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Self-fulfilling and Self-reinforcing Mechanisms: A Deterministic Approach

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Abstract: This paper provides a macro-dynamic model of Keynesian type. The model possesses a self-reinforcing mechanism by which an initial pessimism (resp. optimism) is gradually reinforced. Even if the initial pessimism (resp. optimism) is small, through such a mechanism the economy is led to a chronic slump (resp. sustained boom). In such a case, the initial expectation realizes. In our model, the self-reinforcing mechanism operates accompanied by a self-fulfilling mechanism. Our model possesses the parameters such as the difference between real income and expected income, the adjustment coefficient of aggregate supply and the propensities to consume concerning income and expected income. We analyze the effect of such parameters on the operation of self-reinforcing mechanism.

Keywords: Keynesian Macro-dynamic Model; Pessimism; Optimism; Chronic Slump; Sustained Boom; Self-fulfilling Mechanism; Self-reinforcing Mechanism.

1. Introduction

Dohtani (2016, 2019) consider the Keynesian business cycle models of Kaldor type[1] with the adaptive expectation formation concerning the expected income and show that the pessimistic (resp. optimistic) expectation formation yields nonlinearity in the adjustment function of expected income and leads to slump (resp. boom) fluctuations under the market equilibrium. Moreover, Dohtani (2019) shows that such nonlinearity yields even strange attractors (erratic fluctuations). As stressed in Dohtani (2016), the occurrence of slump (resp. boom) fluctuations implies that the pessimistic (resp. optimistic) expectation realizes and therefore, the slump (resp. boom) fluctuations often persist for a long time. Thus, Dohtani (2016, 2019) make clear a mechanism of self-fulfilling prophecy. We call such a mechanism self-fulfilling mechanism (abbreviated as SFM). In the model of Dohtani (2016, 2019), it is assumed from the beginning that households possess considerably pessimistic (resp. optimistic) expectation formation. That is, it is assumed that the nonlinearity of adjustment coefficient produced by pessimism (resp. optimism) is strong from the beginning. In the assumption, the degree of nonlinearity is assumed to depend on exogenous factors.

However, developing the Keynesian economic dynamic model of Dohtani (2019), we try to construct a model with a self-reinforcing mechanism (abbreviated as SRM)[2]. That is, even if households possess a weak pessimistic expectation formations at the beginning,[3] such pessimistic expectation formations are gradually and endogenously reinforced by SRM. Thus, unlike Dohtani (2016, 2019), the degree of pessimism is endogenously determined. Moreover, in our model, SRM operates accompanied by SFM. We define an index which is closely related to the operation of SRM. Using the index, we consider the effects of parameters on the operation of SRM. Especially, as such parameters, we consider the difference between real income and expected income and the propensities to consume concerning income and expected income. We

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[1] For the Keynesian business-cycle model of Kaldor type, see Kaldor (1941), Chang and Smith (1971), Asada (1987), Gabisch and Lorenz (1987), Owase (1989, 1991), Lorenz (1993), Gandolfo (1996), and Rosser (2000). For the dynamic behaviors of the model, see Kosobud and O'Neill (1972), Boldrin (1984), Herrmann (1985), Grasman and Wentzel (1994), Bischi et. al. (2001), Agliari and Dieci (2005), and Agliari et. al. (2007).

[2] For a different viewpoint of self-fulfilling mechanism, see Arthur (1994).

[3] Needless to say, the weak pessimistic expectations are determined by exogenous factors.

demonstrate that for each parameter, there is a threshold that determines the operation of SRM. That is, if a parameter is larger (resp. smaller) than the threshold, SRM operates (resp. does not operate).

Moreover, our model with SFM and SRM arises the reversal phenomenon of pessimism and optimism. We briefly explain this phenomenon. We suppose that at the starting point, optimism dominates. In the case where the reversal phenomena occur, SRM gradually operates on pessimism and as a result, the economy becomes pessimistic. This phenomenon shows that SRM does not always operate on dominant psychological situation.

This paper is organized as follows. In Section 2, we briefly explain the Keynesian business cycles model of Kaldor type, which includes SFM. In Section 3, we demonstrate that the expected income equals some kind of weighted average of past income. In Section 4, we briefly give an index which estimates the degree of pessimism or optimism. In Section 5, extending the Keynesian business cycles model, we construct a macro-dynamic model including both SFM and SRM. In Section 6, by using numerical examples of the extended model, we analyze dynamic behaviors in detail. In Section 7, we demonstrate the occurrence of the reversal phenomenon of pessimism and optimism. Conclusions and final remarks are given in Section 8.

2. Model with SFM

To analysis psychological features of business fluctuations, Dohtani (2019) considers the following Keynesian business-cycle model of Kaldor type:

$$\Theta_{SFM}: \quad \begin{cases} \dot{Y} = \mu[\alpha Y + \beta Y_e + A + \phi(Y) - jK - Y], \\ \dot{K} = \phi(Y) - (j + \delta)K, \\ \dot{Y}_e = \Psi(Y - Y_e). \end{cases}$$

where Y is income, Y_e expected income, K capital stock, δ depreciation rate, μ the adjustment coefficient of aggregate supply, Ψ the adjustment function of expected income. $\phi' > 0$ and $\Psi' > 0$ are assumed. System Θ_{SFM} is the Kaldor model including expected income, where

$$\text{Consumption}=\alpha Y+\beta Y_e+A, \quad \text{Investment}=\phi(Y)-jK,$$
$$\text{Aggregate demand}=\alpha Y+\beta Y_e+A+\phi(Y)-jK, \quad \text{Aggregate Supply}=Y,$$

In the case where $u=Y-Y_e>0$ (resp. $Y-Y_e<0$), since pessimistic households becomes cautious, the adjustment of Y_{et} is small (resp. large). Thus, as discussed in Dohtani (2016) considerably pessimistic expectations about the future economy yields origin asymmetry and demonstrates that the nonlinearity causes SFM to System Θ_{SFM}. We here provide two examples of the $\Psi-$function possesses such nonlinearity:

Example 1: As a typical smooth example, we have $\Psi(u)=\{a-b\arctan(cu)\}\bullet u$ whose graph is described in Part (1) of Figure 1. In Part (1), we set $a=0.5$, $b=0.25$ and $c=0.4$.■

Example 2: As a piecewise linear example, we have $\Psi(u)=a\{u+|u|\}/2+b\{u-|u|\}/2$ whose graph is described in Part (2) of Figure 1. In Part (2), we set $a=0.11$ and $b=0.9$. Although this function is not of class C^1, the uniqueness and continuity of solution can be proved. See Dohtani (2016) ■.

Figure 1 about here.

In the following, for simplification, we use a piecewise linear $\Psi-$function. The adjustment of expected income is given by the following system:

(1) $$\dot{Y_e}=\Psi(Y-Y_e)=\Psi(u)=\xi(\theta^{+}\frac{u+|u|}{2}+\theta^{-}\frac{u-|u|}{2}).$$

where $\xi>0$ and $\theta^{\pm}>0$. We call ξ the sensitivity of adjustment of expected income and θ^{+} (resp. θ^{-}) the upper (resp. lower) adjustment coefficient. As stated above, if households possess considerably pessimistic (resp. optimistic) expectations about the future economy, θ^{+} becomes very small (resp. large) and θ^{-} very large (resp. small). The blue curves in Parts (1) and (2) of Figure 2 describe the $\Psi-$function in the case where households are considerably pessimistic and optimistic, respectively. In Part (1), we set $\theta^{+}=0.05$ and $\theta^{-}=2$. In Part (1), we set $\theta^{+}=2$ and $\theta^{-}=0.05$. In such an extreme case, System Θ_{SFM} yields (periodic or even chaotic) fluctuations that appear under the market equilibrium. We call such a fluctuation slump fluctuation. For more details, see Dohtani (2016, 2019). We here give a numerical example of a chaotic slump fluctuation. The red curve of Figure 3 describes the time series of a typical chaotic slump fluctuation of System Θ_{SFM}. The blue curve of Figure 3 describes the market equilibrium. For details of Figure 1, see Dohtani (2019). We here make one remark. As demonstrated in Dohtani (2019), the slump fluctuations are attractors and any path converges to the attractor. Therefore, in the case where a slump fluctuation exists, any bubble economy bursts and falls into slump.

Figures 2 and 3 about here.

In the following, by constructing dynamic equations on the degrees of pessimism and optimism, we incorporate SRM into System Θ_{SFM}.

3. Expected Income and Weighted Average Income

Before starting our argument, we here briefly explain that the expected income, which is adjusted by linear $\Psi-$function, can be interpreted as some kind of weighted average income. We assume that the expected income is given by the weighted average of past income:.

$$Y_e(t) \equiv \int_{(-\infty,t]} W(\tau)Y(\tau)d\tau,$$

where $W(\tau)$ is the weight function satisfying the following natural conditions:

$$W'(\tau) < 0 \quad \text{かつ} \quad \int_{(-\infty,t]} W(\tau)d\tau = 1 \quad \text{for any} \quad u \in (-\infty, t].$$

We here consider

$$W(\tau) = \zeta \exp(\zeta(\tau - t)), \quad \zeta > 0.$$

Then, we obtain the following result.

Lemma 1: $\int_{(-\infty,t]} W(\tau)d\tau = 1$ for any $t \in R^1$ and $\dot{Y_e} = \zeta(Y - Y_e)$ ■

Proof: The proof of the first half follows from

$$\int_{(-\infty,t]} \zeta \exp(\zeta(\tau - t)d\tau = \zeta \exp(-\zeta t)\int_{(-\infty,t]} \exp(\zeta\tau)d\tau = \zeta \exp(-\zeta t)\left[\frac{\exp(\zeta t)}{\zeta}\right]_{-\infty}^{t} = 1.$$

Secondly, we prove the latter half. From the definition, we have

$$Y_e(t) = \exp(-\zeta t)\int_{(-\infty,t]} \zeta Y(\tau)\exp(\zeta\tau)d\tau$$

Therefore, we obtain

$$\dot{Y_e} = -\zeta \exp(-\zeta t)\int_{(-\infty,t]} \zeta Y(\tau)\exp(\zeta\tau)d\tau + \exp(-\zeta t)\zeta Y(t)\exp(\zeta t)$$

$$= -\zeta \int_{(-\infty,t]} \zeta Y(\tau)\exp(\zeta(\tau - t))d\tau + \zeta Y(t) = \zeta\{Y(t) - Y_e(t). \text{ ■}$$

We see from Lemma 1 that the expected income, which is adjusted by linear $\Psi -$function, equals the above-mentioned weighted average income with the weight function $W(\tau) = \zeta \exp(\zeta(\tau - t))$.

4. Changes of Degrees of Pessimism and Optimism

In this section, we construct dynamic equations concerning the changes in the degree of pessimism and optimism. Therefore, $\theta^+ = \theta^+(t)$ and $\theta^- = \theta^-(t)$. Without loss of generality, we consider the case where households possess pessimistic. In the following, we assume

Assumption 1: $\theta^{\pm}(t) \in [0.01, 0.7]$ for any $t \ge 0$. ■

The similar argument holds true in the optimistic case. In such a case, we have

$\theta^{+}(t) < \theta^{-}(t)$. It is considered that the value of $\theta^{+}(t)$ becomes smaller and $\theta^{-}(t)$ larger, as the degree of pessimism rises. Using the upper and lower adjustment coefficients $\theta^{\pm}(t)$, we here introduce the following index:

Definition 1: Noting Assumption 1, we define Pessimism-Optimism Index (abbreviated as PO-index):

$$\Pi^{PO}(t) \equiv \frac{\max\{\theta^{+}(t),0.01\} - \max\{\theta^{-}(t)0.01\} - 0.01}{\max\{\theta^{+}(t)0.01\} + \max\{\theta^{-}(t),0.01\} - 0.01}. \blacksquare$$

The PO-index tells us the operation of SRM. We obtain the following lemma:

Lemma 2: We have $\Pi^{PO}(t) \in [-1,1]$. We consider the case where $\theta^{+}(t)$ or $\theta^{-}(t)$ is convergent. We have

(2.1) $\lim_{t\to\infty} \Pi^{PO}(t) = -1$ if and only if $\lim_{t\to\infty} \max\{\theta^{+}(t),0.01\} = 0.01$.

(2.2) $\lim_{t\to\infty} \Pi^{PO}(t) = +1$ if and only if $\lim_{t\to\infty} \max\{\theta^{-}(t),0.01\} = 0.01$.■

Proof: We assume $\lim_{t\to\infty} \min\{\theta^{+}(t),0.01\} = 0.01$. Then, we have

$$\lim_{t\to\infty} \Pi^{PO}(t) = \frac{-\lim_{t\to\infty} \max\{\theta^{+}(t),0.01\}}{\lim_{t\to\infty} \max\{\theta^{+}(t),0.01\}} = -1.$$

Conversely, we assume $\lim_{t\to\infty} \Pi^{PO}(t) = -1$. Then, we have

$$\frac{\max\{\theta^{+}(t),0.01\} - \max\{\theta^{-}(t)0.01\} - 0.01}{\max\{\theta^{+}(t)0.01\} + \max\{\theta^{-}(t),0.01\} - 0.01} = -1.$$

Therefore, we have $\lim_{t\to\infty} \max\{\theta^{+}(t),0.01\} = 0$. Thus, we complete the proof of (2.1). By the same argument, we can prove (2.2).■

As demonstrated in the following numerical examples, the pessimistic case (2.1) (resp. the optimistic case (2.2)) of Lemma 2 shows that if the degree of pessimism (resp. optimism) is large, then $\max\{\theta^{+}(t),0.01\} \to 0.01$ (resp. $\max\{\theta^{-}(t),0.01\} \to 0.01$). On the other hand, if SRM operates, an initial trifling pessimism (resp. optimism) is gradually reinforced and consequently led to a deep pessimism (resp. optimism). From Lemma 2, it is expected that SRM operates if and only if $\lim_{t\to\infty} \Pi^{PO}(t) = \pm 1$. Based on these considerations, we introduce two definitions:

Definition 2: If $\Pi^{PO}(t) \to \pm 1$, we say that SRM operates in System $\Theta_{SFM-SRM}$.■

Definition 3: We say the degree of pessimism as P-degree and the degree of optimism O-degree.■

5. A Dynamic Model with SFM and SRM

Based on the argument in Sections 2 and 4, we provide an intuitive explanation on the effects of $\theta^{\pm}$ on the degrees of pessimism and optimism. For example, we consider the situation where a person is chased by a dangerous dog. We suppose he is afraid that he may be bitten by the dog. As the distance between him and the dangerous dog decreases, the pessimism that he may be bitten becomes more serious. The opposite is the opposite. In the following, the change of distance changes your psychological state. In this case, if the distance gradually decreases (resp. increases), the degree of his pessimism also gradually increases (resp. decreases). This situation is called a negative feedback. Even if the initial pessimism is weak, the negative (resp. positive) feedback operates on his initial weak pessimism and as time goes by, the initial pessimism becomes more and more severe. Such a feedback mechanism is also called a self-reinforcing mechanism. The same mechanism is true for the pessimism and the optimism about the future economy, which appear in the adaptive adjustment of expected income. In the following, we mathematically express such a mechanism of the change of psychological state.

We consider the situation where $Y - Y_e$ increases. This situation implies that real GDP decreases more than expected. In such a case, the degree of pessimism increases. This implies that θ^+ decreases and θ^- increases:

(3) $$\dot{Y} - \dot{Y}_e > 0 \ \ (\text{resp. } \dot{Y} - \dot{Y}_e < 0) \Rightarrow \dot{\theta}^+ > 0 \ \text{and} \ \dot{\theta}^- < 0 \ \ (\text{resp. } \dot{\theta}^+ < 0 \ \text{and} \ \dot{\theta}^- > 0).$$

Based on the diagram (3), we formulate that

$$\dot{\theta}^{+} = f^{+}(\dot{Y}-\dot{Y}_e) = f^{+}(w), \qquad \frac{df^{+}}{dw} > 0,$$

$$\dot{\theta}^{-} = f^{-}(\dot{Y}-\dot{Y}_e) = f^{-}(w), \qquad \frac{df^{-}}{dw} < 0.$$

For simplification, we assume that

$$f^{+}(w) = \tau^{+}w \quad \text{and} \quad f^{-}(w) = -\tau^{-}w.$$

Then, by assuming System Θ_{SFM}, we obtain that the following dynamic equations concerning adjustment coefficient of expected income.

(4.1) $$\dot{\theta}^{+} = \tau^{+} \bullet (\dot{Y}-\dot{Y}_e)$$
$$= \tau^{+}[\mu\{\alpha Y + \beta Y_e + A + \phi(Y) - jK - Y\} - \max\{\theta^{+}, 0.01\}\frac{(Y-Y_e)+|Y-Y_e|}{2} - \max\{\theta^{-}, 0.01\}\frac{(Y-Y_e)-|Y-Y_e|}{2}],$$

(4.2) $$\dot{\theta}^{-} = -\tau^{-} \bullet (\dot{Y}-\dot{Y}_e)$$
$$= -\tau^{-}[\mu\{\alpha Y + \beta Y_e + A + \phi(Y) - jK - Y\} - \max\{\theta^{+} 0.01\}\frac{(Y-Y_e)+|Y-Y_e|}{2} - \max\{\theta^{-}, 0.01\}\frac{(Y-Y_e)-|Y-Y_e|}{2}].$$

Equations (4.1) and (4.2) give dynamic behaviors concerning P-degrees and O-degree.

We now rewrite the adjustment equation of expected income as follows:

$$\dot{Y}_e = \max\{\theta^{+}, 0.01\}\frac{(Y-Y_e)+|Y-Y_e|}{2} + \max\{\theta^{-}, 0.01\}\frac{(Y-Y_e)-|Y(t)-Y_e|}{2}.$$

Accordingly, Equations (5) are rewritten as:

$$\dot{\theta}^{+} = \tau^{+} \bullet (\dot{Y}-\dot{Y}_e)$$
$$= \tau^{+}[\mu\{\alpha Y + \beta Y_e + A + \phi(Y) - jK - Y\} - \max\{\theta^{+}, 0.01\}\frac{(Y-Y_e)+|Y-Y_e|}{2} - \max\{\theta^{-}, 0.01\}\frac{(Y-Y_e)-|Y-Y_e|}{2}]$$

$$\dot{\theta}^- = -\tau^- \bullet (\dot{Y} - \dot{Y}_e)$$

$$= -\tau^-[\mu\{\alpha Y + \beta Y_e + A + \phi(Y) - jK - Y\} - \max\{\theta^+, 0.01\}\frac{(Y - Y_e) + |Y - Y_e|}{2} - \max\{\theta^-, 0.01\}\frac{(Y - Y_e) - |Y - Y_e|}{2}]$$

We incorporate Equations (4) into System Θ_{SFM} :

$$\Theta_{\mathrm{SFM-SRM}}: \begin{cases} \dot{Y} = \mu\{\alpha Y + \beta Y_e + A + \phi(Y) - jK - Y\}, \\ \dot{K} = \phi(Y) - (j + \delta)K, \\ \dot{Y}_e = \max\{\theta^+, 0.01\}\frac{(Y - Y_e) + |Y - Y_e|}{2} + \max\{\theta^-, 0.01\}\frac{(Y - Y_e) - |Y - Y_e|}{2}, \\ \dot{\theta}^+ = \tau^+[\mu\{\alpha Y + \beta Y_e + A + \phi(Y) - jK - Y\} \\ \qquad - \max\{\theta^+, 0.01\}\frac{(Y - Y_e) + |Y - Y_e|}{2} - \max\{\theta^-, 0.01\}\frac{(Y - Y_e) - |Y - Y_e|}{2}], \\ \dot{\theta}^- = -\tau^-[\mu\{\alpha Y + \beta Y_e + A + \phi(Y) - jK - Y\} \\ \qquad - \max\{\theta^+, 0.01\}\frac{(Y - Y_e) + |Y - Y_e|}{2} - \max\{\theta^-, 0.01\}\frac{(Y - Y_e) - |Y - Y_e|}{2}], \end{cases}$$

Since System $\Theta_{SFM-SRM}$ includes the dynamic equations concerning P-degree and O-degree (i.e. the dynamic equations concerning θ^+ and θ^-), it can be expected that System $\Theta_{SFM-SRM}$ possesses SRM. In the next section, we consider the possible occurrence of SRM.

6. Numerical Examples: Unstable Cases

In the following, for simplicity, we assume

Assumption 2: $\delta \bullet \max\{\phi'(u) : u \in R^1\} < (j + \delta)(1 - \alpha - \beta)$.

Then, we have the following:

Lemma 3: System $\Theta_{SFM-SRM}$ possesses the equilibrium set:

$$E \equiv \{Y^*\} \times \{Y_e^*\} \times \{K^*\} \times R^2 \subseteq R^5. \blacksquare$$

Proof: See Appendix.■

Lemma 3 shows that as long as $(Y, Y_e, K) = (Y^*, Y_e^*, K^*)$, System $\Theta_{SFM-SRM}$ is in equilibrium for any $(\theta^+, \theta^-) \in R^2$. Noting this point, we proceed our argument.

In the following, by transforming (Y^*, Y_e^*, K^*) to $(0,0,0)$, we analyze the dynamic behavior of System $\Theta_{SFM-SRM}$. For the purpose, without loss of generality, it is sufficient to assume the following:

$$A = 0, \quad \phi(0) = 0.$$

We assume the same functional form of investment function as the Kaldor model.[4] As such a form, we have

$$\phi(Y) = a\{bY + c\,\mathrm{Arctan}(Y)\}.$$

We start from the proof of the following result.

Lemma 4: Assumption 2 is satisfied under the following condition.

$$a(b+c) < \frac{j+\delta}{\delta}(1-\alpha-\beta). \blacksquare$$

Proof: See Appendix.■

The blue curve of Figure 4 describes the graph of the $\phi-$function with $a = 0.6$, $b = 0.1$ and $c = 1$. In the following, we use this $\phi-$function.

Figure 4 about here.

Then, System $\Theta_{SFM-SRM}$ becomes

4 For such a type of investment function, see Kaldor (1941), Chang and Smith (1971), Gabisch and Lorenz (1987), Owase (1989, 1991), Lorenz (1993), Gandolfo (1996), Dohtani et al. (1996), Rosser (2000), and Dohtani (2016, 2019)

$$\tilde{\Theta}_{SFM-SRM}:\begin{cases} \dot{Y} = \mu[\alpha Y + \beta Y_e + a\{bY + c\text{Arctan}(Y)\} - jK - Y], \\ \dot{K} = a\{bY + c\text{Arctan}(Y)\} - (j+\delta)K, \\ \dot{Y}_e = \xi[\max\{\theta^+, 0.01\}\dfrac{(Y-Y_e)+|Y-Y_e|}{2} + \max\{\theta^-, 0.01\}\dfrac{(Y-Y_e)-|Y-Y_e|}{2}], \\ \dot{\theta}^+ = \tau^+[\mu\{\alpha Y + \beta Y_e + a\{bY + c\text{Arctan}(Y)\} - jK - Y\} \\ \qquad - \max\{\theta^+, 0.01\}\dfrac{(Y-Y_e)+|Y-Y_e|}{2} - \max\{\theta^-, 0.01\}\dfrac{(Y-Y_e)-|Y-Y_e|}{2}], \\ \dot{\theta}^- = -\tau^-[\mu\{\alpha Y + \beta Y_e + a\{bY + c\text{Arctan}(Y)\} - jK - Y\} \\ \qquad + \max\{\theta^+, 0.01\}\dfrac{(Y-Y_e)+|Y-Y_e|}{2} - \max\{\theta^-, 0.01\}\dfrac{(Y-Y_e)-|Y-Y_e|}{2}]. \end{cases}$$

As stated above, to start SRM, we must treat the case where System $\tilde{\Theta}_{SFM-SRM}$ is unstable. To do so, we set $\tau^+ = \tau^- = 0$. Then, we obtain $\theta^+(t) = \tilde{\theta}^+$ (constant) and $\theta^-(t) = \tilde{\theta}^-$ (constant) for any $t \geq 0$. In this case, System $\tilde{\Theta}_{SFM-SRM}$ becomes the business cycle model studied by Dohtani (2019). We call the Kaldor model an included Kaldor model. Dohtani (2019) showed that the included Kaldor model possesses not only periodic paths but also strange attractors. Therefore, it is expected that System $\tilde{\Theta}_{SFM-SRM}$ possesses SRM.

We set

$$\mu = 2, \quad \alpha = 0.6, \quad \beta = 0.25, \quad a = 0.35, \quad b = 0.1, \quad c = 1,$$
$$j = 0.1, \quad \delta = 0.025, \quad \xi = 1.8, \quad \text{and} \quad \tau^+ = \tau^- = 0.$$

In this case, since $\tau^+ = \tau^- = 0$, we have

$$\theta^+(t) = \theta^+(0) \equiv \hat{\theta}^+ \quad \text{and} \quad \theta_t^- = \theta_0^- \equiv \hat{\theta}^- \quad \text{for any} \quad t \geq 0.$$

Then, noting this point, we obtain the following numerical example of System Θ_{SFM}:

$$\tilde{\Theta}_{SFM}:\begin{cases} \dot{Y}_t = 2[0.6Y_t + 0.25Y_{et} + 0.35\{0.1Y_t + \text{Arctan}(Y_t)\} - 0.1K_t - Y_t], \\ \dot{K}_t = 0.35\{0.1Y_t + \text{Arctan}(Y_t)\} - 0.125K_t, \\ \dot{Y}_{et} = 1.8\{\hat{\theta}^+\dfrac{(Y_t - Y_{et}) + |Y_t - Y_{et}|}{2} + \hat{\theta}^-\dfrac{(Y_t - Y_{et}) - |Y_t - Y_{et}|}{2}\}. \end{cases}$$

In the following, we set the initial values as

$$(Y(0),\ K(0),\ Y_{et}(0),\ \hat{\theta}^{+},\ \hat{\theta}^{-}) = (0,\ 1,\ 0,\ 0.1,\ 0.2)$$

In Figure 5, we consider the dynamics of System $\tilde{\Theta}_{FM}$. The blue curve of the figure describes the time series of a typical periodic path in the case where $\hat{\theta}^{+} = 0.2$ and $\hat{\theta}^{-} = 0.2$. Moreover, the red curve of Figure 5 describes the time series of a typical periodic path in the case where $\hat{\theta}^{+} = 0.001$ and $\hat{\theta}^{-} = 0.6$. For the red time series, the degree of pessimism is very large and consequently, the economy falls into a chronic slump. In Dohtani (2019), $\hat{\theta}^{+}$ is assumed to be considerably small and $\hat{\theta}^{-}$ is assumed to be considerably large. In other words, Dohtani (2019) assume that degree of pessimism is considerably large from the beginning. This assumption implies that some external factors operate on the economy and generates such an initial deep pessimism. However, in the present paper, we will show that even if the initial pessimism is weak, $\theta^{\pm}(t)$ change gradually and the initial pessimism gradually becomes deep. As a result, the economy falls into chronic pessimism.

Figure 5 about here.

By incorporating SRM, we relax the assumption of deep pessimism at the beginning. Throughout this section, we set

$$\alpha = 0.6,\quad b = 0.1,\quad c = 1,\quad j = 0.1,\quad \xi = 1.8,\ \text{ and }\ \delta = 0.025.$$

Then, we obtain

$$\tilde{\tilde{\Theta}}_{SFM-SRM}: \begin{cases} \dot{Y} = \mu[0.6Y + \beta Y_e + a\{0.1Y + \mathrm{Arctan}(Y)\} - 0.1K - Y], \\ \dot{K} = a\{0.1Y + c\mathrm{Arctan}(Y)\} - 0.125K, \\ \dot{Y}_e = \xi[\max\{\theta^{+}, 0.01\}\dfrac{(Y - Y_e) + |Y - Y_e|}{2} + \max\{\theta^{-}, 0.01\}\dfrac{(Y - Y_e) - |Y - Y_e|}{2}], \\ \dot{\theta}^{+} = \tau^{+}[\mu\{0.6Y + \beta Y_e + a\{0.1Y + \mathrm{Arctan}(Y)\} - 0.1K - Y\} \\ \qquad - \max\{\theta^{+}, 0.01\}\dfrac{(Y - Y_e) + |Y - Y_e|}{2} - \max\{\theta^{-}, 0.01\}\dfrac{(Y - Y_e) - |Y - Y_e|}{2}], \\ \dot{\theta}^{-} = -\tau^{-}[\mu\{0.6Y + \beta Y_e + a\{0.1Y + \mathrm{Arctan}(Y)\} - 0.1K - Y\} \\ \qquad - \max\{\theta^{+}, 0.01\}\dfrac{(Y - Y_e) + |Y - Y_e|}{2} - \max\{\theta^{-}, 0.01\}\dfrac{(Y - Y_e) - |Y - Y_e|}{2}]. \end{cases}$$

We distinguish between two types of initial pessimism:

Definition 4: If an initial value of expected income is smaller (resp. larger) than the actual income, we say that an initial pessimism (resp. optimism) in expected income exists. If $\theta^{+}(0) < \theta^{-}(0)$ (resp. $\theta^{+}(0) > \theta^{-}(0)$), we say that an initial pessimism (resp. optimism) in adjustment coefficient exists.■

Since $\tau^{\pm} > 0$, the upper and under adjustment coefficient $\theta^{\pm}(t)$ changes over time. It should be noted here that $\theta^{+}(0) < \theta^{-}(0)$. From Definition 4, the parameter values imply that a slightly weak initial pessimism in adjustment coefficient of expected income $\theta^{\pm}(0)$ exists. In the following, we provide numerical examples in which System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ yields chronic slums.

Firstly see Figure 6. The red curve of Part (1) of Figure 6 describes the time series of income of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$. The red time series demonstrates that SRM operates on the initial state and the economy falls into chronic pessimism. The blue curve describes the red time series of Figure 5. In Part (1) of Figure 6, the red time series is located in almost the same place as the blue curves. This implies that income of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with $(\theta^{+}(0),\ \theta^{-}(0)) = (0.1,\ 0.2)$ (weak initial pessimism) falls into the same level of slump as that of System $\tilde{\Theta}_{SFM}$ with $(\theta^{+}(0),\ \theta^{-}(0)) = (0.001,\ 0.5)$ (deep initial pessimism). This implies that SRM operating in System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ is powerful. Moreover, see Part (2) of Figure 6. The blue curve in the figure describes the PO index of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$. The PO-index converges to −1. This also demonstrates that the economy falls into a chronic pessimism.

Secondly, see Figure 7. We consider a slightly smaller initial expected income (i.e., slightly larger initial pessimism) than that of Figure 6:

$$Y_e(0) = -0.66825689\ \ 5407\ .$$

From Part (1) of Figure 7, the PO-index converges to −1. The black curve of Part (2) of Figure 7 describes the time series of System $\tilde{\Theta}_{SFM}$ with a strong initial pessimism:

$$(\theta^{+}(0), \theta^{-}(0)) = (0.002, 0.3)\ .$$

From Part (2) of Figure 7, like the black time series of System $\tilde{\Theta}_{SFM}$, the time series of Y_t in System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (1) does not fluctuate and locates under the market equilibrium. Moreover, it should be noted that the red time series of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ is considerably similar to the black time series of System $\tilde{\Theta}_{SFM}$. Moreover, the red (resp. blue) curve of Part (3) of Figure 6 describes the time series of $\max\{\theta^{+}(t),0\}$ (resp. $\max\{\theta^{-}(t),0\}$). Part (3) of Figure 6 demonstrates that $\max\{\theta^{+}(t),0\}$ and $\max\{\theta^{-}(t),0\}$ fluctuate and both cases of Lemma 2 do not appear. On the other hand, Part (3) of Figure 7 demonstrates that Assumption 2 is satisfied and that $\max\{\theta^{+}(t),0\}$ converges to 0 and $\max\{\theta^{-}(t),0\}$ converges to 1. Parts (1) and (3) of Figure 7 demonstrate that the pessimism case (2.1) of Lemma 2 appears.

Thus, in System $\tilde{\tilde{\Theta}}_{SFM-SRM}$, SRM operates toward a pessimistic economic situation. That is, in the System $\tilde{\Theta}_{SFM}$ with constant $\theta^{\pm}$, System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ generates the change similar to that from the case without pessimism (i.e. with $(\theta^{+}(t),\theta^{-}(t))=(0.3,0.3)$ for any $t\geq 0$) to the case with deep pessimism (i.e. with $(\theta^{+}(t),\theta^{-}(t))=(0.002,0.3)$ for any $t\geq 0$). Thus, we demonstrate that System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ **endogenously generates the psychological change**. The endogenous psychological change implies **the operation of SRM**. Thus, we observe that whether SRM operates depends on initial expected income and a chronic slump occurs.

Figures 6 and 7 about here.

Moreover, see Figures 8 and 9. In the figures, except for the initial expected income Y_{e0}, we set the initial values as

$$(Y(0),\ K(0),\ \theta^{+}(0),\ \theta^{-}(0))=(-0.2,\ -0.3,\ 0.3,\ 0.3).$$

Firstly, see Figure 8. In the figure, we set the initial expected income as

$$Y_e(0)=0.69157190\ 6364.$$

These parameter values imply that an initial optimism in expected income exists but an initial optimism in adjustment coefficient does not exist. Part (1) of Figure 8 describes the time series of PO index of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (1). From Part (1), the PO-index fluctuates between +1 and −1. Moreover, the red curve of Part (2) of Figure 8 describes the time series of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (1). The black curve of Part (2) of Figure 8 describes the time series of System $\tilde{\Theta}_{SFM}$ with

$$(\theta^+, \theta^-) = (0.3, 0.3),$$

Secondly, see Figure 9. We consider a slightly smaller initial expected income (i.e., slightly larger initial optimism) than that of Figure 9:

$$Y_e(0) = 0.69157190\ \ 6365.$$

From Part (1) of Figure 9, the PO-index of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (1) converges to -1. The black curve of Part (2) of Figure 9 describes the time series of System $\tilde{\Theta}_{SFM}$ with an initial deep pessimism:

$$(\theta^+(0), \theta^-(0)) = (0.3, 0.004).$$

From Part (2) of Figure 9, like the black time series of System $\tilde{\Theta}_{SFM}$ with (1), the time series of Y in System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ does not fluctuate and locates above the market equilibrium. Moreover, it should be noted that the red time series of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ is considerably similar to the black time series of System $\tilde{\Theta}_{SFM}$. Moreover, the red (resp. blue) curve of Part (3) of Figure 6 describes the time series of $\max\{\theta^+(t), 0\}$ (resp. $\max\{\theta^+(t), 0\}$). Part (3) in Figure 8 demonstrates that $\max\{\theta^+(t), 0\}$ and $\max\{\theta^+(t), 0\}$ fluctuate and both cases of Lemma 2 do not appear. On the other hand, Part (3) in Figure 9 demonstrates that Assumption 2 is satisfied and that $\max\{\theta^+(t), 0\}$ converges to 1 and $\max\{\theta^+(t), 0\}$ converges to 0. Parts (1) and (3) of Figure 9 demonstrate that the optimistic case (2.1) of Lemma 2 appears.

Thus, in System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (1), SRM operates toward an optimistic economic situation. That is, in the System $\tilde{\Theta}_{SFM}$ with constant $\theta^{\pm}$, System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ generates the change similar to that from (i.e. with $(\theta^+(t), \theta^-(t)) = (0.3, 0.3)$ for any $t \geq 0$) to the case with deep pessimism (i.e. with $(\theta^+(t), \theta^-(t)) = (0.3, 0.004)$ for any $t \geq 0$). Thus, in the same way as in Figure 7, we conclude that SRM operates in System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ and whether SRM operates depends on initial expected income and a chronic slump occurs

Figures 8 and 9 about here.

Through these numerical simulations, we obtain the following:

Observation 1: There exists a threshold value of expected income such that before and after the threshold value, the dynamic behavior of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ largely changes. In the case where the dynamic behavior changes largely (resp. does not change much), SRM operates (resp. does not operate). Moreover, in the case where SRM operates, the PO-index converges to −1 (i.e., $\lim_{t\to\infty}\theta^{+}(t)=0$) or +1 (i.e., $\lim_{t\to\infty}\theta^{-}(t)=0$). If the PO-index converges to −1 (resp. +1), the economy falls into a strong pessimistic (resp. optimistic) situation and a chronic slump (resp. long-term boom). In the case where SRM does not operate, the PO-index fluctuates between −1 and +1 and consequently, the economy also persistently fluctuates. ■

In the following, we consider the case where an initial pessimism occurs. The similar argument holds true for the case where an initial optimism occurs. We demonstrate that the degree of the adjustment coefficient μ of aggregate supply has effects on the operation of SRM. Except for the value of μ, we set the same parameter values as before. Firstly, we set

(1) $$(\mu,\ \alpha,\ \beta)=(1.33498\ ,\ 0.63,\ 0.2).$$

See Figures 10 and 11. In the figures, we set the initial values as

$$(Y(0),\ K(0),\ Y_e(0),\ \theta^{+}(0),\ \theta^{-}(0))=(0.2,\ 0.1,\ -0.1,\ 0.5,\ 0.5).$$

The curves in Figures 10 and 11 describe the same time series as those of Figures 8 and 9. From Part (1) of Figure 10, the PO-index of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (2) persistently fluctuates. From Part (2) of Figure 10, System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ generates persistent fluctuations. Next, see Figure 11. In the figure, we set

(2) $$(\mu,\ \alpha,\ \beta)=(1.33497\ ,\ 0.63,\ 0.2).$$

From Part (1) of Figure 11, the PO-index of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (3) converges to −1. Moreover, from Part (3) of Figure 11, we see that the economy of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (3) falls into a chronic slump. Through the same argument as before, we see from Parts (1) and Lemma 2 that Assumption 2 is satisfied and that the pessimistic case (2.1) of Lemma 2 appears. Thus, we obtain the following:

Observation 2: There is a threshold value of adjustment coefficient such that if the adjustment coefficient is smaller (resp. larger) than the threshold value, then the fluctuation disappears (resp. appears) and SRM operates (resp. does not operate) and the PO-index converges to −1 (resp. also fluctuates). Then, the pessimistic case (2.1) of Lemma 2 appears (resp. neither case of Lemma 2 does not happen).■

Figures 10 and 11 about here.

Moreover, we demonstrate that the degree of the propensity to consume α has effects on the operation of SRM. Except for the value of α , we set the same parameter values as before. Firstly, we set

(3) $$(\mu,\ \alpha,\ \beta) = (2,\ 0.67148594\ 78,\ 0.2).$$

See Figures 12 and 13. In the figures, we set the initial values as

$$(Y(0),\ K(0),\ Y_e(0),\ \theta^{+}(0),\ \theta^{-}(0)) = (0.5,\ 0,\ -0.2,\ 0.2,\ 0.2).$$

The curves in Figures 12 and 13 describe the same time series as those of Figures 8 and 9. From Part (1) of Figure 12, the PO-index of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (4) persistently fluctuates. From Part (2) of Figure 12, System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ generates persistent fluctuations. Next, see Figure 13. In the figure, we set

(4) $$(\mu,\ \alpha,\ \beta) = (2,\ 0.67148594\ 79,\ 0.2).$$

From Part (1) of Figure 13, the PO-index of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (5) converges to −1. Moreover, from Part (2) of Figure 13, we see that the economy of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (5) falls into a chronic slump. Through the same argument as before, we see from Parts (1) and Lemma 2 that Assumption 2 is satisfied and that the pessimistic case (2.1) of Lemma 2 appears. Thus, we obtain the following:

Observation 3: There is a threshold value of the propensity to consume such that if the propensity is larger (resp. smaller) than the threshold value, then the fluctuation disappears (resp. appears) and SRM operates (resp. does not operate) and the PO-index converges to −1 (resp. also fluctuates). Then, the pessimistic case (2.1) of

Lemma 2 appears (resp. neither case of Lemma 2 does not happen).■

Figures 12 and 13 about here.

Although we omit the argument, the same result for propensity to consume (concerning income) holds true for the propensity to consume concerning expected income β:

Observation 4: There is a threshold value of the propensity to consume concerning expected income such that if the propensity is larger (resp. smaller) than the threshold value, then the fluctuation disappears (resp. appears) and SRM operates (resp. does not operate) and the PO-index converges to −1 (resp. also fluctuates). Then, the pessimistic case (2.1) of Lemma 2 appears (resp. neither case of Lemma 2 does not happen).■

We here consider the relation between SRM and the initial value of the adjustment coefficient of expected income, $\theta^{\pm}(0)$. We set the same parameter values as those of Figure before. Firstly, we set

(5) $$(\mu,\ \alpha,\ \beta) = (2,\ 0.6,\ 0.27887084\ 38).$$

See Figures 14 and 15. In the figures, we set the initial values as

$$(Y(0),\ K(0),\ Y_e(0),\ \theta^{+}(0),\ \theta^{-}(0)) = (0.15,\ 0.3,\ -0.1,\ 0.1,\ 0.3).$$

The red curves in Figures 14 and 15 describe the same red time series as those of Figures 8 and 9. From Part (1) of Figure 14, the PO-index of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (6) persistently fluctuates. From Part (2) of Figure 14, System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ generates persistent fluctuations. Next, see Figure 15. In the figure, we set

(6) $$(\mu,\ \alpha,\ \beta) = (2,\ 0.6,\ 0.27887084\ 39).$$

From Part (1) of Figure 15, the PO-index of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (6) converges to −1. Moreover, from Part (2) of Figure 13, we see that the economy of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with (6) falls into a chronic slump. Through the same argument as before, we see from Parts (1) and Lemma 2 that Assumption 2 is satisfied and that the pessimistic case (2.1) of Lemma 2 appears. Thus, we obtain the following:

Observation 5: There is a threshold value of the propensity to consume concerning expected income such that if the propensity is larger (resp. smaller) than the threshold value, then the fluctuation disappears (resp. appears) and SRM operates (resp. does not operate) and the PO-index converges to −1 (resp. also fluctuates). Then, the pessimistic case (2.1) of Lemma 2 appears (resp. neither case of Lemma 2 does not happen).■

Figures 14 and 15 about here.

As the final discussion in this section, we demonstrate that the depth of slump depends on the depth of initial pessimism. See Figure 16. In the figures, we set

$$(Y(0),\ \ K(0),\ \ \theta^{+}(0),\ \ \theta^{-}(0)) = (0.15,\ \ 0.3,\ \ 0.3,\ \ 0.3)\,.$$

The blue and the red curves in Figures 16 respectively describe the time series of $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with

$$Y_e(0) = 0.6 \ \text{ and } \ Y_e(0) = 0.7.$$

From Figure 16, we demonstrate that as the initial pessimism of expected income (abbreviated as IPEI) is deeper, the bottom of the occurring slump is deeper.

Next, we see Figure 17. In the figures, we set

$$(Y(0),\ \ K(0),\ \ Y_e(0)) = (0.3,\ \ 0.3,\ \ -0.4)\,.$$

The blue and the red curves in Figures 17 respectively describe the time series of $\tilde{\tilde{\Theta}}_{SFM-SRM}$ with

$$(\theta^{+}(0),\ \ \theta^{-}(0)) = (0.2, 0.3) \ \text{ and } \ (\theta^{+}(0),\ \ \theta^{-}(0)) = (0.03, 0.5)$$

From Figure 17, we demonstrate that as the initial pessimism of adjustment coefficient concerning expected income (abbreviated as IPACEI) is deeper, the bottom of the occurring slump is deeper. Thus, we obtain the following:

Observation 5: As IPEI and/or IPACEI is deeper, the bottom of the occurring slump is deeper.■

Figures 16 and 17 about here.

7. Reversal Phenomena of Expectation

In this section, we consider the possible occurrence of reversal phenomena from pessimism to optimism or the opposite. Without loss of generality, we consider the reversal from pessimism. In this section, we set

$$\alpha = 0.61, \quad \beta = 0.23 \quad \mu = 1.3, \quad a = 0.5, \quad b = 0.12, \quad c = 0.9 \quad j = 0.125,$$
$$\tau^{+} = \tau^{-} = 3.$$

In Figure 18, we set

$$(Y(0), \quad K(0), \quad Y_e(0)) = (0, \quad -0.3, \quad 0)$$

This shows that IPEY does not exist. On the other hand, in Figure 18, we consider the case where IPACEY is deep:

$$(\theta^{+}(0), \quad \theta^{-}(0)) = (0.0001, \quad 0.4) .$$

The red curve of Figure 18 describes the time series of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$. Figure 18 describes that the economy with the deep IPACEY converges to a long-run boom. This demonstrates the possible occurrence of the reversal from pessimism to optimism. In Figure 19, The red (resp. blue) curve describes the time series of $\max\{\theta^{+}(t),0\}$ (resp. $\max\{\theta^{-}(t),0\}$) of System $\tilde{\tilde{\Theta}}_{SFM-SRM}$. See Figure 19. The black dashed line points to 0.6378. Therefore, Figure 19 shows that

$$\lim_{t\to\infty} \max\{\theta^{+}(t),0\} \approx 0.6378 > \lim_{t\to\infty} \max\{\theta^{-}(t),0\} \approx 0.$$

Therefore, the adjustment coefficient concerning expected income, θ^{+}, changes from 0.0001 to 0.6378 and θ^{-} from 0.4 to 0. Thus, Figure 19 demonstrates the psychological reversal phenomena concerning the magnitude relation of adjustment coefficient concerning expected income. This phenomenon is yielded by SRM. To summarize these arguments, we obtain the following.

Observation 6: SRM does not always reinforce the initial psychological situation. Through SRM, System $\tilde{\tilde{\Theta}}_{SFM-SRM}$ can generate the reversal phenomenon that leads the economy from an initial psychological situation to the different psychological one (e.g. from an initial pessimistic situation to optimistic one).■

Figures 18 and 19 about here.

8. Conclusions and Final Remarks

The important features of our model are as follows. Dohtani (2016) constructed a macro model which possesses a self-fulfilling mechanism. Such a model describes a self-fulfilling economic situation in which a pessimistic expectation produces a chronic slump. Dohtani (2016) assumed that households possess a deep pessimism (resp. optimism) from the beginning. Moreover, by incorporating a self-reinforcing mechanism, we construct a model that describes an economic situation that a weak pessimism (resp. optimism) at the beginning is gradually reinforced over time. As a variable, the model possesses expected income that is adaptively adjusted. Moreover, the model includes a dynamic equation in which the adjustment coefficient of the adjustment equation also changes over time. We demonstrated that a self-reinforcing mechanism such a dynamic equation. Since the weak pessimism at the beginning is reinforced, the model possesses not only SRM but also SFM.. We defined the PO-index that is closely related to the operation of SRM. We show that if the PO-index converges to -1 (resp. 1), SRM concerning pessimism (resp. optimism) operates.

We analyzed the effects of parameters on the operation of self-reinforcing mechanism. Firstly, we showed that if the difference between real income and expected income is larger than a threshold, the PO-index converges to -1 or 1 and SRM operates. Secondly, we showed that the adjustment coefficient of aggregate supply is smaller than a threshold, the PO-index converges to -1 or 1 and SRM operates. Thirdly, we showed that either of propensity to consume concerning income and expected income is larger than a threshold, the PO-index converges to -1 or 1 and SRM operates.

We here make one important remark. Although in business cycles theories psychological expectations have been discussed, they have not been on the front stage. In many cases, such psychological expectations are groundless. Therefore, they are too fickle to be the subject to scientific research. However, in the case where SRM operates, the economy follows the negative or positive feedback process in which a psychological expectation is reinforced while being realized again and again. Therefore, if a psychological expectation is reinforced through SRM, then it becomes reliable and is rationalized through such a feedback process. For this feedback process, see Krugman (2008, Chapter 8). In the chapter, Krugman says that

> *The basic answer is that while many economists were aware of the elements of this story –everyone understood that the feedback from confidence, to financial markets, to the real economy, and back again to confidence existed in principle - nobody realized just how powerful that feedback process would be in practice. And as a result nobody realized how explosive the circular logic of crises could be.*

In this paper, from the Krugman viewpoint of demand-side macroeconomics, we tried to give a simple mathematical representation to the circular logic. Through the feedback process, the psychological expectation becomes a rational expectation. Thus, in the case where SRM operates, the psychological expectations can be the subject to scientific research. In this paper, we must emphasize this point.

The necessary conditions for the operation of SRM seem to be considerably complicated. It seems to be possible that our model yields a variety of dynamic phenomena. The examples that we treat will be only a small part of them. To reveal them is left for future research.

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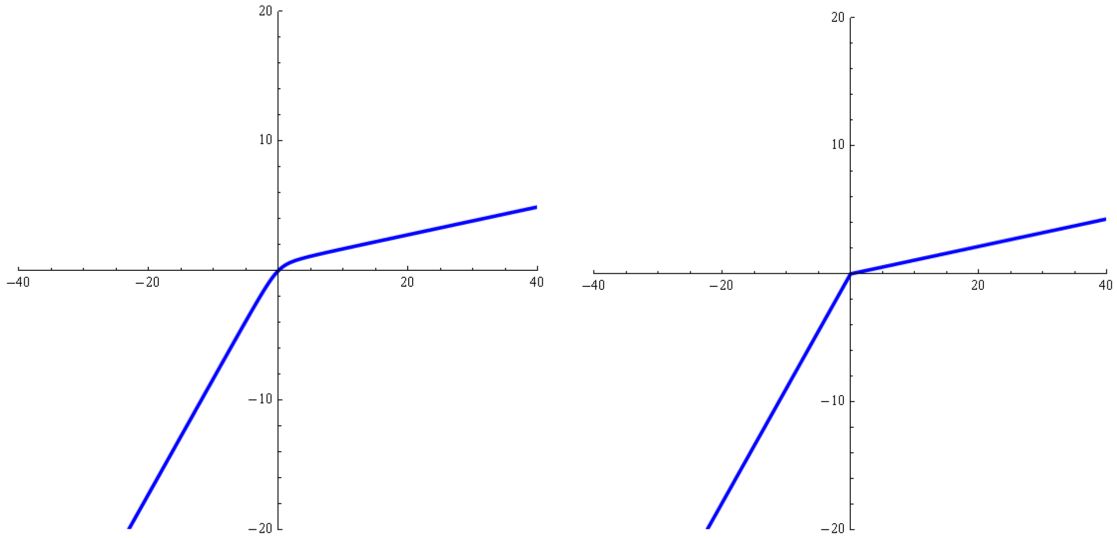
Figures



(1) **(2)**

Figure 1

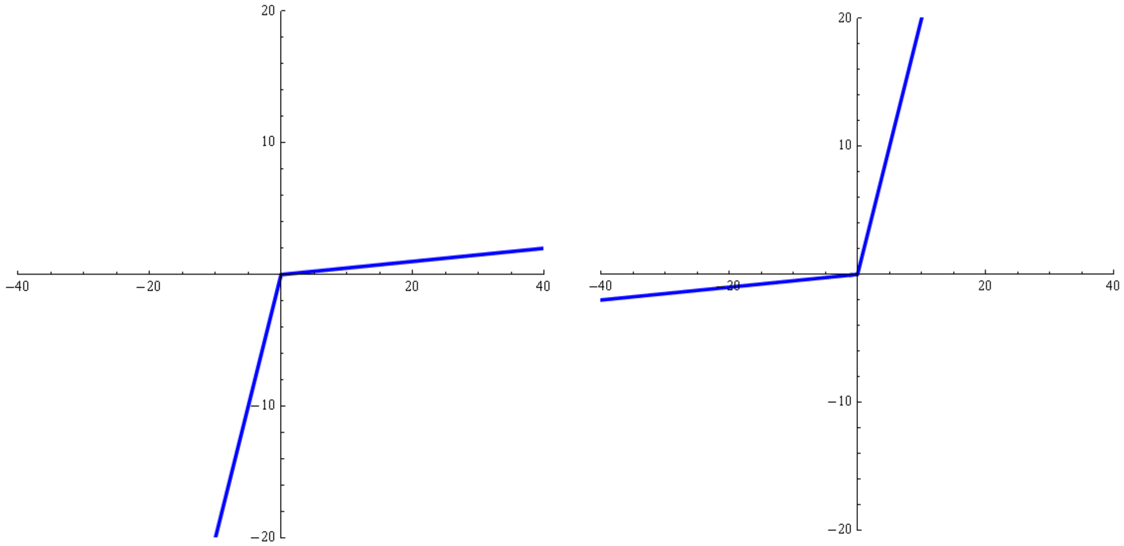


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Figure 2

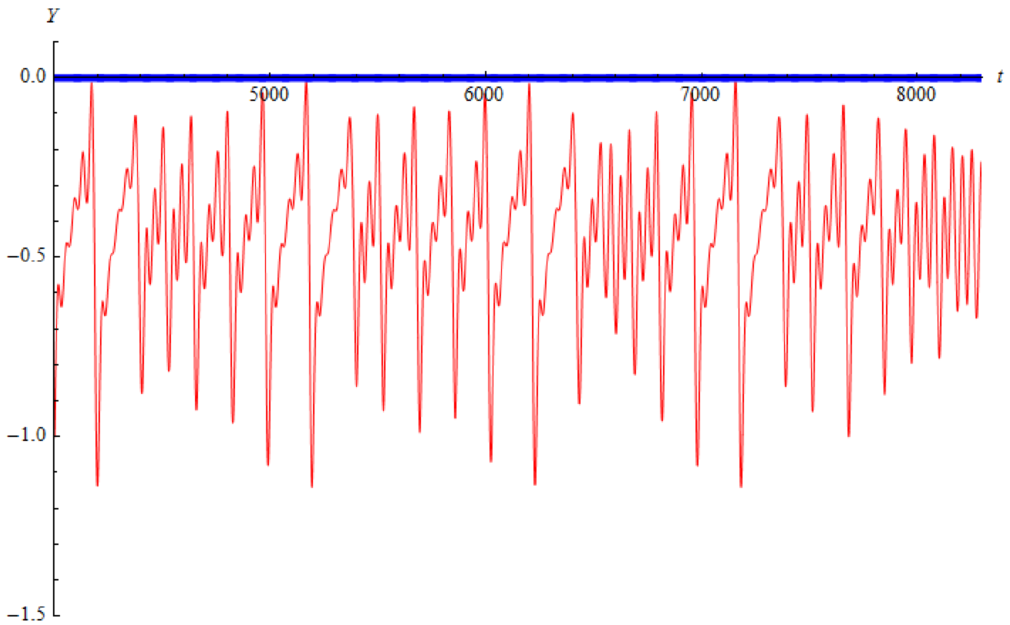


Figure 3

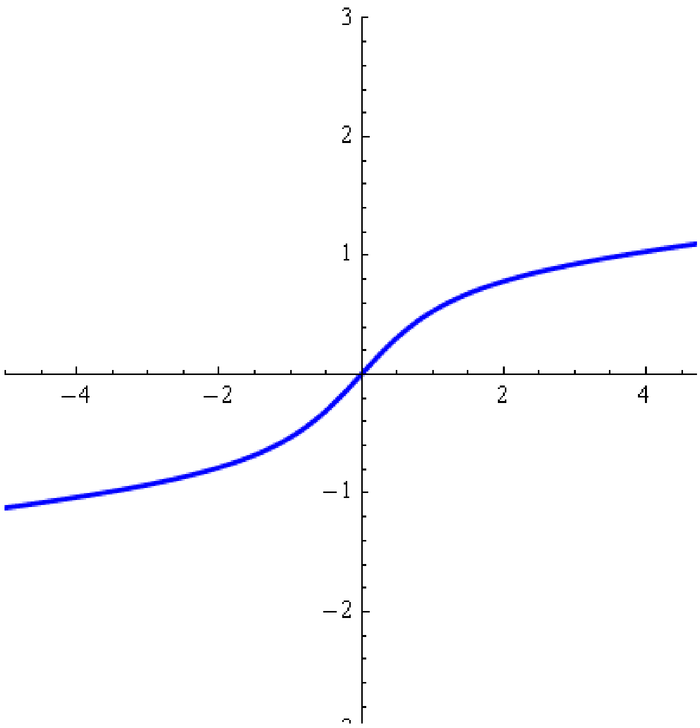


Figure 4

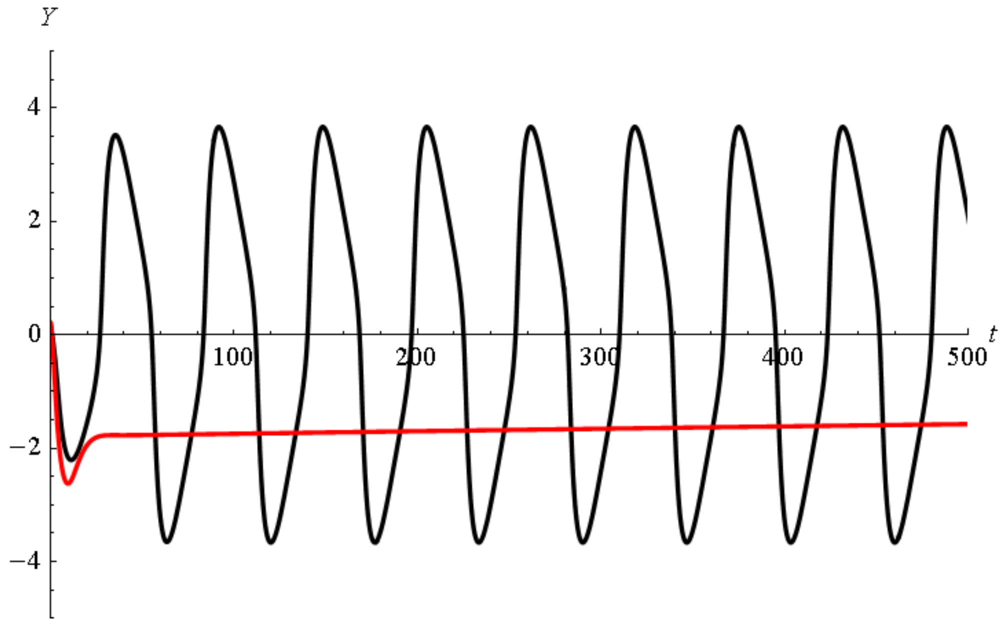
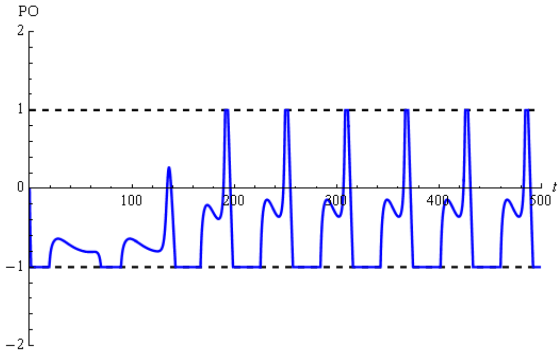
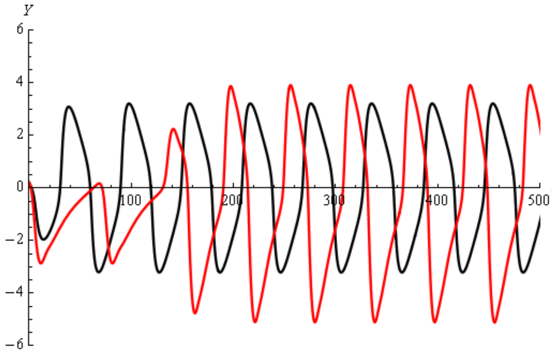


Figure 5





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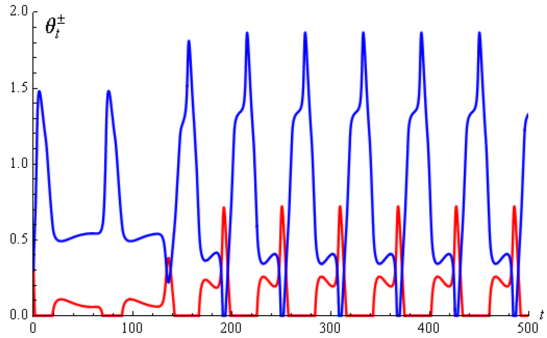
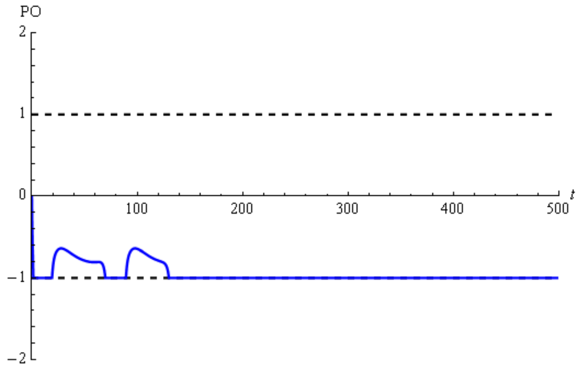
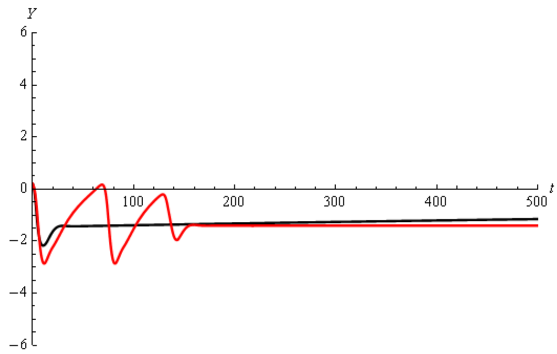


(3)

Figure 6





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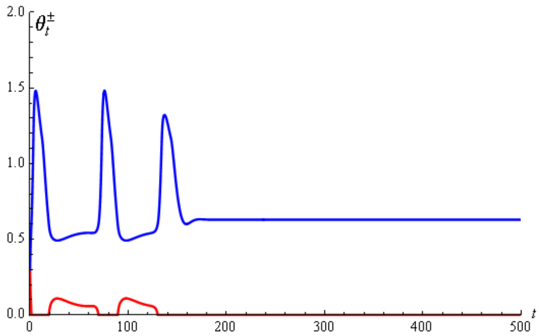
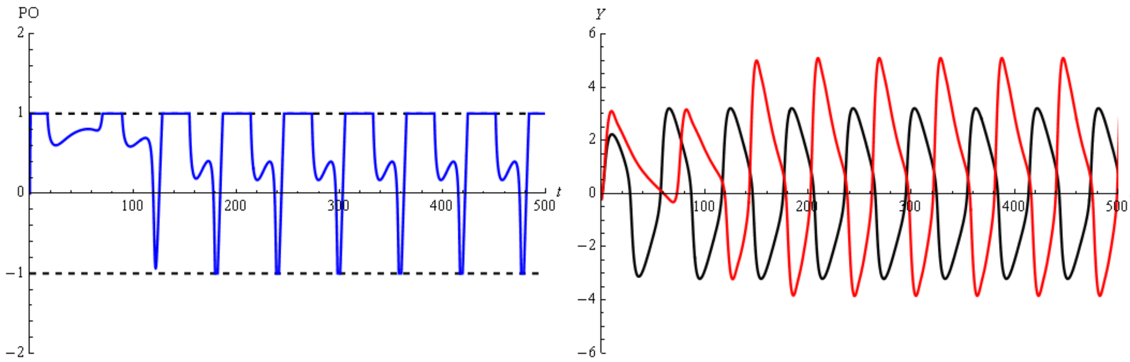


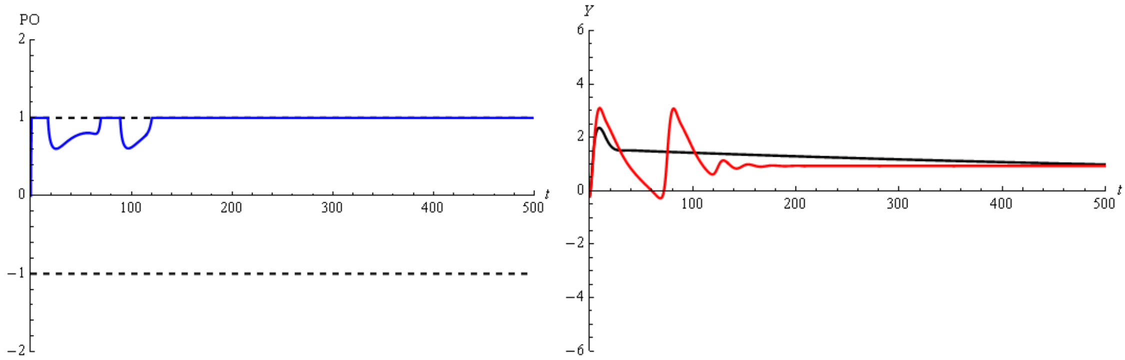
(3)

Figure 7



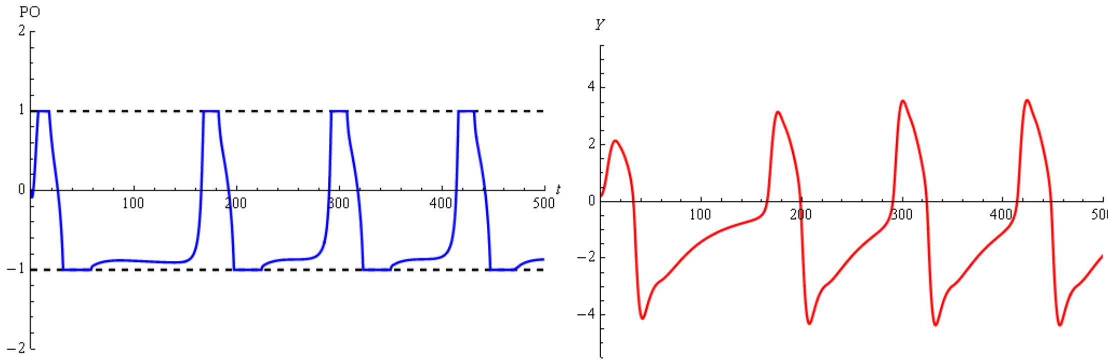
(1) **(2)**

Figure 8



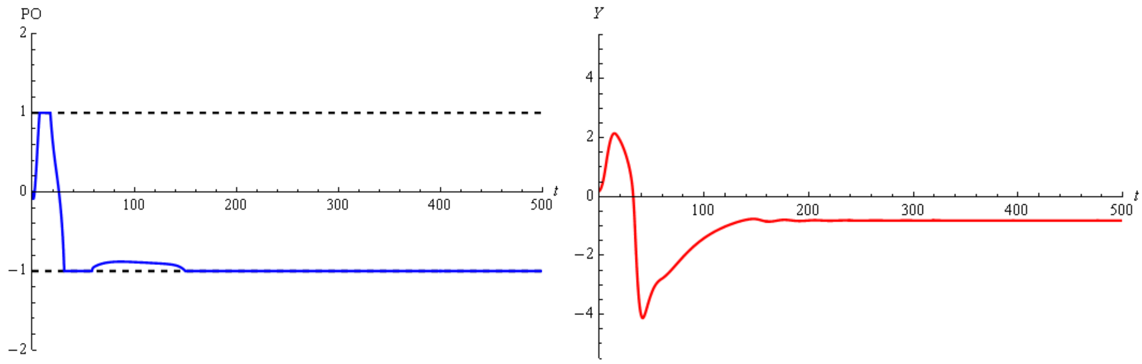
(1) **(2)**

Figure 9



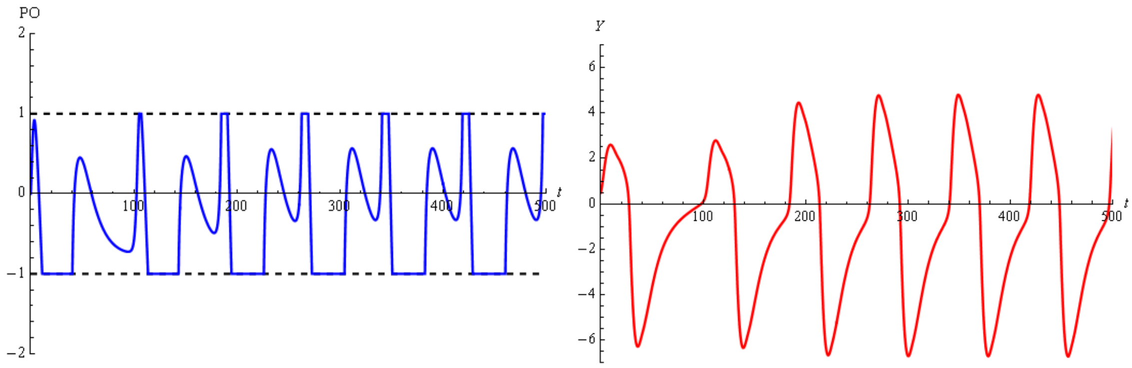
(1) **(2)**

Figure 10



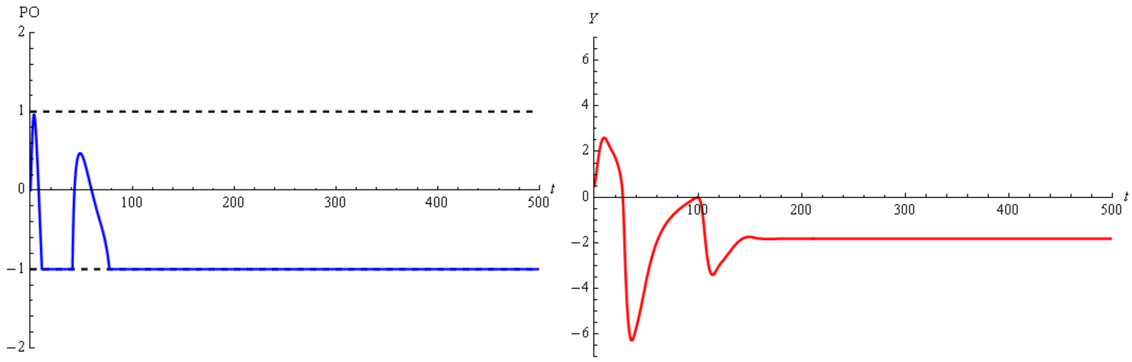
(1) **(2)**

Figure 11



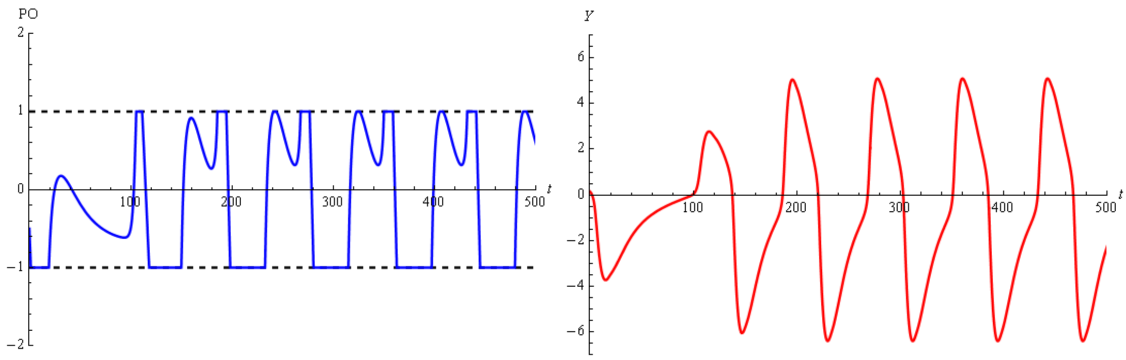
(1) **(2)**

Figure 12



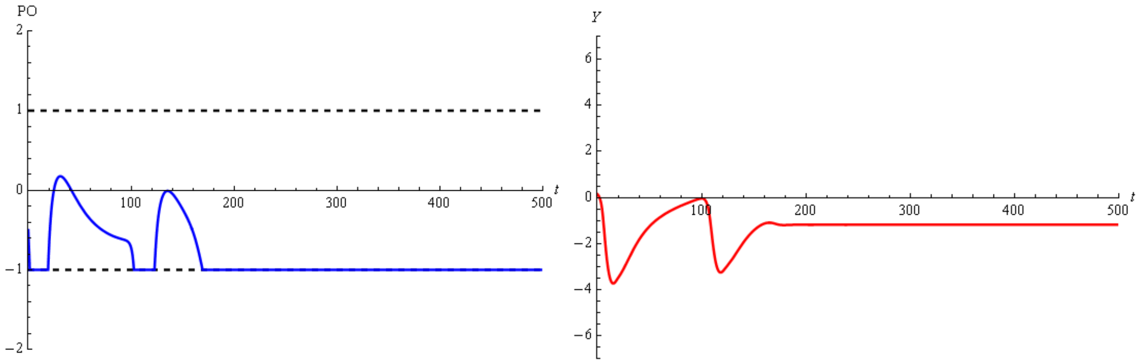
(1) **(2)**

Figure 13



(1) **(2)**

Figure 14



(1) **(2)**

Figure 15

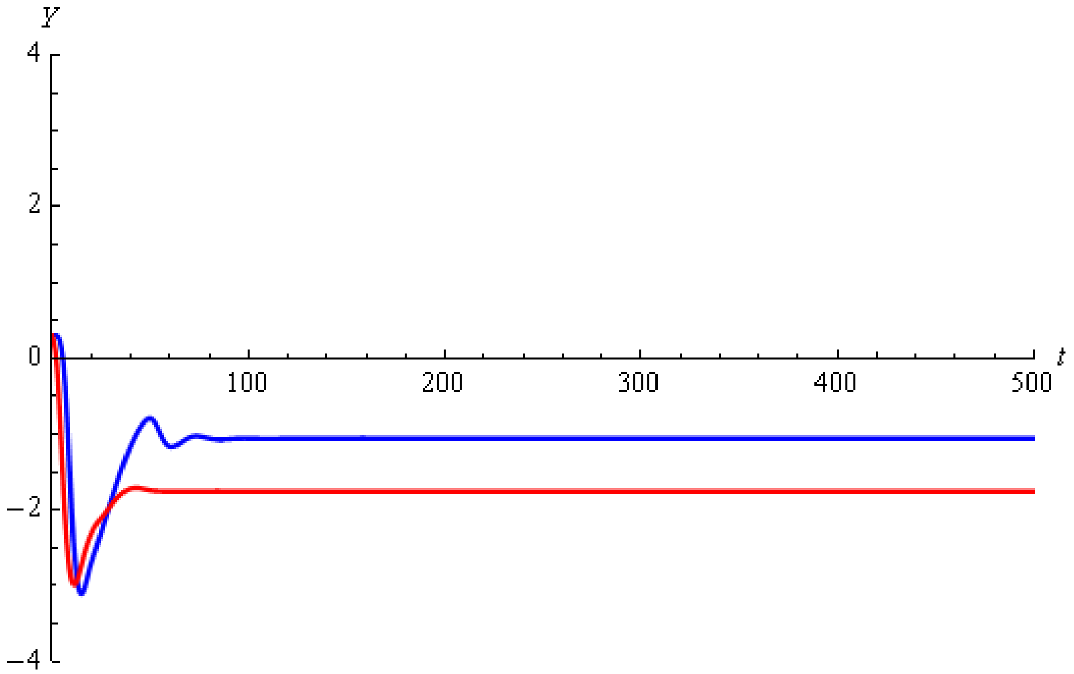


Figure 16

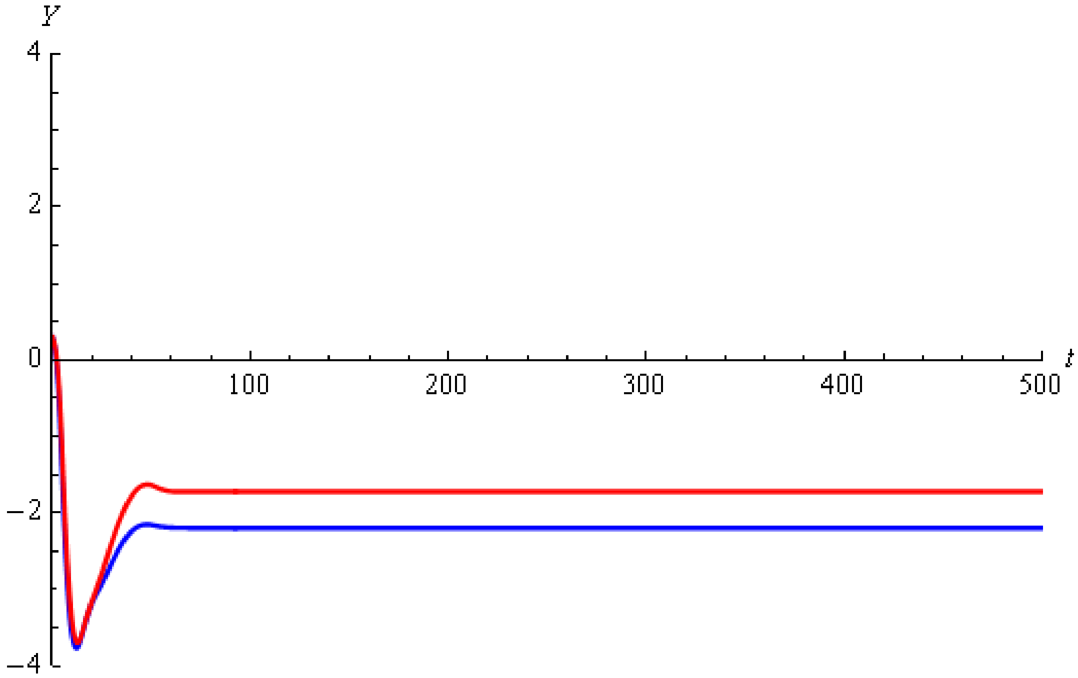


Figure 17

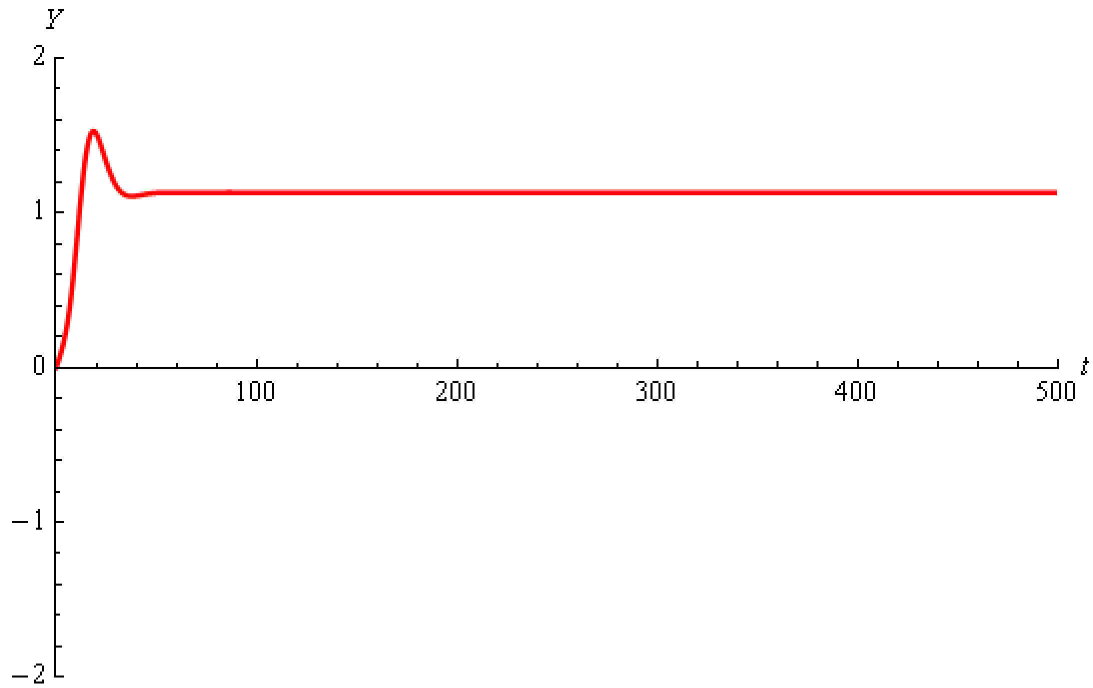


Figure 18

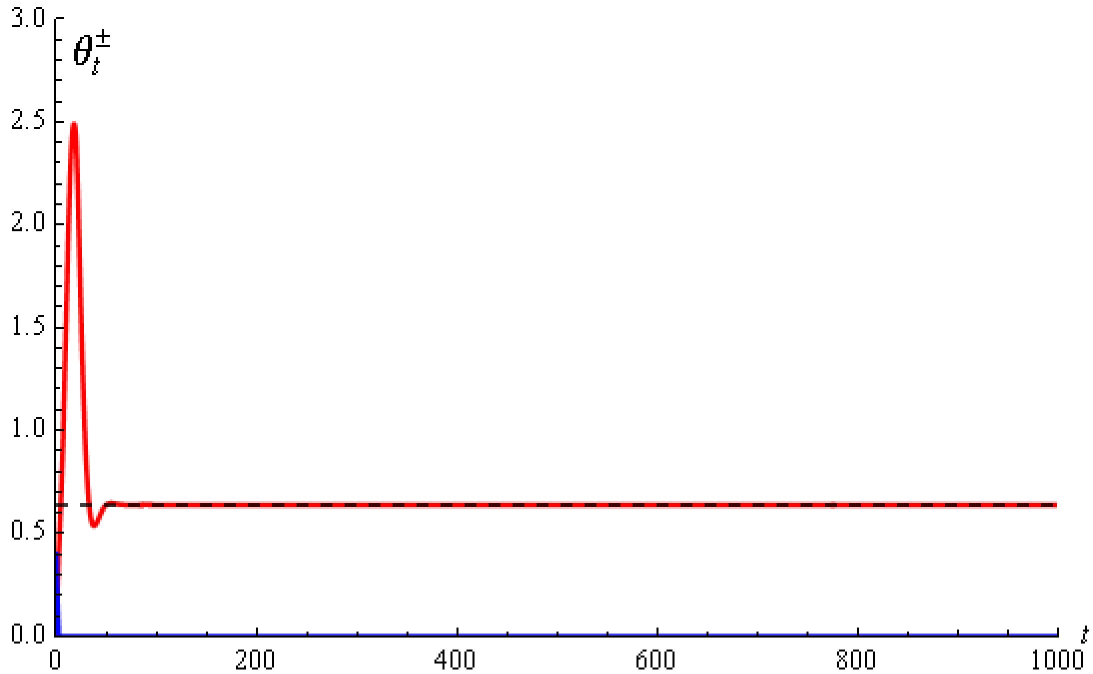


Figure 19