted by* $C^V_i\left(\mathbf{p}^V_{i,t}, \mathbf{q}_{i,t}, \mathbf{x}^F_{i,t}, \mathbf{z}^Q_{i,t}, \tau_{i,t}\right)$, *is given by*

$$C^V_i\left(\mathbf{p}^V_{i,t}, \mathbf{q}_{i,t}, \mathbf{x}^F_{i,t}, \mathbf{z}^Q_{i,t}, \tau_{i,t}\right) = \min_{\boldsymbol{x}^V_{i,t}} \left\{ \sum_{j=1}^{M_V} p^V_{i,j,t} \cdot x^V_{i,j,t} \,\middle|\, \phi_i\left(\mathbf{q}_{i,t}, \mathbf{x}_{i,t}, \mathbf{z}^Q_{i,t}, \tau_{i,t}\right) = 0 \right\}. \tag{5}$$

In the transformation function (4), as mentioned above, it is important that not all of the components of the real balance vector $\mathbf{q}_{i,t}$ be inputs. In the case of the variable cost function (5), elements of the real balance vector $\mathbf{q}_{i,t}$ may be outputs or fixed inputs, but not all can be fixed inputs.

As in the CURM, let $\mathbf{q}_{i,t}^O = \left(q_{i,1,t}^O, \cdots, q_{i,N_O,t}^O\right)'$ denote the output vector of real balances of the i-th financial firm in period t, and let $\mathbf{q}_{i,t}^F = \left(q_{i,1,t}^F, \cdots, q_{i,N_F,t}^F\right)'$ be the fixed input vector. Both vectors include all elements of $\mathbf{q}_{i,t}$.[6] Due to the duality between transformation functions and variable cost functions, the variable cost function C_i^V is strictly increasing in $\mathbf{p}_{i,t}^V$ and $\mathbf{q}_{i,t}^O$, strictly decreasing in $\mathbf{x}_{i,t}^F$ and $\mathbf{q}_{i,t}^F$, and homogeneous of degree one and strictly concave in $\mathbf{p}_{i,t}^V$. In addition to these conditions, it is assumed that C_i^V is twice continuously differentiable in all its arguments and strictly convex in $\mathbf{q}_{i,t}$ and $\mathbf{x}_{i,t}^F$. These assumptions become necessary when the dynamically uncertain behavior of financial firms is considered.

In the financial industry, the scale of real resource fixed inputs such as physical capital is smaller than in the manufacturing industry, and the times needed to adjust these inputs are shorter. For this reason, as in the CURM, it is assumed that gross investment achieves instantaneous productivity and the adjustment cost associated with installing capital is zero. Let $I_{i,j,t}$ denote gross investment, and let $\delta_{i,j,t}$ be the depreciation rate, which is defined as a constant and assumed to be given. Capital accumulation can then be defined as follows.

Definition 6 *Capital accumulation of the j-th real resource fixed input of the i-th financial firm at time t, denoted by $x_{i,j,t}^F$, is given by*

$$x_{i,j,t}^F = I_{i,j,t} + (1 - \delta_{i,j,t}) \cdot x_{i,j,t-1}^F;\ j = 1, \cdots, M_F. \tag{6}$$

2.3 Quasi Short-Run Profits, Equity Capital, and Utility Functions

As in the CURM, the profits of a financial firm are defined as the net cash flow generated by employing or issuing financial goods, minus the real resource

[6] In this case, $\mathbf{q}_t = \left(\mathbf{q}_t^{V\prime}, \mathbf{q}_t^{F\prime}\right)'$ and $N_O + N_F = N_A + N_L$ are satisfied.

costs given by the sum of real resource variable and fixed costs. This result gives the "quasi" short-run profit, which differs from the usual short-run profit of a static model in that revenues from financial goods are not expressed by the sum of the product of an output and its price, and fixed costs for financial goods are not represented by the sum of the product of a fixed input and its price. The present quasi short-run profit also differs from that defined in the CURM in that the new definition includes the components of uncertainty in the SEHRR and the SEHCR, upon which the quasi short-run profit is based.

Definition 7 *The quasi short-run profit of the i-th financial firm during period t, denoted by $\pi_{i,t}^{QS}$, is defined as follows:*

$$\begin{aligned}
\pi_{i,t}^{QS} &= \sum_{j=1}^{N_A+N_L} q_{i,j,t}^{NCF} - C_i^V\left(\mathbf{p}_{i,t}^V, \mathbf{q}_{i,t}, \mathbf{x}_{i,t}^F, \mathbf{z}_{i,t}^Q, \tau_{i,t}\right) - \sum_{j=1}^{M_F} p_{i,j,t}^F \cdot I_{i,j,t} \\
&= \sum_{j=1}^{N_A+N_L} b_j \cdot \left[\left\{1 + h_{i,j}\left(Q_{j,t-1}, \mathbf{z}_{i,j,t-1}^H, \zeta_{i,j,t}\right)\right\} \cdot p_{G,t-1} \cdot q_{i,j,t-1} - p_{G,t} \cdot q_{i,j,t}\right] \\
&\quad - C_i^V\left(\mathbf{p}_{i,t}^V, \mathbf{q}_{i,t}, \mathbf{x}_{i,t}^F, \mathbf{z}_{i,t}^Q, \tau_{i,t}\right) - \sum_{j=1}^{M_F} p_{i,j,t}^F \cdot \left[x_{i,j,t}^F - (1 - \delta_{i,j,t}) \cdot x_{i,j,t-1}^F\right] \\
&= \sum_{j=1}^{N_A+N_L} b_j \cdot \left[\left\{1 + b_C \cdot h_{i,j}^R\left(Q_{j,t-1}, \mathbf{z}_{i,j,t-1}^H\right) + \zeta_{i,j,t}\right\} \cdot p_{G,t-1} \cdot q_{i,j,t-1} - p_{G,t} \cdot q_{i,j,t}\right] \\
&\quad - C_i^V\left(\mathbf{p}_{i,t}^V, \mathbf{q}_{i,t}, \mathbf{x}_{i,t}^F, \mathbf{z}_{i,t}^Q, \tau_{i,t}\right) - \sum_{j=1}^{M_F} p_{i,j,t}^F \cdot \left[x_{i,j,t}^F - (1 - \delta_{i,j,t}) \cdot x_{i,j,t-1}^F\right] \\
&= \sum_{j=1}^{N_A+N_L} b_j \cdot \left[\left\{1 + b_C \cdot h_{i,j}^R\left(Q_{j,t-1}, \mathbf{z}_{i,j,t-1}^H\right)\right\} \cdot p_{G,t-1} \cdot q_{i,j,t-1} - p_{G,t} \cdot q_{i,j,t}\right] \\
&\quad - C_i^V\left(\mathbf{p}_{i,t}^V, \mathbf{q}_{i,t}, \mathbf{x}_{i,t}^F, \mathbf{z}_{i,t}^Q, \tau_{i,t}\right) - \sum_{j=1}^{M_F} p_{i,j,t}^F \cdot \left[x_{i,j,t}^F - (1 - \delta_{i,j,t}) \cdot x_{i,j,t-1}^F\right] \\
&\quad + p_{G,t-1} \cdot \sum_{j=1}^{N_A+N_L} b_j \cdot q_{i,j,t-1} \cdot \zeta_{i,j,t}, \qquad (7)
\end{aligned}$$

where $p_{i,j,t}^F$ $(j = 1, \cdots, M_F)$ are the prices of real resource fixed inputs.

The first term of the right-hand side of the last equality (showing the sum of net cash flows) represents the total net revenue of financial goods. The second term denotes the variable cost of real resource variable inputs such as labor and materials, the third term represents the total expenditure on investments, and the last term consists of the uncertain or unpredictable components of the SEHRR and the SEHCR. As described in Section 2.1, following the interpretation that the certain or predictable components of the SEHRR and the SEHCR are equivalent to the EHRR and the EHCR, the first through third terms on the right-hand side correspond to the quasi short-run profit in the CURM, and the fourth term is the difference from the CURM definition.

Financial firms such as banks are characteristically confronted with various risks. Pyle (1997) defined these risks as reductions in firm value due to changes in the business environment. Based on this definition, in the case of banks, these risks were categorized into market risk, credit risk, operational risk, and performance risk. Market risk is the change in net asset value due to changes in underlying economic factors such as interest rates, exchange rates, and equity and commodity prices. Credit risk is the change in net asset value due to changes in the perceived ability of counter-parties to meet their contractual obligations. Operational risk results from costs incurred through mistakes made in carrying out transactions such as settlement failures, failures to meet regulatory requirements, and untimely collections. Performance risk encompasses losses resulting from the failure to properly monitor employees or use appropriate methods (including "model risk").

If these risks are realized, financial firms suffer losses in loans and other assets, in the worst case leading to bankruptcy. In this situation, equity capital plays a role as a cushion against losses, and hence acts as protection against financial distress. According to Berger et al. (1995), financial distress occurs when the financial firm is expected to have difficulty honoring its commitments. Financial distress costs therefore include the costs of bankruptcy such as the costs of transferring ownership of the firm from shareholders to creditors, and any loss in value that may occur as a result of the perception that bankruptcy may be imminent, even if bankruptcy may ultimately be avoided. The latter may result from the loss of talented employees, demands for more timely payments by suppliers, declines in revenues from credit-risk-sensitive products such as long-term swaps and guarantees, and potentially suboptimal operating, investment, and financing decisions due to conflicts of interest between shareholders and creditors.

An increase in equity capital reduces the risk of the burden of financial distress costs, and also reduces the funding costs that may arise due to declining funding rates, provided that creditors are lead to believe that the risk of their assets is low. However, the preservation of equity capital also has disadvantages, such as higher opportunity costs, transaction costs, and agency costs, and lower earnings prospects. Opportunity costs represent the increase in quasi short-run profits over that acquired in the absence of equity capital. Transactions costs for equity capital are higher than those for debt. For example, the costs of issuing equity are higher than those for issuing a bond or debt. Higher agency costs arise due to conflicts of interest between shareholders and managers by reducing managers' incentives to work harder, resulting in poorer expense preference behavior and investment decision making. Finally, the earnings prospects of financial markets decrease with increasing equity capital, putting downward pressure on the stock price of the financial firm. Nevertheless, despite these disadvantages, many financial firms consider the advantages of equity capital to be worth pursuing.

Considering the importance attached to equity capital by real financial firms, the utility function defined to deal explicitly with the attitude of financial firms to risk is assumed to depend on not only the quasi short-run profit ($\pi_{i,t}^{QS}$, Eq. (7)), but also the equity capital $q_{e,i,t}$, which is defined as follows.[7,8]

Definition 8 *The utility function of the i-th financial firm during period t*

[7] Although the utility function indirectly accounts for distress possibility, without an explicitly specified model for bankruptcy, it may be difficult to make any relevant inference about distress costs. The introduction of this model is an important task for the future.

[8] Although the utility function is not a widely used tool for modeling firm behavior in finance literature, circumstances may arise in which a firm may act in a risk-averse or risk-loving manner. For example, when a firm has a positive net worth (capital) and there are costs of financial distress, the firm may act in a risk-averse manner. However, when the net worth of the firm is negative, the firm may act in a risk-loving manner. In this case, the firm could gamble (risk-shift) in order to increase the likelihood that the firm will regain positive net worth before being shut down by regulators.

is defined as

$$u_i\left(\pi^{QS}_{i,t}, q_{e,i,t}\right), \tag{8}$$

where

$$q_{e,i,t} = \sum_{j=1}^{N_A} p_{G,t} \cdot q_{i,j,t} + \sum_{j=1}^{M_F} p^F_{i,j,t} \cdot x^F_{i,j,t} - \sum_{j=N_A+1}^{N_A+N_L} p_{G,t} \cdot q_{i,j,t}. \tag{9}$$

The accounting definition of equity capital is the total asset value minus the total liability value. As in the usual utility function, it is assumed that the utility function is strictly increasing, twice continuously differentiable, and strictly concave in $\pi^{QS}_{i,t}$ and $q_{e,i,t}$.

3 Derivation of Generalized User-Revenue Prices and Extension of Generalized Lerner Indices

The CURM was extended to include uncertainties with respect to the EHRR and the EHCR, giving the SEHRR and the SEHCR and a new formulation of the utility function of financial firms that accounts for both quasi short-run profits and equity capital. The inclusion of uncertainty through definition of the SEHRR and the SEHCR introduces risk-adjustment effects into the CURM. These effects are expressed as the covariance of uncertain factors in the SEHRR or the SEHCR, according to a stochastic discount factor, and allow the volatility risk of quasi short-run profits to be considered explicitly. The redefinition of the utility function introduces equity capital effects, which are expressed as the marginal rate of substitution between equity capital and quasi short-run profits. The new utility function thus makes it possible to indirectly evaluate the risk of bearing the cost of financial distress, as well as the opportunity costs of equity capital. In this section, these extensions are discussed in detail, the SURP and the CURP are extended to the GURP, and the GLI is extended to the EGLI. These additional modifications effectively incorporate the relevant risks into the proposed GURM.

The decision of financial firms is formulated here as a stochastic dynamic program, and stochastic Euler equations are transformed into a form that expresses risk corrections explicitly. The GURP is also derived, capital effects and risk-adjustment effects are defined, and relationship between the GURP and the CURP is clarified. Based on this relationship, the EGLI is derived and the relationship between the EGLI and the GLI is shown.

3.1 Dynamic-Uncertainty Behavior and Stochastic Euler Equations

The formulation of the decisions of a financial firm as a stochastic dynamic programming (SDP) problem is derived from the same considerations as in the CURM. Two specifications of the problem exist, for which the primary difference is in the relative timing of decision-making periods and the realization of uncertainty. In the first specification, the decision is made after the uncertainty is realized, such that in each period the decision maker chooses the state variable of the next period directly. In the second specification, the decision is made before the uncertainty is realized, in which case the decision maker chooses the control variable of the current period, and the state variable of the next period then becomes a function of the chosen control variable and the state variable of the current period. The adjustment cost of stock variables is assumed to be zero, as described in Section 2, and more reliable information on the decision leads to a rise in firm value. The first specification is therefore assumed to be similar to that in the original CURM, that is, the financial firm chooses the state variable of the next period directly.

In the case of SDP, the state variables are classified as endogenous and exogenous state variables. As in the CURM, the endogenous state variable vectors $\mathbf{y}_{i,t}$ $(t \geq 0)$ are the vectors of real balances of financial goods $\mathbf{q}_{i,t}$, and the vectors of real resource fixed inputs $\mathbf{x}^F_{i,t}$, that is,

$$\mathbf{y}_{i,t} = \left(\mathbf{q}'_{i,t}, \mathbf{x}^{F\prime}_{i,t}\right)' = \left(q_{i,1,t}, \cdots, q_{i,N_A+N_L,t}, x^F_{i,1,t}, \cdots, x^F_{i,M_F,t}\right)' (t \geq 0) .$$

The exogenous state variable vectors $\mathbf{z}_{i,t}$ $(t \geq 0)$, are similarly defined as

$$\mathbf{z}_{i,t} = \left(\mathbf{z}^{H\prime}_{i,t-1}, \boldsymbol{\zeta}'_{i,t}, p_{G,t}, \mathbf{p}^{V\prime}_{i,t}, \tau_{i,t}, \mathbf{p}^{F\prime}_{i,t}, \boldsymbol{\delta}'_{i,t}\right)' (t \geq 0),$$

where $\mathbf{z}^{H}_{i,t-1} = \left(\mathbf{z}^{H\prime}_{i,1,t-1}, \cdots, \mathbf{z}^{H\prime}_{i,N_A+N_L,t-1}\right)'$ $(t \geq 0)$ are the exogenous variable vectors, which consist of the certain or predictable components of the SEHRR and the SEHCR in the period $t-1$ (≥ -1). At $t = 0$, $\mathbf{z}^{H}_{i,-1} = \mathbf{z}^{H}_{i,0} = \left(\mathbf{z}^{H\prime}_{i,1,0}, \cdots, \mathbf{z}^{H\prime}_{i,N_A+N_L,0}\right)'$. As described in Section 2.1, $\mathbf{z}^{H}_{i,j,t-1}$ $(j = 1, \cdots, N_A + N_L)$, the components of $\mathbf{z}^{H}_{i,t-1}$, are given by

$$\mathbf{z}^{H}_{i,j,t-1} = \left(\mathbf{z}^{R\prime}_{i,j,t-1}, \mathbf{z}^{Q\prime}_{i,j,t-1}, \mathbf{z}^{S\prime}_{i,j,t-1}, h^{C}_{i,j,t-1}, \mathbf{z}^{D\prime}_{i,j,t-1}\right)' (j = 1, \cdots, N_A)$$

for financial assets, and

$$\mathbf{z}^{H}_{i,j,t-1} = \left(\mathbf{z}^{R\prime}_{i,j,t-1}, \mathbf{z}^{Q\prime}_{i,j,t-1}, \mathbf{z}^{I\prime}_{i,j,t-1}, \mathbf{z}^{S\prime}_{i,j,t-1}, r^{D}_{i,t-1}, \kappa_{i,j,t-1}\right)' (j = N_A + 1, \cdots, N_A + N_L)$$

for liabilities. In the equation for $\mathbf{z}_{i,t}$, $\boldsymbol{\zeta}_{i,t} = \left(\zeta_{i,1,t}, \cdots, \zeta_{i,N_A+N_L,t}\right)'$ $(t \geq 0)$ are vectors of the uncertain or unpredictable components of the SEHRR and the SEHCR, and $p_{G,t}$ $(t \geq 0)$ are the general price indices. As described in Section 2.2, $\mathbf{p}^{V}_{i,t} = \left(p^{V}_{i,1,t}, \cdots, p^{V}_{i,M_V,t}\right)'$ $(t \geq 0)$ are the vectors of variable input prices, and $\tau_{i,t}$ $(t \geq 0)$ are the indices of exogenous technical change. In addition, $\mathbf{p}^{F}_{i,t} = \left(p^{F}_{i,1,t}, \cdots, p^{F}_{i,M_F,t}\right)'$ $(t \geq 0)$ are the vectors of fixed input prices, and $\boldsymbol{\delta}_{i,t} = (\delta_{i,1,t}, \cdots, \delta_{i,M_F,t})'$ $(t \geq 0)$ are the vectors of depreciation rates. Among these exogenous state variables, the vectors of the exogenous state variables with respect to the variable cost function are defined as $\mathbf{z}^{C}_{i,t} = \left(\mathbf{p}^{V\prime}_{i,t}, \mathbf{z}^{Q\prime}_{i,t}, \tau_{i,t}\right)'$ $(t \geq 0)$, where $\mathbf{z}^{Q}_{i,t} = \left(\mathbf{z}^{Q\prime}_{i,1,t}, \cdots, \mathbf{z}^{Q\prime}_{i,N_A+N_L,t}\right)'$ $(t \geq 0)$ are the corresponding vectors that affect the quality of financial goods. The vectors with respect to the quasi short-run profit in period t (≥ 0) are defined as

$$\mathbf{z}^{\pi}_{i,t} = \left(\mathbf{z}^{H\prime}_{i,t-1}, \boldsymbol{\zeta}'_{i,t}, p_{G,t-1}, p_{G,t}, \mathbf{z}^{C\prime}_{i,t}, \mathbf{p}^{F\prime}_{i,t}, \boldsymbol{\delta}'_{i,t}\right)' (t \geq 0),$$

and in the case of $t = 0$, $\mathbf{z}^{\pi}_{i,0} = \left(\mathbf{z}^{H\prime}_{i,0}, \boldsymbol{\zeta}'_{i,0}, p_{G,0}, \mathbf{p}^{V\prime}_{i,0}, \tau_{i,0}, \mathbf{p}^{F\prime}_{i,0}, \boldsymbol{\delta}'_{i,0}\right)'$. The vectors with respect to equity capital are defined as $\mathbf{z}^{e}_{i,t} = \left(p_{G,t}, \mathbf{p}^{F\prime}_{i,t}\right)'$ $(t \geq 0)$.

As in the CURM, in considering the uncertainties faced by the financial firm, it is assumed that the stochastic process $\{\mathbf{z}_{i,t}\}_{t\geq 0}$ follows a stationary Markov process. Let $(Z, \boldsymbol{B}_Z)$ be a measurable space, where Z is a set of $\mathbf{z}_{i,t}$, and $\boldsymbol{B}_Z$ is a σ-algebra of its subsets. In this case, the stochastic properties of the exogenous state variables can be expressed as a stationary transition function: $Q : Z \times \boldsymbol{B}_Z \rightarrow [0, 1]$.[9] The interpretation of this definition is that $Q(\mathbf{z}_{i,t}, A_{i,t+1})$ is the probability that the state of the next period lies in the set $A_{i,t+1}$, given that the current state is $\mathbf{z}_{i,t}$. The product space of $(Z, \boldsymbol{B}_Z)$ is expressed as $\left(Z^t, \boldsymbol{B}_Z^t\right) = (Z \times \cdots \times Z, \boldsymbol{B}_Z \times \cdots \times \boldsymbol{B}_Z)$, and $\mathbf{z}_{i,0}$ $(\in Z)$ is given.

Definition 9 *The probability measures on $(Z, \boldsymbol{B}_Z)$, $\mu^t(\mathbf{z}_{i,0}, \cdot) : \boldsymbol{B}_Z^t \rightarrow [0, 1]$ $(t \geq 1)$, are defined as follows.*[10] *For any rectangle $A_i^t = A_{i,1} \times \cdots \times A_{i,t} \in \boldsymbol{B}_Z^t$:*

$$\mu^t\left(\mathbf{z}_{i,0}, A_i^t\right) = \int_{A_{i,1}} \cdots \int_{A_{i,t-1}} \int_{A_{i,t}} Q(\mathbf{z}_{i,t-1}, \mathbf{dz}_{i,t})\, Q(\mathbf{z}_{i,t-2}, \mathbf{dz}_{i,t-1}) \cdots Q(\mathbf{z}_{i,0}, \mathbf{dz}_{i,1}). \tag{10}$$

The probability measure $\mu^t(\mathbf{z}_{i,0}, \cdot)$ satisfies the properties of measures and $\mu^t(\mathbf{z}_{i,0}, Z^t) = 1$.

As described in the CURM, the decision to be carried out in period t can depend upon the information that will be available at that time. This information can be expressed as a sequence of vectors of the exogenous state variables. Let $\mathbf{z}_i^t = (\mathbf{z}_{i,1}, \cdots, \mathbf{z}_{i,t})$ $(\in Z^t)$ denote the partial history in periods 1 through t, and let $(Y, \boldsymbol{B}_Y)$ be a measurable space, where Y is a set of vectors of the endogenous state variables $\mathbf{y}_{i,t}$, and $\boldsymbol{B}_Y$ is a σ-algebra of its subsets. A plan $\mathbf{y}_i^p$ is then defined as the set of a value $\mathbf{y}_{i,0}^p$ $(\in Y)$ and a sequence of functions $\mathbf{y}_{i,t}^p : Z^t \rightarrow Y$ $(t \geq 1)$, where $\mathbf{y}_{i,t}^p(\mathbf{z}_i^t) = \left(\mathbf{q}_{i,t}^p(\mathbf{z}_i^t)', \mathbf{x}_{F,i,t}^p(\mathbf{z}_i^t)'\right)'$ is the value of $\mathbf{y}_{i,t+1} = \left(\mathbf{q}_{i,t+1}', \mathbf{x}_{i,t+1}^{F\prime}\right)'$ that will be chosen in period t if the partial history of the exogenous state variables in periods 1 through t is $\mathbf{z}_i^t$.

In the remainder of the present paper, as in the CURM, the financial firm is assumed to choose a plan that maximizes the expected value of the

[9] For further details of the stationary transition function, see Stokey and Lucas (1989: p.212).

[10] For a full account of the probability measures, see Stokey and Lucas (1989: pp.220–225).

discounted intertemporal utility of its profits stream. The intertemporal utility function is also assumed to be additively separable. In this case, the optimization problem of the i-th financial firm is given by

$$\max_{\mathbf{y}_i^p} u_i \left[\pi_i^{QS}\left(\mathbf{y}_{i,0}, \mathbf{y}_{i,0}^p\left(\mathbf{z}_{i,0}\right), \mathbf{z}_{i,0}^\pi\right), q_{e,i}^p\left(\mathbf{y}_{i,0}^p\left(\mathbf{z}_{i,0}\right), \mathbf{z}_{i,0}^e\right)\right]$$

$$+\lim_{T\to\infty}\sum_{t=1}^{T}\int_{Z^t}\beta_i^t\cdot u_i\left[\pi_i^{QS}\left(\mathbf{y}_{i,t-1}^p\left(\mathbf{z}_i^{t-1}\right), \mathbf{y}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^\pi\right), q_{e,i}^p\left(\mathbf{y}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^e\right)\right]\mu^t\left(\mathbf{z}_{i,0}, \mathbf{dz}_i^t\right), \tag{11}$$

where $\beta_i^t = \prod_{s=0}^{t-1}\beta_{i,s} = \prod_{s=0}^{t-1}\dfrac{1}{1+r_{i,s}^D}$ is the cumulative discount factor and $r_{i,s}^D$ is the subjective rate of time preference.[11] Here, $\pi_i^{QS}\left(\mathbf{y}_{i,0}, \mathbf{y}_{i,0}^p\left(\mathbf{z}_{i,0}\right), \mathbf{z}_{i,0}^\pi\right)$ and $\pi_i^{QS}\left(\mathbf{y}_{i,t-1}^p\left(\mathbf{z}_i^{t-1}\right), \mathbf{y}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^\pi\right)$ $(t\geq 1)$ are the planned quasi short-run profits, which are defined by Eq. (7) as follows:

$$\begin{aligned}&\pi_i^{QS}\left(\mathbf{y}_{i,t-1}^p\left(\mathbf{z}_i^{t-1}\right), \mathbf{y}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^\pi\right)\\&=\sum_{j=1}^{N_A+N_L} b_j\cdot\left[\left\{1+b_C\cdot h_{i,j}^R\left(Q_{j,t-1}^p, \mathbf{z}_{i,j,t-1}^H\right)+\zeta_{i,j,t}\right\}\cdot p_{G,t-1}\cdot q_{i,j,t-1}^p\left(\mathbf{z}_i^{t-1}\right)-p_{G,t}\cdot q_{i,j,t}^p\left(\mathbf{z}_i^t\right)\right]\\&-C_i^V\left(\mathbf{y}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^C\right)-\sum_{j=1}^{M_F} p_{i,j,t}^F\cdot\left[x_{F,i,j,t}^p\left(\mathbf{z}_i^t\right)-\left(1-\delta_{i,j,t}\right)\cdot x_{F,i,j,t-1}^p\left(\mathbf{z}_i^{t-1}\right)\right]\ (t\geq 1),\end{aligned} \tag{12}$$

$$\begin{aligned}&\pi_i^{QS}\left(\mathbf{y}_{i,0}, \mathbf{y}_{i,0}^p\left(\mathbf{z}_{i,0}\right), \mathbf{z}_{i,0}^\pi\right)\\&=\sum_{j=1}^{N_A+N_L} b_j\cdot\left[\left\{1+b_C\cdot h_{i,j}^R\left(Q_{j,0}, \mathbf{z}_{i,j,0}^H\right)+\zeta_{i,j,0}\right\}\cdot p_{G,0}\cdot q_{i,j,0}-p_{G,0}\cdot q_{i,j,0}^p\left(\mathbf{z}_{i,0}\right)\right]\\&\quad -C_i^V\left(\mathbf{y}_{i,0}^p\left(\mathbf{z}_{i,0}\right), \mathbf{z}_{i,0}^C\right)-\sum_{j=1}^{M_F} p_{i,j,0}^F\cdot\left[x_{F,i,j,0}^p\left(\mathbf{z}_{i,0}\right)-\left(1-\delta_{i,j,0}\right)\cdot x_{i,j,0}^F\right].\end{aligned} \tag{13}$$

In addition, $q_{e,i}^p\left(\mathbf{y}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^e\right)$ $(t\geq 0)$ are the planned equity capital, which,

[11] For details of this optimization problem, see Stokey and Lucas (1989: pp.241–254).

based on Eq. (9), is given by

$$q^p_{e,i}\left(\mathbf{y}^p_{i,t}\left(\mathbf{z}^t_i\right),\mathbf{z}^e_{i,t}\right)=\sum_{j=1}^{N_A}p_{G,t}\cdot q^p_{i,j,t}\left(\mathbf{z}^t_i\right)+\sum_{j=1}^{M_F}p^F_{i,j,t}\cdot x^p_{F,i,j,t}\left(\mathbf{z}^t_i\right)-\sum_{j=N_A+1}^{N_A+N_L}p_{G,t}\cdot q^p_{i,j,t}\left(\mathbf{z}^t_i\right)\ (t\geq 0). \tag{14}$$

As described in the CURM, the necessary conditions for stochastic optimization problems in sequence form can be found by adopting a variational approach. Such conditions are represented by stochastic Euler equations, which for the above optimization problem (11) are expressed as

$$-\frac{\partial u^*_{i,t}}{\partial \pi^{QS*}_{i,t}}\cdot\left(b_j\cdot p_{G,t}+\frac{\partial C^{V*}_{i,t}}{\partial q^{p*}_{i,j,t}}\right)+b_j\cdot p_{G,t}\cdot\frac{\partial u^*_{i,t}}{\partial q^{p*}_{e,i,t}}$$
$$+\beta_{i,t}\cdot b_j\cdot p_{G,t}\cdot\int_Z\left\{1+b_C\cdot\left(h^{R*}_{i,j,t}+\frac{\partial h^{R*}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}}\right)+\zeta_{i,j,t+1}\right\}\cdot\frac{\partial u^*_{i,t+1}}{\partial \pi^{QS*}_{i,t+1}}Q\left(\mathbf{z}_{i,t},\mathbf{dz}_{i,t+1}\right)=0;$$
$$j=1,\cdots,N_A+N_L, \tag{15}$$

$$-\frac{\partial u^*_{i,t}}{\partial \pi^{QS*}_{i,t}}\cdot\left(p^F_{i,j,t}+\frac{\partial C^{V*}_{i,t}}{\partial x^{p*}_{F,i,j,t}}\right)+p^F_{i,j,t}\cdot\frac{\partial u^*_{i,t}}{\partial q^{p*}_{e,i,t}}$$
$$+\beta_{i,t}\cdot\int_Z p^F_{i,j,t+1}\cdot\left(1-\delta_{i,j,t+1}\right)\cdot\frac{\partial u^*_{i,t+1}}{\partial \pi^{QS*}_{i,t+1}}Q\left(\mathbf{z}_{i,t},\mathbf{dz}_{i,t+1}\right)=0;$$
$$j=1,\cdots,M_F, \tag{16}$$

where $\pi^{QS*}_{i,t}=\pi^{QS}_i\left(\mathbf{y}^{p*}_{i,t-1}\left(\mathbf{z}^{t-1}_i\right),\mathbf{y}^{p*}_{i,t}\left(\mathbf{z}^t_i\right),\mathbf{z}^\pi_{i,t}\right)$, $q^{p*}_{e,i,t}=q^p_{e,i}\left(\mathbf{y}^{p*}_{i,t}\left(\mathbf{z}^t_i\right),\mathbf{z}^e_{i,t}\right)$, $u^*_{i,t}=u_i\left(\pi^{QS*}_{i,t},q^{p*}_{e,i,t}\right)$, $C^{V*}_{i,t}=C^V_i\left(\mathbf{y}^{p*}_{i,t}\left(\mathbf{z}^t_i\right),\mathbf{z}^C_{i,t}\right)$, and $h^{R*}_{i,j,t}=h^R_{i,j}\left(Q^{p*}_{j,t},\mathbf{z}^H_{i,j,t}\right)$.

As described in Section 2.1, b_j is used to distinguish between financial assets and liabilities, and b_C is used to distinguish cash from other financial assets. In the above equations, $q^{p*}_{i,j,t}=q^{p*}_{i,j,t}\left(\mathbf{z}^t_i\right)$ $(j=1,\cdots,N_A+N_L)$ denote the optimal levels for financial goods, $x^{p*}_{F,i,j,t}=x^{p*}_{F,i,j,t}\left(\mathbf{z}^t_i\right)$ $(j=1,\cdots,M_F)$ represent the optimal levels for real resource fixed inputs, and $\mathbf{y}^{p*}_{i,t}=\mathbf{y}^{p*}_{i,t}\left(\mathbf{z}^t_i\right)=\left(\mathbf{q}^{p*}_{i,t}\left(\mathbf{z}^t_i\right)',\mathbf{x}^{p*}_{F,i,t}\left(\mathbf{z}^t_i\right)'\right)'$ are the optimal levels for the endogenous state variables.

As in the CURM, if the utility function $u_{i,t}^{*}$ is concave and continuously differentiable in $\mathbf{y}_{i,t-1}^{p*} = \left(\mathbf{q}_{i,t-1}^{p*\prime}, \mathbf{x}_{F,i,t-1}^{p*\prime}\right)'$ and $\mathbf{y}_{i,t}^{p*}$ and is integrable[12], then if each of the partial derivatives of $u_{i,t}^{*}$ with respect to $\mathbf{y}_{i,t-1}^{p*}$ are absolutely integrable[13], the stochastic Euler equations (15) and (16) with the transversality conditions

$$\lim_{t\to\infty} \beta_i^t \cdot \int_Z \frac{\partial u_{i,t+1}^{*}}{\partial \pi_{i,t+1}^{QS*}} \cdot \frac{\partial \pi_{i,t+1}^{QS*}}{\partial y_{i,j,t}^{p*}} \cdot y_{i,j,t}^{p*}\, Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}\right) = 0;\ j = 1, \cdots, N_A + N_L + M_F, \tag{17}$$

are sufficient conditions for an optimal plan $\mathbf{y}_i^{p*} = \left\{\mathbf{q}_{i,0}^{p*}, \mathbf{x}_{F,i,0}^{p*}, \left\{\mathbf{q}_{i,t}^{p*}, \mathbf{x}_{F,i,t}^{p*}\right\}_{t=1}^{\infty}\right\}$, where $y_{i,j,t}^{p*}$ means $q_{i,j,t}^{p*}$ or $x_{F,i,j,t}^{p*}$.

The difference between Eq. (15) and the stochastic Euler equations concerning financial goods in the CURM is the inclusion of term expressing the marginal utility of equity capital $(b_j \cdot p_{G,t} \cdot \partial u_{i,t}^{*} \left/ \partial q_{e,i,t}^{p*}\right.)$ and the uncertain components of the SEHRR and the SEHCR $(\zeta_{i,j,t+1};\ j = 1, \cdots, N_A + N_L)$ in Eq. (15). The extensions are simply additions to the stochastic Euler equations in the original CURM and allow the effects of equity capital and uncertainties in the SEHRR and the SEHCR to be considered explicitly.

3.2 Risk Corrections

The influence of uncertainties in the SEHRR and the SEHCR is resolved explicitly by transforming Eq. (15) into the form of an expression of risk correction. This is similar to the treatment in the CCAPM.

Theorem 1 *Under the assumption that* $\partial u_{i,t}^{*} \left/ \partial \pi_{i,t}^{QS*}\right. \neq 0$ *and* $E\left[\zeta_{i,j,t+1} \middle| \mathbf{z}_{i,t}\right] = 0$*, Eq. (15) can be transformed into the form of an expression of risk correc-*

[12] The integrability of $u_{i,t}^{*}$ means $\int_Z u_{i,t}^{*}\, Q\left(\mathbf{z}_{i,t-1}, \mathbf{dz}_{i,t}\right) < \infty$.

[13] The absolute integrability of $\frac{\partial u_{i,t}^{*}}{\partial y_{i,j,t-1}^{p*}}$ is defined as $\int_Z \left|\frac{\partial u_{i,t}^{*}}{\partial y_{i,j,t-1}^{p*}}\right| Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}\right) < \infty$.

tion as follows:

$$
\begin{aligned}
&- b_j \cdot p_{G,t} - MC^{V*}_{i,j,t} + b_j \cdot p_{G,t} \cdot MRS^{\pi*}_{e,i,t} \\
&\quad + \beta_{i,t} \cdot b_j \cdot p_{G,t} \cdot \left\{1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t}\right)\right\} \cdot E\left[IMRS^*_{\pi,i,t+1} \left| \mathbf{z}_{i,t}\right.\right] \\
&\quad + \beta_{i,t} \cdot b_j \cdot p_{G,t} \cdot \frac{cov\left(\zeta_{i,j,t+1}, \partial u^*_{i,t+1} \Big/ \partial \pi^{QS*}_{i,t+1} \Big| \mathbf{z}_{i,t}\right)}{\partial u^*_{i,t} \Big/ \partial \pi^{QS*}_{i,t}} = 0; \\
&\qquad\qquad j = 1, \cdots, N_A + N_L, \qquad (18)
\end{aligned}
$$

where $MC^{V*}_{i,j,t} = \partial C^{V*}_{i,t} \big/ \partial q^{p*}_{i,j,t}$, $MRS^{\pi*}_{e,i,t} = \left(\partial u^*_{i,t} \big/ \partial q^{p*}_{e,i,t}\right) \Big/ \left(\partial u^*_{i,t} \Big/ \partial \pi^{QS*}_{i,t}\right)$ [14], $\eta^*_{i,j,t} = \partial h^{R*}_{i,j,t} \big/ \partial \ln q^{p*}_{i,j,t}$, $IMRS^*_{\pi,i,t+1} = \left(\partial u^*_{i,t+1} \Big/ \partial \pi^{QS*}_{i,t+1}\right) \Big/ \left(\partial u^*_{i,t} \Big/ \partial \pi^{QS*}_{i,t}\right)$ [15], *and* $E\left[\cdot \left| \mathbf{z}_{i,t}\right.\right] = \int_Z \cdot Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}\right)$.

---

[14] This term is the marginal rate of substitution (MRS) of quasi short-run profits for equity capital. This MRS is a measure of the rate at which the financial firm is just willing to substitute quasi short-run profits for equity capital, or in other words, a measure of the opportunity costs of equity capital.

[15] This term represents the intertemporal marginal rate of substitution (IMRS) with respect to quasi short-run profits, and is a measure of the rate at which the financial firm is just willing to substitute quasi short-run profits in period t for profits in period t + 1. In the case that the financial firm is risk-averse, the marginal utility of quasi short-run profits is a decreasing function of quasi short-run profits. The IMRS therefore declines if quasi short-run profits increase from the current period to the next period and rises if profits decrease.

Proof. Both sides of Eq. (15) are divided by $\partial u_{i,t}^* \Big/ \partial \pi_{i,t}^{QS*}$, provided $\partial u_{i,t}^* \Big/ \partial \pi_{i,t}^{QS*} \neq 0$, which gives

$$
\begin{aligned}
& - b_j \cdot p_{G,t} - \frac{\partial C_{i,t}^{V*}}{\partial q_{i,j,t}^{p*}} + b_j \cdot p_{G,t} \cdot \frac{\partial u_{i,t}^* \big/ \partial q_{e,i,t}^{p*}}{\partial u_{i,t}^* \Big/ \partial \pi_{i,t}^{QS*}} \\
& \quad + \beta_{i,t} \cdot b_j \cdot p_{G,t} \cdot \int_Z \left\{ 1 + b_C \cdot \left(h_{i,j,t}^{R*} + \frac{\partial h_{i,j,t}^{R*}}{\partial \ln q_{i,j,t}^{p*}} \right) + \zeta_{i,j,t+1} \right\} \\
& \quad \cdot \frac{\partial u_{i,t+1}^* \Big/ \partial \pi_{i,t+1}^{QS*}}{\partial u_{i,t}^* \Big/ \partial \pi_{i,t}^{QS*}} Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}\right) = 0;\ j = 1, \cdots, N_A + N_L. \quad (19)
\end{aligned}
$$

To simplify the expressions, the notation of Theorem 1 is used. Equation (19) can then be rewritten as

$$
\begin{aligned}
& - b_j \cdot p_{G,t} - MC_{i,j,t}^{V*} + b_j \cdot p_{G,t} \cdot MRS_{e,i,t}^{\pi*} \\
& + \beta_{i,t} \cdot b_j \cdot p_{G,t} \cdot E\left[\left\{ 1 + b_C \cdot \left(h_{i,j,t}^{R*} + \eta_{i,j,t}^* \right) + \zeta_{i,j,t+1} \right\} \cdot IMRS_{\pi,i,t+1}^* \left| \mathbf{z}_{i,t} \right. \right] = 0; \\
& \qquad j = 1, \cdots, N_A + N_L. \quad (20)
\end{aligned}
$$

To transform these equations into a form in which risk corrections are expressed explicitly, the expectation in the third term of the left-hand side of Eq. (20) is transformed by the same method as employed in the CCAPM. Let $w_{i,j,t+1}^* = 1 + b_C \cdot \left(h_{i,j,t}^{R*} + \eta_{i,j,t}^* \right) + \zeta_{i,j,t+1}$. The expectation in the third term is then expressed as $E\left[w_{i,j,t+1}^* \cdot IMRS_{\pi,i,t+1}^* \middle| \mathbf{z}_{i,t} \right]$. As in the CCAPM, the covariance of $w_{i,j,t+1}^*$ with respect to $IMRS_{\pi,i,t+1}^*$, $\operatorname{cov}\left(w_{i,j,t+1}^*, IMRS_{\pi,i,t+1}^* \middle| \mathbf{z}_{i,t} \right)$, is the focus of attention. Using the property of covariance

$$
\begin{aligned}
\operatorname{cov}\left(w_{i,j,t+1}^*, IMRS_{\pi,i,t+1}^* \middle| \mathbf{z}_{i,t} \right) &= E\left[w_{i,j,t+1}^* \cdot IMRS_{\pi,i,t+1}^* \middle| \mathbf{z}_{i,t} \right] \\
& \quad - E\left[w_{i,j,t+1}^* \middle| \mathbf{z}_{i,t} \right] \cdot E\left[IMRS_{\pi,i,t+1}^* \middle| \mathbf{z}_{i,t} \right],
\end{aligned}
$$

$E\left[w^*_{i,j,t+1} \cdot IMRS^*_{\pi,i,t+1} \middle| \mathbf{z}_{i,t} \right]$ can be written as

$$\begin{aligned} E\left[w^*_{i,j,t+1} \cdot IMRS^*_{\pi,i,t+1} \middle| \mathbf{z}_{i,t} \right] &= E\left[w^*_{i,j,t+1} \middle| \mathbf{z}_{i,t} \right] \cdot E\left[IMRS^*_{\pi,i,t+1} \middle| \mathbf{z}_{i,t} \right] \\ &\quad + \mathrm{cov}\left(w^*_{i,j,t+1}, IMRS^*_{\pi,i,t+1} \middle| \mathbf{z}_{i,t} \right). \end{aligned} \tag{21}$$

Substituting $w^*_{i,j,t+1} = 1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t} \right) + \zeta_{i,j,t+1}$ for $E\left[w^*_{i,j,t+1} \middle| \mathbf{z}_{i,t} \right]$, under the assumption that $E\left[\zeta_{i,j,t+1} \middle| \mathbf{z}_{i,t} \right] = 0$, leads to

$$E\left[w^*_{i,j,t+1} \middle| \mathbf{z}_{i,t} \right] = 1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t} \right). \tag{22}$$

Substituting $w^*_{i,j,t+1} = 1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t} \right) + \zeta_{i,j,t+1}$ and $IMRS^*_{\pi,i,t+1} = \left(\partial u^*_{i,t+1} \middle/ \partial \pi^{QS*}_{i,t+1} \right) \middle/ \left(\partial u^*_{i,t} \middle/ \partial \pi^{QS*}_{i,t} \right)$ for $\mathrm{cov}\left(w^*_{i,j,t+1}, IMRS^*_{\pi,i,t+1} \middle| \mathbf{z}_{i,t} \right)$, the property of covariance gives the following:

$$\begin{aligned} \mathrm{cov}\left(w^*_{i,j,t+1}, IMRS^*_{\pi,i,t+1} \middle| \mathbf{z}_{i,t} \right) &= \mathrm{cov}\left(\zeta_{i,j,t+1}, IMRS^*_{\pi,i,t+1} \middle| \mathbf{z}_{i,t} \right) \\ &= \frac{\mathrm{cov}\left(\zeta_{i,j,t+1}, \partial u^*_{i,t+1} \middle/ \partial \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)}{\partial u^*_{i,t} \middle/ \partial \pi^{QS*}_{i,t}} \end{aligned} \tag{23}$$

Substituting Eqs. (22) and (23) for Eq. (21), the expectation in the third term of the left-hand side of Eq. (20) can be transformed into a form expressing risk corrections explicitly, as follows:

$$\begin{aligned} & E\left[\left\{ 1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t} \right) + \zeta_{i,j,t+1} \right\} \cdot IMRS^*_{\pi,i,t+1} \left| \mathbf{z}_{i,t} \right. \right] \\ &= \left\{ 1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t} \right) \right\} \cdot E\left[IMRS^*_{\pi,i,t+1} \left| \mathbf{z}_{i,t} \right. \right] \\ &\quad + \frac{\mathrm{cov}\left(\zeta_{i,j,t+1}, \partial u^*_{i,t+1} \middle/ \partial \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)}{\partial u^*_{i,t} \middle/ \partial \pi^{QS*}_{i,t}}. \end{aligned} \tag{24}$$

Substituting Eq. (24) into Eq. (20) thus adds a risk-adjustment term, as given by Eq. (18). ■

The second term of the right-hand side of Eq. (24), $\mathrm{cov}\left(\zeta_{i,j,t+1}, \partial u^*_{i,t+1} \middle/ \partial \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right) \middle/ \left(\partial u^*_{i,t} \middle/ \partial \pi^{QS*}_{i,t} \right)$, that is, the ratio of the covariance of uncertain components of the SEHRR and the SEHCR with respect to the marginal utility of quasi short-run profits in period $t+1$ to the same marginal utility in period t, is a risk-adjustment term. In the case that the risk attitude of financial firms is averse, the marginal utility of quasi short-run profits is a decreasing function of its profits. Therefore, $\mathrm{cov}\left(\zeta_{i,j,t+1}, \pi_{i,t+1}^{QS*}\middle| \mathbf{z}_{i,t}\right)$ is positive if $\mathrm{cov}\left(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{*} \middle/ \partial\pi_{i,t+1}^{QS*}\middle| \mathbf{z}_{i,t}\right)$ is negative, and vice versa. In this case, the variance of quasi short-run profits in the next period increases if a financial asset in the current period increases, while the same variance decreases if a liability in the current period increases, and vice versa. For example, if ξ $(0 < \xi < 1)$ of the jth financial good in period t increases, then from Eq. (7), the quasi short-run profit in the next period becomes $\pi_{i,t+1}^{QS} + b_j \cdot \left\{1 + b_C \cdot h_{i,j}^{R}\left(Q_{j,t}, \mathbf{z}_{i,j,t}^{H}\right) + \zeta_{i,j,t+1}\right\} \cdot p_{G,t} \cdot \xi$. In this case, its variance can be expressed as

$$
\begin{aligned}
&\mathrm{var}\left(\pi_{i,t+1}^{QS} + b_j \cdot \left\{1 + b_C \cdot h_{i,j}^{R}\left(Q_{j,t}, \mathbf{z}_{i,j,t}^{H}\right) + \zeta_{i,j,t+1}\right\} \cdot p_{G,t} \cdot \xi \middle| \mathbf{z}_{i,t}\right) \\
&= \mathrm{var}\left(\pi_{i,t+1}^{QS}\middle| \mathbf{z}_{i,t}\right) + 2 \cdot b_j \cdot p_{G,t} \cdot \xi \cdot \mathrm{cov}\left(\zeta_{i,j,t+1}, \pi_{i,t+1}^{QS}\middle| \mathbf{z}_{i,t}\right) \\
&\qquad + \left(b_j \cdot p_{G,t} \cdot \xi\right)^2 \cdot \mathrm{var}\left(\zeta_{i,j,t+1}\middle| \mathbf{z}_{i,t}\right). \qquad (25)
\end{aligned}
$$

Thus, if ξ is sufficiently small, then the third term of the right-hand side of this equation is much smaller than the second term. The sign of the second term, $\mathrm{cov}\left(\zeta_{i,j,t+1}, \pi_{i,t+1}^{QS}\middle| \mathbf{z}_{i,t}\right)$, determines whether this variance is greater than $\mathrm{var}\left(\pi_{i,t+1}^{QS}\middle| \mathbf{z}_{i,t}\right)$. Thus, in the case that the j-th financial good is a financial asset (i.e., $b_j = 1$), the variance is greater than $\mathrm{var}\left(\pi_{i,t+1}^{QS}\middle| \mathbf{z}_{i,t}\right)$ if the sign of $\mathrm{cov}\left(\zeta_{i,j,t+1}, \pi_{i,t+1}^{QS}\middle| \mathbf{z}_{i,t}\right)$ is positive. Similarly, in the case that the j-th financial good is a liability (i.e., $b_j = -1$), this variance is greater than $\mathrm{var}\left(\pi_{i,t+1}^{QS}\middle| \mathbf{z}_{i,t}\right)$ if the sign of $\mathrm{cov}\left(\zeta_{i,j,t+1}, \pi_{i,t+1}^{QS}\middle| \mathbf{z}_{i,t}\right)$ is negative.

In the CCAPM, $\beta_{i,t} \cdot IMRS_{\pi,i,t+1}^{*}$ and $1 \big/ E\left[\beta_{i,t} \cdot IMRS_{\pi,i,t+1}^{*} \left| \mathbf{z}_{i,t}\right.\right]$ are regarded as a stochastic discount factor and a risk-free rate. Using this risk-free rate, the Eq. (18) can be expressed as the following corollary.

Corollary 1 *Equation (18) can be expressed as follows:*

$$
\begin{aligned}
&1 + (b_j \cdot p_{G,t})^{-1} \cdot MC_{i,j,t}^{V*} - MRS_{e,i,t}^{\pi*} \\
&= \left\{ 1 + b_C \cdot \left(h_{i,j,t}^{R*} + \eta_{i,j,t}^{*} \right) \right\} \cdot \overline{\beta}_{i,t}^{S*} + \beta_{i,t} \cdot \frac{cov\left(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{*} \Big/ \partial \pi_{i,t+1}^{QS*} \middle| \mathbf{z}_{i,t} \right)}{\partial u_{i,t}^{*} \Big/ \partial \pi_{i,t}^{QS*}} \\
&\qquad = \frac{1 + b_C \cdot \left(h_{i,j,t}^{R*} + \eta_{i,j,t}^{*} \right)}{\overline{R}_{i,t}^{F*}} + \beta_{i,t} \cdot \frac{cov\left(\zeta_{i,j,t+1}, \partial u_{i,t+1}^{*} \Big/ \partial \pi_{i,t+1}^{QS*} \middle| \mathbf{z}_{i,t} \right)}{\partial u_{i,t}^{*} \Big/ \partial \pi_{i,t}^{QS*}}; \\
&\qquad\qquad j = 1, \cdots, N_A + N_L, \quad (26)
\end{aligned}
$$

where $\beta_{i,t}^{S*} = \beta_{i,t} \cdot IMRS_{\pi,i,t+1}^{*}$, $\overline{\beta}_{i,t}^{S*} = E\left[\beta_{i,t}^{S*} \left| \mathbf{z}_{i,t} \right. \right]$, *and* $\overline{R}_{i,t}^{F*} = 1 \Big/ \overline{\beta}_{i,t}^{S*}$.

Proof. Divide Eq. (18) by $-b_j \cdot p_{G,t}$ and substitute $E\left[\beta_{i,t} \cdot IMRS_{\pi,i,t+1}^{*} \left| \mathbf{z}_{i,t} \right. \right] = \overline{\beta}_{i,t}^{S*} = 1 \Big/ \overline{R}_{i,t}^{F*}$ into Eq. (18). ■

In the case that the j-th financial good is a financial asset, the left-hand side of Eq. (26) can be interpreted as the net costs required to increase the financial asset in period t by one unit. These are referred to as the net marginal costs, which include not only the money for one unit required to increase the financial asset by one unit, but also the change in real resource variable costs $(b_j \cdot p_{G,t})^{-1} \cdot MC_{i,j,t}^{V*}$ and an increase in utility associated with an increase in equity capital based on the marginal utility of quasi short-run profits. In the case that the j-th financial good is a liability, the left-hand side of Eq. (26) can be interpreted as the net funds obtained by increasing the liability in period t by one unit. Similar to the net marginal costs, these net marginal funds account for not only the money for one unit obtained by increasing the liability by one unit, but also the change in real resource variable costs $(b_j \cdot p_{G,t})^{-1} \cdot MC_{i,j,t}^{V*}$ and a decrease in utility associated with a decrease in equity capital based on the marginal utility of quasi short-run profits. As explained in Eq. (22), $1 + b_C \cdot \left(h_{i,j,t}^{R*} + \eta_{i,j,t}^{*} \right)$ in the first term of the right-hand side of Eq. (26) represents the expectation of $w_{i,j,t+1}^{*} = 1 + b_C \cdot \left(h_{i,j,t}^{R*} + \eta_{i,j,t}^{*} \right) + \zeta_{i,j,t+1}$, that is, $E\left[w_{i,j,t+1}^{*} \left| \mathbf{z}_{i,t} \right. \right]$. $w_{i,j,t+1}^{*}$

is the net revenue obtained by employing the financial asset of one unit if the j-th financial good is a financial asset (i.e., $b_j = 1$), and the total cost repaid for the liability of one unit if the j-th financial good is a liability (i.e., $b_j = -1$).[16] In both cases, as in the CCAPM, $w^*_{i,j,t+1}$ is the payoff. The first term of the right-hand side of the first equality in Eq. (26) is the product of these payoffs and the stochastic discount factor, and that of the second equality is the payoff discounted by the risk-free rate, or in other words, the present discounted value of expected payoff. The second term of the right-hand side of Eq. (26) is the product of the risk-adjustment term and the subjective discount factor. Similar to Eq. (23), this term corresponds to the covariance of the payoffs with stochastic discount factor $\mathrm{cov}\left(w^*_{i,j,t+1}, \beta^{S*}_{i,t}\middle|\, \mathbf{z}_{i,t}\right)$ $\left(=\mathrm{cov}\left(w^*_{i,j,t+1}, \beta_{i,t} \cdot IMRS^*_{\pi,i,t+1}\middle|\, \mathbf{z}_{i,t}\right)\right)$. Therefore, Eq. (26) expresses the net marginal costs or funds as being equivalent to the sum of the present discounted value of expected payoffs and the covariance of the payoffs with the stochastic discount factor.

Under these definitions, if quasi short-run profits are constant or the financial firm is risk-neutral, then net marginal costs or funds are equivalent to the present discounted value of expected payoffs. If the financial firm is risk-averse and $\mathrm{cov}\left(w^*_{i,j,t+1}, \pi^{QS*}_{i,t+1}\middle|\, \mathbf{z}_{i,t}\right) > 0$, then the net marginal costs or funds are smaller than the present discounted value of expected payoffs, because $\mathrm{cov}\left(w^*_{i,j,t+1}, \beta^{S*}_{i,t}\middle|\, \mathbf{z}_{i,t}\right) < 0$. If the financial firm is risk-averse and $\mathrm{cov}\left(w^*_{i,j,t+1}, \pi^{QS*}_{i,t+1}\middle|\, \mathbf{z}_{i,t}\right) < 0$, then the net marginal costs or funds are greater than the present discounted value of expected payoffs, because $\mathrm{cov}\left(w^*_{i,j,t+1}, \beta^{S*}_{i,t}\middle|\, \mathbf{z}_{i,t}\right) > 0$. As shown by Eq. (25), if the j-th financial good is a financial asset (i.e., $b_j = 1$) and $\mathrm{cov}\left(w^*_{i,j,t+1}, \pi^{QS*}_{i,t+1}\middle|\, \mathbf{z}_{i,t}\right) > 0$, then

$$\mathrm{var}\left(\pi^{QS*}_{i,t+1} + b_j \cdot \left\{1 + b_C \cdot h^R_{i,j}\left(Q^{p*}_{j,t}, \mathbf{z}^H_{i,j,t}\right) + \zeta_{i,j,t+1}\right\} \cdot p_{G,t} \cdot \xi \middle|\, \mathbf{z}_{i,t}\right)$$
$$> \mathrm{var}\left(\pi^{QS*}_{i,t+1}\middle|\, \mathbf{z}_{i,t}\right).$$

[16] The sign of $\eta^*_{i,j,t}$ is considered to be negative if the j-th financial good is a financial asset (i.e., $b_j = 1$) and is considered to be positive if a liability (i.e., $b_j = -1$). Therefore, $w^*_{i,j,t+1}$ is the net revenue in the former case and is the total cost in the latter case.

Hence, the volatility risk of quasi short-run profits increases because $\text{cov}\left(w^*_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right) = \text{cov}\left(\zeta_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)$. In this case, if the net marginal costs do not decrease, then the financial assets of the financial firm would not increase, resulting in a decrease in net marginal costs. Conversely, if $\text{cov}\left(w^*_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right) < 0$, then the financial assets of the financial firm would increase even if the marginal cost increases slightly, as an increase in financial assets reduces the volatility risk of quasi short-run profits and thereby increases net marginal costs. On the other hand, if the j-th financial good is a liability (i.e., $b_j = -1$) and $\text{cov}\left(w^*_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right) > 0$, then the financial firm would increase the liability even if the net marginal funds decrease slightly, since an increase in the liability reduces the volatility risk of quasi short-run profits and thereby reduces net marginal funds. Conversely, if $\text{cov}\left(w^*_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right) < 0$, then the financial firm would not increase the liability if the net marginal funds do not increase because an increase in the liability increases the volatility risk of quasi short-run profits, which would result in an increase in net marginal funds.

Corollary 2 *Similarly to Theorem 1, provided that $E\left[\zeta_{i,j,t+1} \middle| \mathbf{z}_{i,t} \right] = 0$, Eq. (26) can be expressed as follows:*

$$\begin{aligned} E\left[R^{\zeta*}_{i,j,t+1} \middle| \mathbf{z}_{i,t} \right] &= \overline{R}^{F*}_{i,t} - \overline{R}^{F*}_{i,t} \cdot cov\left(R^{\zeta*}_{i,j,t+1}, \beta^{S*}_{i,t} \middle| \mathbf{z}_{i,t} \right) \\ &= \overline{R}^{F*}_{i,t} - \frac{cov\left(R^{\zeta*}_{i,j,t+1}, \partial u^*_{i,t+1} \middle/ \partial \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)}{E\left[\partial u^*_{i,t} \middle/ \partial \pi^{QS*}_{i,t} \middle| \mathbf{z}_{i,t} \right]}; \\ &\qquad j = 1, \cdots, N_A + N_L, \quad (27) \end{aligned}$$

where

$$R^{\zeta*}_{i,j,t+1} = \frac{1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t} \right) + \zeta_{i,j,t+1}}{1 + \left(b_j \cdot p_{G,t} \right)^{-1} \cdot MC^{V*}_{i,j,t} - MRS^{\pi*}_{e,i,t}}.$$

Proof. As in the CCAPM, dividing both sides of Eq. (26) by

$$1 + \left(b_j \cdot p_{G,t} \right)^{-1} \cdot MC^{V*}_{i,j,t} - MRS^{\pi*}_{e,i,t}$$

and substituting

$$\beta_{i,t} \cdot \frac{\operatorname{cov}\left(\zeta_{i,j,t+1}, \partial u^*_{i,t+1} \Big/ \partial \pi^{QS*}_{i,t+1} \Big| \mathbf{z}_{i,t}\right)}{\partial u^*_{i,t} \Big/ \partial \pi^{QS*}_{i,t}} = \operatorname{cov}\left(\zeta_{i,j,t+1}, \beta^{S*}_{i,t} \big| \mathbf{z}_{i,t}\right)$$

leads to the following expression:

$$1 = \frac{1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t}\right)}{1 + (b_j \cdot p_{G,t})^{-1} \cdot MC^{V*}_{i,j,t} - MRS^{\pi*}_{e,i,t}} \cdot \overline{\beta}^{S*}_{i,t} + \frac{\operatorname{cov}\left(\zeta_{i,j,t+1}, \beta^{S*}_{i,t} \big| \mathbf{z}_{i,t}\right)}{1 + (b_j \cdot p_{G,t})^{-1} \cdot MC^{V*}_{i,j,t} - MRS^{\pi*}_{e,i,t}};$$

$$j = 1, \cdots, N_A + N_L. \quad (28)$$

For simplification, the following notation is used:

$$R^{\zeta*}_{i,j,t+1} = \frac{1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t}\right) + \zeta_{i,j,t+1}}{1 + (b_j \cdot p_{G,t})^{-1} \cdot MC^{V*}_{i,j,t} - MRS^{\pi*}_{e,i,t}}.$$

Similar to Theorem 1, provided that $E\left[\zeta_{i,j,t+1} \big| \mathbf{z}_{i,t}\right] = 0$, the following equation holds:

$$E\left[R^{\zeta*}_{i,j,t+1} \Big| \mathbf{z}_{i,t}\right] = \frac{1 + b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t}\right)}{1 + (b_j \cdot p_{G,t})^{-1} \cdot MC^{V*}_{i,j,t} - MRS^{\pi*}_{e,i,t}}.$$

In addition, using the property of covariance, the following equation applies:

$$\operatorname{cov}\left(R^{\zeta*}_{i,j,t+1}, \beta^{S*}_{i,t} \Big| \mathbf{z}_{i,t}\right) = \frac{\operatorname{cov}\left(\zeta_{i,j,t+1}, \beta^{S*}_{i,t} \big| \mathbf{z}_{i,t}\right)}{1 + (b_j \cdot p_{G,t})^{-1} \cdot MC^{V*}_{i,j,t} - MRS^{\pi*}_{e,i,t}}.$$

In this case, Eq. (28) can be expressed as

$$1 = E\left[R^{\zeta*}_{i,j,t+1} \Big| \mathbf{z}_{i,t}\right] \cdot \overline{\beta}^{S*}_{i,t} + \operatorname{cov}\left(R^{\zeta*}_{i,j,t+1}, \beta^{S*}_{i,t} \Big| \mathbf{z}_{i,t}\right), \; j = 1, \cdots, N_A + N_L. \quad (29)$$

Substituting $\overline{\beta}^{S*}_{i,t} = 1 \Big/ \overline{R}^{F*}_{i,t}$ into the above equation then yields Eq. (27). ■

In the case that the j-th financial good is a financial asset, $R^{\zeta*}_{i,j,t+1}$ is the payoff of the financial asset divided by the net marginal costs, corresponding to the returns referred to in the CCAPM and referred to similarly in this case.

In the case that the j-th financial good is a liability, $R^{\zeta*}_{i,j,t+1}$ is the payoff of the liability divided by the net marginal funds, representing repayments. In Eq. (27), the expected returns or repayments are equivalent to the difference between the risk-free rate and the covariance of returns or repayments considering the marginal utility of quasi short-run profits in period $t+1$ divided by the same marginal utility in period t.

In the case that the financial firm is risk-averse, the sign of the second term of the left-hand side of Eq. (27) is consistent with the sign of $\mathrm{cov}\left(R^{\zeta*}_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)$, since the marginal utility of quasi short-run profits is a decreasing function of quasi short-run profits and is usually positive. The sign of $\mathrm{cov}\left(R^{\zeta*}_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)$ is also consistent with the sign of $\mathrm{cov}\left(w^{*}_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)$ (=$\mathrm{cov}\left(\zeta_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)$), because, based on the property of covariance,

$$\begin{aligned}
\mathrm{cov}\left(R^{\zeta*}_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right) &= \frac{\mathrm{cov}\left(w^{*}_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)}{1 + (b_j \cdot p_{G,t})^{-1} \cdot MC^{V*}_{i,j,t} - MRS^{\pi*}_{e,i,t}} \\
&= \frac{\mathrm{cov}\left(\zeta_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)}{1 + (b_j \cdot p_{G,t})^{-1} \cdot MC^{V*}_{i,j,t} - MRS^{\pi*}_{e,i,t}}
\end{aligned}$$

holds and the sign of the denominator of the right-hand side of this equation is usually positive. Consequently, if the financial firm is risk-averse, the j-th financial good is a financial asset ($b_j = 1$), and $\mathrm{cov}\left(R^{\zeta*}_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right) > 0$, then the financial firm would not increase the financial asset if the expected returns do not increase so as to avoid an increase in the volatility risk of quasi short-run profits, resulting in an increase in expected returns. Conversely, if $\mathrm{cov}\left(R^{\zeta*}_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right) < 0$, then the financial firm would increase the financial asset even if the expected return decreases slightly, since an increase in the financial asset reduces the volatility risk of quasi short-run profits. Thus, the expected returns would decrease. If the financial firm is risk-averse, the j-th financial good is a liability ($b_j = -1$), and $\mathrm{cov}\left(R^{\zeta*}_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right) > 0$, then the financial firm would increase the liability even if the expected repayment increases slightly, as an increase in the liability reduces the volatility risk of quasi short-run profits, in which case the expected repayments would increase. Conversely, if $\operatorname{cov}\left(R^{\zeta *}_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right) < 0$, then the financial firm would not increase the liability if the expected repayments do not decrease because an increase in the liability increases the volatility risk of quasi short-run profits, resulting in a decrease in expected repayments. If quasi short-run profits are constant or the financial firm is risk-neutral, then the expected returns or repayments are equivalent to the risk-free rate.

3.3 Equity Capital Effects, Risk-Adjustment Effects, and Generalized User-Revenue Prices

Equation (18) represents a stochastic Euler equation with respect to financial goods, extended from that in the original CURM to incorporate consideration of the effects of equity capital and the volatility risk of quasi short-run profits. By transforming these equations, the GURP is derived as an extension of the SURP and the CURP.

Corollary 3 *Equation (18) can be expressed as follows:*

$$MC^{V*}_{i,j,t} = b_j \cdot p_{G,t} \cdot \left[\left(b_C \cdot h^{R*}_{i,j,t} - r^{F*}_{i,t} \right) / \left(1 + r^{F*}_{i,t} \right) + b_C \cdot \eta^{*}_{i,j,t} / \left(1 + r^{F*}_{i,t} \right) \right.$$
$$\left. + MRS^{\pi *}_{e,i,t} + \varpi^{*}_{i,j,t} \right] ; \; j = 1, \cdots, N_A + N_L, \quad (30)$$

where $r^{F*}_{i,t} = \overline{R}^{F*}_{i,t} - 1$ *and*

$$\varpi^{*}_{i,j,t} = \beta_{i,t} \cdot \frac{cov\left(\zeta_{i,j,t+1}, \partial u^{*}_{i,t+1} \middle/ \partial \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)}{\partial u^{*}_{i,t} \middle/ \partial \pi^{QS*}_{i,t}}.$$

Proof. Transformation of Eq. (18) with respect to $MC^{V*}_{i,j,t}$ and rearrangement then gives

$$MC^{V*}_{i,j,t} = b_j \cdot p_{G,t} \cdot \left[\left\{ b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t} \right) - \left(1 \middle/ \overline{\beta}^{S*}_{i,t} - 1 \right) \right\} \cdot \overline{\beta}^{S*}_{i,t} + MRS^{\pi*}_{e,i,t} \right.$$

$$\left. + \beta_{i,t} \cdot \frac{\operatorname{cov}\left(\zeta_{i,j,t+1}, \partial u^*_{i,t+1} \middle/ \partial \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)}{\partial u^*_{i,t} \middle/ \partial \pi^{QS*}_{i,t}} \right]$$

$$= b_j \cdot p_{G,t} \cdot \left[\left\{ b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t} \right) - \left(\overline{R}^{F*}_{i,t} - 1 \right) \right\} \middle/ \overline{R}^{F*}_{i,t} + MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t} \right]$$

$$= b_j \cdot p_{G,t} \cdot \left[\left\{ b_C \cdot \left(h^{R*}_{i,j,t} + \eta^*_{i,j,t} \right) - r^{F*}_{i,t} \right\} / \left(1 + r^{F*}_{i,t} \right) + MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t} \right]$$

$$= b_j \cdot p_{G,t} \cdot \left[\left(b_C \cdot h^{R*}_{i,j,t} - r^{F*}_{i,t} \right) / \left(1 + r^{F*}_{i,t} \right) + b_C \cdot \eta^*_{i,j,t} / \left(1 + r^{F*}_{i,t} \right) + MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t} \right];$$

$$j = 1, \cdots, N_A + N_L.$$

■

The right-hand side of this equation is then the price of the j-th financial good, i.e., is equivalent to $MC^{V*}_{i,j,t}$. This is thus used as the definition for the GURP.

Definition 10 *The generalized user-revenue price of the i-th financial firm during period t, denoted by $p^{GUR}_{i,j,t}$, is defined as*

$$p^{GUR}_{i,j,t} = b_j \cdot p_{G,t} \cdot \left[\left(b_C \cdot h^{R*}_{i,j,t} - r^{F*}_{i,t} \right) / \left(1 + r^{F*}_{i,t} \right) + b_C \cdot \eta^*_{i,j,t} / \left(1 + r^{F*}_{i,t} \right) \right.$$
$$\left. + MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t} \right]; \; j = 1, \cdots, N_A + N_L. \quad (31)$$

From this definition and Corollary 3, the following remark follows immediately.

Remark 1 *From Corollary 3 and Definition 10,*

$$MC^{V*}_{i,j,t} = p^{GUR}_{i,j,t}; \; j = 1, \cdots, N_A + N_L \quad (32)$$

holds, and thus the classification of financial goods into inputs and outputs *based on the sign of each GURP is consistent with the classification based on the sign of each partial derivative of the variable cost function with respect to financial goods. The sign of the partial derivative of the variable cost function is the same as the sign of the GURP, indicating that a financial good is an output if positive and a fixed input if negative.*

As described in the CURM, $\eta^*_{i,j,t}$ in the second term of the right-hand side of Eq. (31) reflects the effects of the market structure of the j-th financial good and the strategic interdependence of financial firms, as expressed by

$$\begin{aligned} \eta^*_{i,j,t} &= \frac{\partial h^{R*}_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}} = \frac{q^{p*}_{i,j,t}}{Q^{p*}_{j,t}} \cdot \frac{\partial h^{R*}_{i,j,t}}{\partial \ln Q^{p*}_{j,t}} \cdot \left(1 + \sum\nolimits_{k \neq i}^{N_F} \frac{\partial q^{p*}_{k,j,t}}{\partial q^{p*}_{i,j,t}} \right) \\ &= s^*_{i,j,t} \cdot \eta^{Q*}_{i,j,t} \cdot \left(1 + CV^*_{i,j,t} \right); \, j = 1, \cdots, N_A + N_L, \end{aligned} \tag{33}$$

where $s^*_{i,j,t}$ $(= q^{p*}_{i,j,t} / Q^{p*}_{j,t})$ is the ratio of the real balance of the j-th financial good of the i-th financial firm to the total balance in the market for the j-th financial good. The range of $s^*_{i,j,t}$ is $0 < s^*_{i,j,t} \leq 1$, and $s^*_{i,j,t} = 1$ if the i-th financial firm has a monopoly. Here, $\eta^{Q*}_{i,j,t}$ $(= \partial h^{R*}_{i,j,t} / \partial \ln Q^{p*}_{j,t})$ is the elasticity of the certain or predictable components of the SEHRR or the SEHCR for the j-th financial good with respect to the total balance in the market, and represents the fractional change in the former due to a 1% increase in the latter. From Eqs. (2) and (3), $\eta^{Q*}_{i,j,t}$ can be expressed as

$$\eta^{Q*}_{i,j,t} = \frac{\partial h^{R*}_{i,j,t}}{\partial \ln Q^{p*}_{j,t}} = \begin{cases} \dfrac{\partial r_{i,j,t}}{\partial \ln Q^{p*}_{j,t}} + \dfrac{\partial r^Q_{i,j,t}}{\partial \ln Q^{p*}_{j,t}} + \dfrac{\partial h^S_{i,j,t}}{\partial \ln Q^{p*}_{j,t}} - \dfrac{\partial h^D_{i,j,t}}{\partial \ln Q^{p*}_{j,t}} \\ \qquad (j = 2, \cdots, N_A), \\ \dfrac{\partial r_{i,j,t}}{\partial \ln Q^{p*}_{j,t}} + \dfrac{\partial r^Q_{i,j,t}}{\partial \ln Q^{p*}_{j,t}} + \dfrac{\partial h^I_{i,j,t}}{\partial \ln Q^{p*}_{j,t}} - \dfrac{\partial h^S_{i,j,t}}{\partial \ln Q^{p*}_{j,t}} \\ \qquad (j = N_A + 1, \cdots, N_A + N_L). \end{cases} \tag{34}$$

From these equations, if the j-th financial good is a financial asset (other than cash), that is, if $j = 2, \cdots, N_A$, then the elasticity of the certain or predictable components of the SEHRR with respect to the total balance in the market corresponds to the sum of the same elasticities of the collected interest rate, the uncollected interest rate, and the service charge rate, minus the same elasticity of the default rate. If the j-th financial good is a liability, then the elasticity of the certain or predictable components of the SEHCR with respect to the total balance in the market corresponds to the sum of the same elasticities of the paid interest rate, the unpaid interest rate, and the insurance premium rate, minus the same elasticity of the service charge rate. The sign of the elasticity of the certain or predictable component of the collected interest rate with respect to the total balance in the market is usually negative, and the sign of the same elasticity of the paid interest rate is usually positive. However, the sign of the other elasticities can be both positive or both negative.

The conjectural derivative $CV^*_{i,j,t}$ $(= \sum_{k \neq i}^{N_F} \partial q^{p*}_{k,j,t} / \partial q^{p*}_{i,j,t})$ describes how the i-th financial firm regards the changes in the j-th financial good of other firms with respect to the change in the j-th financial good of the i-th financial firm in period t. As described in the CURM, if $s^*_{i,j,t} = 1$ and $CV^*_{i,j,t} = 0$, then the i-th financial firm has a monopoly in the j-th financial good market in period t. If $CV^*_{i,j,t} = 0$, then the i-th financial firm is a Cournot firm, that is, the outputs of all other financial firms are no expected to change as the output of the i-th financial firm changes. If $CV^*_{i,j,t} = -1$, then the i-th financial firm is a competitive firm, that is, the price-marginal cost margin is zero. Higher values of $CV^*_{i,j,t}$ correspond to larger gaps between price and marginal cost, and thus represent less intense competition. The second term of the right-hand side of Eq. (31), $p_{G,t}$ times $\eta^*_{i,j,t}$ divided by $1 + r^{F*}_{i,t}$, thus represents the market structure and conduct effects.

As described in footnote 14, $MRS^{\pi*}_{e,i,t}$ in the third term of the right-hand side of Eq. (31) represents the marginal rate of substitution of quasi short-run profits for equity capital. The parameter thus provides a measure of the rate at which the financial firm is willing to substitute quasi short-run profits for equity capital as an index of the importance of equity capital, which directly reflects the financial firm's subjective evaluation of equity capital and associated opportunity costs. As described in Section 2.3, in the case that the financial firm encounters financial distress, equity capital acts as a cushion against losses and reduces the burden of financial distress costs.

The $MRS^{\pi *}_{e,i,t}$ parameter therefore indirectly reflects the subjective value of a decrease in the risk of bearing financial distress costs through an increase in equity capital. In other words, it provides a subjective evaluation of the occurrence of the risk of bearing financial distress costs due to a lack of equity capital. From Eq. (14), equity capital in period t increases due to an increase in a financial asset or a decrease in a liability in period t. In this case, quasi short-run profits in period t decrease. In the case that the financial firm is risk-averse, the marginal utility of equity capital in period t, given by $\partial u^*_{i,t} \big/ \partial q^{p*}_{e,i,t}$, is a decreasing function of equity capital. Similarly, the marginal utility of quasi short-run profits in period t, given by $\partial u^*_{i,t} \Big/ \partial \pi^{QS*}_{i,t}$, is also a decreasing function of quasi short-run profits. Therefore, if equity capital in period t is large, then the denominator of $MRS^{\pi *}_{e,i,t}$ is large and the numerator is small, resulting in a small value for the $MRS^{\pi *}_{e,i,t}$. Therefore, in the case that the financial firm is risk-averse, if the equity capital is large, then the risk of bearing financial distress costs is small, and the subjective value of a decrease in the risk of bearing financial distress costs through an increase in equity capital is also small. In this case, from Eq. (31), the GURP decreases if the j-th financial good is a financial asset (i.e., $b_j = 1$) and increases if the j-th financial good is a liability ($b_j = -1$). In the case of a financial asset, the third term of the right-hand side of Eq. (31), $b_j \cdot p_{G,t} \cdot MRS^{\pi *}_{e,i,t}$, is positive, as an increase in a financial asset increases equity capital. In the case of a liability, the term is negative because an increase in a liability reduces equity capital. For a liability, contrary to the case for a financial asset, the risk of bearing financial distress costs increases with a decrease in equity capital, and this subject value is negative. Similar to the case for a financial asset, however, if the equity capital is large, then the risk of bearing financial distress costs is small, and the absolute value of this subjective value is small. The third term of the right-hand side of Eq. (31) thus represents equity capital effects.

As described Section 3.2, the multiplicand

$$\beta_{i,t} \cdot \operatorname{cov}\left(\zeta_{i,j,t+1}, \partial u^*_{i,t+1} \Big/ \partial \pi^{QS*}_{i,t+1} \,\Big|\, \mathbf{z}_{i,t}\right) \Big/ \left(\partial u^*_{i,t} \Big/ \partial \pi^{QS*}_{i,t}\right)$$

in the fourth term of the right-hand side of Eq. (31), denoted $\varpi^*_{i,j,t}$ for simplicity, represents the effects of the volatility risk of quasi short-run profits. The sign of $\varpi^*_{i,j,t}$ is determined by the sign of $\mathrm{cov}\left(\zeta_{i,j,t+1}, \partial u^*_{i,t+1} \middle/ \partial \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)$, because the sign of $\partial u^*_{i,t} \middle/ \partial \pi^{QS*}_{i,t}$ is usually positive. As the marginal utility of quasi short-run profits is a decreasing function of quasi short-run profits if the financial firm is risk-averse, the sign of $\mathrm{cov}\left(\zeta_{i,j,t+1}, \partial u^*_{i,t+1} \middle/ \partial \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)$ is opposite that of $\mathrm{cov}\left(\zeta_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)$. The sign of $\varpi^*_{i,j,t}$ is thus opposite that of $\mathrm{cov}\left(\zeta_{i,j,t+1}, \pi^{QS*}_{i,t+1} \middle| \mathbf{z}_{i,t} \right)$. In this case, an increase in a financial asset reduces the variance of quasi short-run profits in period $t+1$, that is, reduces the volatility risk of quasi short-run profits if the j-th financial good is a financial asset (i.e., $b_j = 1$), and an increase in a liability increases the volatility risk of quasi short-run profits if the j-th financial good is a liability (i.e., $b_j = -1$). From Eq. (31), the GURP of the financial asset thus increases, while the GURP of the liability decreases.[17] In the case that the financial firm is risk-averse, the financial firm desires the volatility risk of quasi short-run profits to decrease. In this scenario, the financial goods that when increased reduce the risk are rated high, while those that when increased also increase the risk are rated low. The GURP of the former financial good should therefore increase, while that of the latter should decrease. The fourth term of the right-hand side of Eq. (31), which is the product of $\varpi^*_{i,j,t}$ and $p_{G,t}$, thus represents the risk-adjustment effects.

As defined in the CURM, the SURP and the CURP are expressed as the following definitions.

Definition 11 *The stochastic user-revenue price of the i-th financial firm during period t, denoted by $p^{SUR}_{i,j,t}$, is defined as*

$$p^{SUR}_{i,j,t} = b_j \cdot p_{G,t} \cdot \left(b_C \cdot h^{R*}_{i,j,t} - r^{F*}_{i,t} \right) / \left(1 + r^{F*}_{i,t} \right) \; ; \, j = 1, \cdots, N_A + N_L. \quad (35)$$

[17] If the GURP is negative, i.e., the financial good is a fixed input, a positive value of $\varpi^*_{i,j,t}$ corresponds to a decrease in the absolute value of the GURP.

Definition 12 *The conjectural user-revenue price of the i-th financial firm during period t, denoted by* $p_{i,j,t}^{CUR}$*, is defined as*

$$\begin{aligned} p_{i,j,t}^{CUR} &= b_j \cdot p_{G,t} \cdot \left[\left(b_C \cdot h_{i,j,t}^{R*} - r_{i,t}^{F*} \right) / \left(1 + r_{i,t}^{F*} \right) + b_C \cdot \eta_{i,j,t}^{*} / \left(1 + r_{i,t}^{F*} \right) \right] \\ &= p_{i,j,t}^{SUR} + b_j \cdot p_{G,t} \cdot b_C \cdot \eta_{i,j,t}^{*} / \left(1 + r_{i,t}^{F*} \right) \; ; \; j = 1, \cdots, N_A + N_L. \quad (36) \end{aligned}$$

Remark 2 *Using the SURP or the CURP, the GURP can be then expressed as*

$$\begin{aligned} p_{i,j,t}^{GUR} &= p_{i,j,t}^{SUR} + b_j \cdot p_{G,t} \cdot \left[b_C \cdot \eta_{i,j,t}^{*} / \left(1 + r_{i,t}^{F*} \right) + MRS_{e,i,t}^{\pi*} + \varpi_{i,j,t}^{*} \right] \\ &= p_{i,j,t}^{CUR} + b_j \cdot p_{G,t} \cdot \left[MRS_{e,i,t}^{\pi*} + \varpi_{i,j,t}^{*} \right] \; ; \; j = 1, \cdots, N_A + N_L. \quad (37) \end{aligned}$$

This equation shows that the GURP takes into account the SURP, as well as market structure and conduct effects, equity capital effects, and risk-adjustment effects. The GURP is therefore equivalent to the CURP with the addition of equity capital effects and risk-adjustment effects, that is, the SURP is extended to include explicit consideration of market structure and conduct effects, equity capital effects, and risk-adjustment effects. If the equity capital effects and risk-adjustment effects are zero, that is, if the effects cancel or are both zero, then the GURP is fully equivalent to the CURP. If the market structure and conduct effects are zero, then the GURP is fully equivalent to the SURP. As described in the CURM, if the financial firm is risk-neutral, then the GURP corresponds to the UCP of the UCM.

3.4 Extended Generalized-Lerner Indices

The EGLI, an extension of the GLI in the CURM, can be derived using Eqs. (30) and (32), which represent the relationship between the GURP and marginal variable costs, and Eq. (37), which give the relationships among the SURP, the CURP, and the GURP. In concrete terms, as in the CURM, dividing the discrepancy between the SURP and the marginal variable costs by the SURP of Eq. (35) gives the EGLI. The SURP is a price in which market structure and conduct effects, equity capital effects, and risk-adjustment effects are zero. The discrepancy between the SURP and marginal variable costs therefore takes these effects into account. In this section, the case of a positive SURP and positive marginal variable costs is considered with respect to the j-th financial good as an output.

Remark 3 *From Eqs. (32) and (37), the discrepancy between the SURP and marginal variable costs can be expressed as*

$$p_{i,j,t}^{SUR}-MC_{i,j,t}^{V*}=-b_j\cdot p_{G,t}\cdot\left(\gamma_{i,j,t}^{*}+MRS_{e,i,t}^{\pi *}+\varpi_{i,j,t}^{*}\right);\; j=1,\cdots,N_A+N_L, \quad (38)$$

where

$$\gamma_{i,j,t}^{*}=b_C\cdot\eta_{i,j,t}^{*}\big/\left(1+r_{i,t}^{F*}\right)\;;\; j=1,\cdot\cdot\cdot,N_A+N_L. \quad (39)$$

The EGLI is defined by dividing both sides of Eq. (38) by the SURP given by Eq. (35).

Definition 13 *The extended generalized-Lerner index of the j-th financial good of the i-th firm in period t, denoted by $EGLI_{i,j,t}$, is defined as*

$$\begin{aligned}EGLI_{i,j,t}&=\frac{p_{i,j,t}^{SUR}-MC_{i,j,t}^{V*}}{p_{i,j,t}^{SUR}}\\&=-\frac{b_C\cdot\eta_{i,j,t}^{*}+\left(MRS_{e,i,t}^{\pi *}+\varpi_{i,j,t}^{*}\right)\cdot\left(1+r_{i,t}^{F*}\right)}{b_C\cdot h_{i,j,t}^{R*}-r_{i,t}^{F*}};\\&\qquad j=1,\cdot\cdot\cdot,N_A+N_L. \quad (40)\end{aligned}$$

Under the assumption that the j-th financial good is an output, the sign of $b_C\cdot h_{i,j,t}^{R*}-r_{i,t}^{F*}$ is positive if the j-th financial good is a financial asset, and negative if the j-th financial good is a liability. If the sign of $\eta_{i,j,t}^{*}$ is determined by the sign of $\partial r_{i,j,t}\big/\partial\ln Q_{j,t}^{p*}$, then the sign of $\eta_{i,j,t}^{*}$ is negative if the j-th financial good is a financial asset and positive if the j-th financial good is a liability. The sign of $MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t}$ can be both positive and negative.

Remark 4 *Even if $\eta^*_{i,j,t} = 0$, the SURP is greater than the marginal variable costs if $MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t} < 0$ for financial assets or $MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t} > 0$ for liabilities and thus the EGLI has a positive value.*

The inequality $MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t} < 0$ holds for financial assets if the subjective value of an increase in the risk of bearing financial distress costs due to an increase in the volatility risk of quasi short-run profits is greater than the subjective value of a decrease in the risk of bearing financial distress costs due to an increase in equity capital. Similarly, $MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t} > 0$ applies for liabilities if an increase in the liability increases the volatility risk of quasi short-run profits, or even if the risk is reduced, the subject value of a decrease in the risk of bearing financial distress costs is smaller than the subject value of an increase in the risk due to a decrease in equity capital. In these cases, it is thus understood that the EGLI has a positive value even if the market for the j-th financial good is competitive, that is, $\eta^*_{i,j,t} = 0$.

Remark 5 *Even if the market is uncompetitive, that is, $\eta^*_{i,j,t} < 0$ for financial assets and $\eta^*_{i,j,t} > 0$ for liabilities, then the degree by which the SURP exceeds the marginal variable costs is restrained, making the EGLI smaller than the GLI. Here, $MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t} > 0$ and $\left(MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t}\right) \cdot \left(1 + r^{F*}_{i,t}\right) \leq \left|\eta^*_{i,j,t}\right|$ for financial assets, and $MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t} < 0$ and $\left|\left(MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t}\right) \cdot \left(1 + r^{F*}_{i,t}\right)\right| \leq \eta^*_{i,j,t}$ for liabilities. In the case that $\eta^*_{i,j,t} + \left(MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t}\right) \cdot \left(1 + r^{F*}_{i,t}\right) = 0$, the SURP is equivalent to the marginal variable costs and the EGLI is zero.*

The inequality $MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t} > 0$ holds for financial assets if an increase in the financial asset reduces the volatility risk of quasi short-run profits, or even if the risk increases, the subject value of an increase in the risk of bearing financial distress costs is smaller than the subject value of a decrease in the risk due to an increase in equity capital. Similarly, $MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t} < 0$ applies for liabilities if the subjective value of a decrease in the risk of bearing financial distress costs due to a decrease in the volatility risk of quasi short-run profits is greater than the subjective value of an increase in the risk of bearing financial distress costs due to a decrease in equity capital. In these cases, it is understood that the EGLI is smaller than the GLI, or that the EGLI is zero, even if the market for the j-th financial good is uncompetitive.

As defined in the CURM, the GLI is defined as follows.

Definition 14 *The generalized-Lerner index of the j-th financial good of the i-th firm in period t, denoted by $GLI_{i,j,t}$, is defined as*

$$GLI_{i,j,t} = -\frac{b_C \cdot \eta^*_{i,j,t}}{b_C \cdot h^{R*}_{i,j,t} - r^{F*}_{i,t}};\ j = 1, \cdots, N_A + N_L. \tag{41}$$

Remark 6 *Using the GLI, the EGLI can be then expressed as*

$$EGLI_{i,j,t} = GLI_{i,j,t} - \frac{\left(MRS^{\pi*}_{e,i,t} + \varpi^*_{i,j,t}\right) \cdot \left(1 + r^{F*}_{i,t}\right)}{b_C \cdot h^{R*}_{i,j,t} - r^{F*}_{i,t}};\ j = 1, \cdots, N_A + N_L. \tag{42}$$

The EGLI thus represents an extension of the GLI to include consideration of equity capital effects and risk-adjustment effects in the discrepancy between the SURP and marginal variable costs. If these effects cancel or are both zero, the EGLI is fully equivalent to the GLI.

4 Conclusion

In the present paper a generalized user-revenue model was constructed as an extension of the conjectural user-revenue model. The proposed GURM takes into account the volatility risk of quasi short-run profits and equity capital effects reflecting the risk of bearing the costs of financial distress. This extension was achieved by introducing the principles of the consumption-based capital asset pricing model into the CURM proposed by Homma and Souma (2005). Specifically, uncertainties were added to the endogenous holding-revenue and holding-cost rates, and the definition of the utility function of financial firms was extended to incorporate both quasi short-run profits and equity capital. Risk-adjustment effects were introduced by expressing the covariance of uncertain factors in the stochastic endogenous holding-revenue and holding-cost rates with a stochastic discount factor, allowing for explicit consideration of the volatility risk of quasi short-run profits. Equity capital effects were introduced by expressing the marginal rate of substitution between equity capital and quasi short-run profits, making it possible to subjectively evaluate the accrual of equity capital by financial firms considering both the opportunity costs and the risk of bearing the cost of financial distress. The stochastic and conjectural user-revenue prices were derived as a generalized user-revenue price, and the extended generalized-Lerner index was proposed to incorporate these extensions. The modifications of the CURM allow the analysis to account for these risks.

The generalized user-revenue price adds explicit consideration of market structure and conduct effects and equity capital and risk-adjustment effects to the stochastic user-revenue price. The GURP is also an extension of the conjectural user-revenue price to account explicitly for equity capital and risk-adjustment effects. If the sum of equity capital and risk-adjustment effects is zero, that is, either the two effects cancel or both are zero, then the GURP is exactly equivalent to the CURP. Similarly, if the sum of the market structure and conduct effects is zero, then the GURP is exactly equivalent to the SURP. If the financial firm is risk-neutral, then the GURP is also equivalent to the user-cost price of the user-cost model. The extended GLI incorporates consideration of the effects of equity capital and risk adjustment on the discrepancy between the SURP and marginal variable costs. Thus, if both effects are zero or cancel, then the EGLI is exactly equivalent to the GLI.

The present definition of the EGLI clarified two key points of particular importance for industrial organization. First, even if the market for a financial good is competitive, the SURP is greater than the marginal variable costs, resulting in a positive EGLI value. In this case, for financial assets, the subjective value of an increase in the risk of bearing financial distress costs due to an increase in the volatility risk of quasi short-run profits is greater than the subjective value of a decrease in the risk of bearing financial distress costs due to an increase in equity capital. For liabilities, an increase in the liability increases the volatility risk of quasi short-run profits, or alternatively, even if the risk is reduced, the subjective value of a decrease in the risk of bearing financial distress costs is smaller than the subjective value of an increase in the risk due to a decrease in equity capital. Second, even if the market for a financial good is uncompetitive, the degree by which the SURP exceeds the marginal variable costs is restrained. Thus, the EGLI is smaller than the GLI, or if the SURP is equivalent to the marginal variable costs, the EGLI is zero. In this case, for financial assets, an increase in the financial asset reduces the volatility risk of quasi short-run profits, or alternatively, even if the risk is increased, the subjective value of an increase in the risk of bearing financial distress costs is smaller than the subjective value of a decrease in the risk due to an increase in equity capital. For liabilities, the subjective value of a decrease in the risk of bearing financial distress costs due to a decrease in the volatility risk of quasi short-run profits is greater than the subjective value of an increase in the risk of bearing financial distress costs due to a decrease in equity capital.

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