A Conjectural User-Revenue Model of Financial Firms under Dynamic Uncertainty:

A Theoretical Approach[1]

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Abstract

This paper extends the user-cost approach of Hancock (1985, 1991) in two ways. First, our model allows financial firms to behave strategically as well as competitively. Second, we do not assume that financial firms are risk-neutral.

Our main object is to derive the index of the degree of competition under dynamic uncertainty using this extended model. In our model, the classification of financial goods into inputs and outputs is always consistent with the classification based on the sign of each of the partial derivatives of the variable cost function with respect to financial goods.

1 Introduction

The user-cost approach has been used to determine whether a financial product is an input or an output on the basis of its net contribution to the revenues of the financial institution (Hancock, 1985, 1991).[1] This approach has found wide-ranging application and has been applied mainly to measuring efficiency, productivity, economies of scale, and economies of scope in the financial industry without assuming *a priori* that loans are outputs and deposits are inputs (Hancock, 1991). Homma et al. (1996) were the first to apply the user-cost method to the Japanese banking industry, and they estimated a stochastic profit frontier function for panel data during the High Growth Era. Ōmori and Nakajima (2000) estimated total factor productivity and economies of scope in the Japanese banking industry using data from 1987 to 1995. Other papers applied this approach to measure the value of financial services in the national income accounts (Fixler and Zieschang, 1991, 1992). Nagano (2001) and Utsunomiya (2002) measured the nominal value of financial services in Japan using this approach. No one has applied this approach to measure the degree of competition in financial services.

Although the user-cost approach has the advantage of an unambiguous classification of financial goods into inputs and outputs based on the sign of each of the user-cost prices, this classification is not always consistent with the classification based on the sign of each of the partial derivatives of the variable cost function with respect to financial goods. In addition, for Hancock's user-cost prices to provide meaningful estimates of prices of financial goods, strong assumptions must be maintained, including a risk-neutral attitude of the financial firm, competitive markets for financial goods, and informational symmetry between buyers and sellers. As a consequence, if any of these assumptions is violated, the price measures based on Hancock's user-cost prices will yield biased estimates of prices of financial goods, and the classification of financial goods into inputs and outputs will be incorrect.

In the light of these problems, in this paper, we derive generalizations

[1] Three principal approaches have been used to measure outputs in the financial services sector: the asset or intermediation approach, the user-cost approach, and the value-added approach (see Berger and Humphrey, 1992).

of Hancock's user-cost prices. The first generalization is the stochastic user-revenue price that does not assume the financial firm's attitude toward risk to be neutral. This price allows us to consider the case that the attitude toward risk is averse or loving. The second generalization is the conjectural user-revenue price that does not assume that the financial goods markets are competitive nor that there is informational symmetry between buyers and sellers in addition to the assumption of a risk-neutral attitude of the financial firm. Using this price, we can take into account the case that financial firms are strategically interdependent and there are informational asymmetries between buyers and sellers that lead to adverse selection. The relation between the stochastic and conjectural user-revenue prices is used to generalize the Lerner index of monopoly power to the financial firms' oligopoly under dynamic uncertainty.

Barnett and Zhou (1994) and Barnett et al. (1995) were the first to analyze the user-cost approach under dynamic uncertainty. This is likely to lead to generalizations similar to analysis of the stochastic user-revenue price. Unfortunately, their purpose is the pursuit of more desirable monetary aggregation, and thus they not only do not derive a generalized user-cost price such as the stochastic user-revenue price but also do not consider the case where financial firms are strategically interdependent and there are informational asymmetries between buyers and sellers. Furthermore, the formulation of the dynamic-uncertainty model in their papers is less rigorous in terms of the stochastic properties of the exogenous state variables than in this paper. The contribution we make in this paper is, therefore, significant.

The ultimate purpose of this paper is to derive an index of the degree of competition in the financial industry by using the generalized user-cost approach. There are some papers using other approaches that estimate first-order conditions for profit-maximizing oligopolies to measure the degree of competition and collusion in Japanese financial industries.[2] Souma and Tsutsui (2000) examined a change in the level of competition in the Japanese life insurance industry for the period 1986–1997 using the asset approach and

[2] The asset or intermediation approach treats financial service firms as pure financial intermediaries that borrow funds and transform the resulting liabilities into assets.

found that the industry was not very competitive but became more competitive from 1995 when the New Insurance Industry Law came into force. Uchida and Tsutsui (2005) applied an asset approach, similar to that of Souma and Tsutsui (2000), to the Japanese banking industry and estimated the degree of competition from 1974 to 2000. They found that the market had become more competitive in the 1970s, and judged that the Japanese banking sector faced perfect competition by the middle of the 1990s. By using the H-statistic, Kamesaka and Tsutsui (2002) found that the Japanese securities industry was in monopoly equilibrium in the 1980s and in monopolistic competition equilibrium in the 1990s.

In sections 2.1–2.5, we generalize the Hancock (1985, 1991) user-cost approach. In section 2.6, using this generalized model, we derive the stochastic and conjectural user-revenue price and our index of the degree of competition in the financial industry. In section 3, we sketch the empirical research procedure based on this model. Section 4 concludes.

2 Theoretical Specification

Throughout this section, we assume that time is divided into discrete periods. The periods are sufficiently short that variations in exogenous (state) variables within the period can be neglected. The exogenous variables remain constant within each period, but can change discretely at the boundaries of periods. The process of adjustment is essentially instantaneous so that we can ignore stock adjustment problems.

2.1 Net Cash Flow

All financial transactions are assumed to take place at the boundaries between intervals. Each financial firm holds an inventory consisting of stocks of financial assets and liabilities during each time period. We note that the net cash flow of a financial firm includes the cost or revenue of holding this inventory no less importantly than the cost of real resource inputs.

Let $p_{G,t}$ be the general price index in period t, which is used to deflate nominal units to real. Let $q_{i,j,t}$ be the real balance of the jth financial good of the ith firm and $h_{i,j,t}$ be the holding cost, or revenue per yen, where $j = 1, \cdots, N_A$ for assets, and $j = N_A + 1, \cdots, N_A + N_L$ for liabilities. Holding costs or revenues are contracted at the beginning of each period, but paid or received at the end of the period.

The net cash flow produced by financial good j during period t is

$$q_{i,j,t}^{NCF} = b_j \cdot (h_{i,j,t-1} \cdot p_{G,t-1} \cdot q_{i,j,t-1} + p_{G,t-1} \cdot q_{i,j,t-1} - p_{G,t} \cdot q_{i,j,t}), \qquad (1)$$

where $b_j = 1$ if the financial good is an asset, $j = 1, \cdots, N_A$, and $b_j = -1$ if the financial good is a liability, $j = N_A + 1, \cdots, N_A + N_L$. In the case of an asset such as a loan, with the exception of cash, the first term in equation (1) is holding revenues and the last two terms represent the change in the nominal asset. This change is positive (or negative) if principal payments from borrowers are larger (or smaller) than new loans. These terms represent the net cash flow from employing an asset. What has to be noticed is that the holding revenue of cash, $h_{i,1,t}$, is zero because the holding of cash which is the first asset, $j = 1$, does not yield revenue. On a liability such as a deposit, the first term is holding cost and the last two terms represent the nominal liability change, which is positive if new deposits are larger than withdrawals. These terms represent the net cash flow from issuing a liability.

2.2 Endogenous Holding Revenue or Cost

Holding revenue is the net revenue generated from holding an asset per time period. Net revenue is defined to equal default losses subtracted from the sum of interest revenues, service charges, and capital gains or losses. Interest revenue is the sum of collected and uncollected interest. The amount of uncollected interest is interpreted as one measure of asset quality. The service charge includes late loan payments and stand-by charges. The default loss includes assets marked down or written off, interest payments forgiven, and collection costs.

We assume that there is oligopolistic interdependence among financial firms and informational asymmetries between borrowers and lenders that lead to adverse selection, so that the components of holding revenue are determined endogenously. For example, the prime determinants of loan interest revenue are the total of loans in the market, institutional interest rate, credit risk, and the financial condition of borrowers.

Let $r_{i,j,t}$ denote the collected interest rate of the jth asset of the ith financial firm in period t, $r^{Q}_{i,j,t}$ the uncollected interest rate, $h^{S}_{i,j,t}$ the service charge rate, $h^{C}_{i,j,t}$ capital gains or losses, $h^{D}_{i,j,t}$ the default rate, $Q_{j,t}$ the total assets in the market, $\mathbf{z}^{k}_{i,j,t}$ $(k = R, Q, S, D)$ the vectors of exogenous (state) variables affecting each component of the holding revenue, and $\mathbf{z}^{H}_{i,j,t} = \left(\mathbf{z}^{R\prime}_{i,j,t}, \mathbf{z}^{Q\prime}_{i,j,t}, \mathbf{z}^{S\prime}_{i,j,t}, h^{C}_{i,j,t}, \mathbf{z}^{D\prime}_{i,j,t}\right)^{\prime}$. The holding revenue per yen for the ith financial firm's jth asset in period t is represented by

$$\begin{aligned} h_{i,j,t} &= r_{i,j,t} + r^{Q}_{i,j,t} + h^{S}_{i,j,t} + h^{C}_{i,j,t} - h^{D}_{i,j,t} \\ &= r_{i,j}\left(Q_{j,t}, \mathbf{z}^{R}_{i,j,t}\right) + r^{Q}_{i,j}\left(Q_{j,t}, \mathbf{z}^{Q}_{i,j,t}\right) + h^{S}_{i,j}\left(Q_{j,t}, \mathbf{z}^{S}_{i,j,t}\right) + h^{C}_{i,j,t} \\ &\quad - h^{D}_{i,j}\left(Q_{j,t}, \mathbf{z}^{D}_{i,j,t}\right) \\ &= h_{i,j}\left(Q_{j,t}, \mathbf{z}^{H}_{i,j,t}\right), \text{for } j = 1, \cdots, N_{A}. \qquad (2) \end{aligned}$$

Note that capital gains or losses $h^{C}_{i,j,t}$ are assumed to be determined exogenously.

Holding cost is the net cost of holding a liability per time period. The net cost is defined as the sum of interest payments, insurance premiums, and an implicit reserve tax, less service charges. The interest payment is the sum of paid and unpaid interest. The amount of unpaid interest is regarded as a measure of liability quality. Consider a deposit. An average deposit yields service charges per time period from depositors. The financial firm, on the other hand, pays interest on deposits, deposit insurance premiums, and the implicit tax imposed by the reserve requirement. The reserve requirement is a tax because it requires financial firms to hold deposits that do not bear interest with their central bank. The tax is the forgone interest on uninvested required reserves. With respect to bond obligations or borrowed funds, insurance premiums and the implicit reserve tax are zero because these liabilities are not subject to insurance premiums or reserve requirements.

As for holding cost, the components of holding cost are assumed to be determined endogenously because of oligopolistic interdependence among financial firms and informational asymmetries between depositors and financial firms. For instance, the deposit interest payment depends on total deposits in the market, the default risk of the financial firm, the current wealth of depositors, and their expectation of future wealth.

Let $r_{i,j,t}$ be the paid interest rate of the jth liability of the ith financial firm in period t, $r^Q_{i,j,t}$ the unpaid interest rate, $h^I_{i,j,t}$ the insurance premium rate, $h^S_{i,j,t}$ the service charge rate, $r^D_{i,t}$ the subjective rate of time preference, $\kappa_{i,j,t}$ the required reserve ratio, $Q_{j,t}$ the total liabilities in the market, $\mathbf{z}^k_{i,j,t}$ $(k = R, Q, I, S)$ the vectors of exogenous (state) variables affecting each component of holding cost, and $\mathbf{z}^H_{i,j,t} = \left(\mathbf{z}^{R\prime}_{i,j,t}, \mathbf{z}^{Q\prime}_{i,j,t}, \mathbf{z}^{I\prime}_{i,j,t}, \mathbf{z}^{S\prime}_{i,j,t}, r^D_{i,t}, \kappa_{i,j,t}\right)'$. The holding cost per yen for the ith financial firm's jth liability in period t is given by

$$\begin{aligned} h_{i,j,t} &= r_{i,j,t} + r^Q_{i,j,t} + h^I_{i,j,t} + r^D_{i,t} \cdot \kappa_{i,j,t} - h^S_{i,j,t} \\ &= r_{i,j}\left(Q_{j,t}, \mathbf{z}^R_{i,j,t}\right) + r^Q_{i,j}\left(Q_{j,t}, \mathbf{z}^Q_{i,j,t}\right) + h^I_{i,j}\left(Q_{j,t}, \mathbf{z}^I_{i,j,t}\right) + r^D_{i,t} \cdot \kappa_{i,j,t} \\ &\quad - h^S_{i,j}\left(Q_{j,t}, \mathbf{z}^S_{i,j,t}\right) \\ &= h_{i,j}\left(Q_{j,t}, \mathbf{z}^H_{i,j,t}\right), \text{for } j = N_A + 1, \cdots, N_A + N_L. \end{aligned} \tag{3}$$

Note that $r^D_{i,t} \cdot \kappa_{i,j,t}$ means the implicit tax rate imposed by the reserve requirement and is assumed to be determined exogenously.

2.3 Production Technology

Let $\mathbf{q}_{i,t} = (q_{i,1,t}, \cdots, q_{i,N_A+N_L,t})'$ denote the vector of real balances of financial goods of the ith financial firm in period t, $\mathbf{x}_{i,t} = (x_{i,1,t}, \cdots, x_{i,M,t})'$ the vector of real resource inputs such as labor, physical capital, and materials, $\mathbf{z}^Q_{i,t} =$

$\left(\mathbf{z}^{Q\prime}_{i,1,t},\cdots,\mathbf{z}^{Q\prime}_{i,N_A+N_L,t}\right)'$ the vector of exogenous (state) variables affecting the quality of financial goods, and $\tau_{i,t}$ an index of (exogenous) technical change. We assume that the efficient production technology of the ith financial firm in period t can be represented by the following transformation function:

$$\phi_i\left(\mathbf{q}_{i,t},\mathbf{x}_{i,t},\mathbf{z}^{Q}_{i,t},\tau_{i,t}\right)=0. \tag{4}$$

The important point to note is that some elements of the real balance vector $\mathbf{q}_{i,t}$ can be outputs or inputs, but all of them cannot be inputs. The transformation function ϕ_i satisfies the appropriate regularity conditions. That is, ϕ_i is strictly convex in $(\mathbf{q}_{i,t},\mathbf{x}_{i,t})$ and $\partial\phi_i/\partial q_{i,j,t}>0$ if $q_{i,j,t}$ is an output, $\partial\phi_i/\partial q_{i,j,t}<0$ if $q_{i,j,t}$ is an input, and $\partial\phi_i/\partial x_{i,j,t}<0$, since $\mathbf{x}_{i,t}$ is an input vector.

Taking the intertemporal optimization problem of financial firms into consideration, some inputs are optimized, and others remain constant within a single period. The former are variable inputs and the latter are fixed (or quasi-fixed) ones. Let $\mathbf{x}^V_{i,t}=\left(x^V_{i,1,t},\cdots,x^V_{i,M_V,t}\right)'$ be the vector of the real resource variable inputs, such as labor and materials, of the ith financial firm in period t, $\mathbf{p}^V_{i,t}=\left(p^V_{i,1,t},\cdots,p^V_{i,M_V,t}\right)'$ the vector of variable input prices, and $\mathbf{x}^F_{i,t}=\left(x^F_{i,1,t},\cdots,x^F_{i,M_F,t}\right)'$ the vector of real resource fixed inputs such as physical capital and human capital.

As far as a single period is concerned, we assume that the financial firm takes variable input prices as given and minimizes variable cost with respect to variable inputs subject to equation (4). Consequently, the following variable cost function is obtained:

$$C^V_i\left(\mathbf{p}^V_{i,t},\mathbf{q}_{i,t},\mathbf{x}^F_{i,t},\mathbf{z}^Q_{i,t},\tau_{i,t}\right)=\min_{\boldsymbol{x}^V_{i,t}}\left\{\sum_{j=1}^{M_V}p^V_{i,j,t}\cdot x^V_{i,j,t}\middle|\,\phi_i\left(\mathbf{q}_{i,t},\mathbf{x}_{i,t},\mathbf{z}^Q_{i,t},\tau_{i,t}\right)=0\right\}. \tag{5}$$

It is important to note that some elements of the real balance vector $\mathbf{q}_{i,t}$ can be outputs or fixed inputs, but all of them cannot be fixed inputs.

Let $\mathbf{q}^O_{i,t}=\left(q^O_{i,1,t},\cdots,q^O_{i,N_O,t}\right)'$ denote the output vector of real balances of the ith financial firm in period t and let $\mathbf{q}^F_{i,t}=\left(q^F_{i,1,t},\cdots,q^F_{i,N_F,t}\right)'$ be the fixed-input vector. Both vectors include all elements of $\mathbf{q}_{i,t}$. Because of duality between transformation functions and variable cost functions, the variable cost function C_i^V is strictly increasing in $\mathbf{p}_{i,t}^V$ and $\mathbf{q}_{i,t}^O$, strictly decreasing in $\mathbf{x}_{i,t}^F$ and $\mathbf{q}_{i,t}^F$, and homogeneous of degree one and strictly concave in $\mathbf{p}_{i,t}^V$. Added to these conditions, we assume that C_i^V is twice continuously differentiable in all its arguments and strictly convex in $\mathbf{q}_{i,t}$ and $\mathbf{x}_{i,t}^F$ because the analysis below needs these assumptions.

Now, consider the jth real resource stock of the ith financial firm at time t, $x_{i,j,t}^F$. Let $I_{i,j,t}$ denote gross investment and let $\delta_{i,j,t}$ be the depreciation rate. We assume that gross investment becomes productive instantaneously and the adjustment cost associated with installing capital is zero. The depreciation rate $\delta_{i,j,t}$ is constant and assumed to be given. Capital accumulation is given by

$$x_{i,j,t}^F = I_{i,j,t} + (1 - \delta_{i,j,t}) \cdot x_{i,j,t-1}^F, \text{ for } j = 1, \cdots, M_F. \tag{6}$$

2.4 Profit and Utility

The financial firm receives a profit, namely, a return on a financial undertaking after all operating expenses have been met. The profit of the ith financial firm during period t is represented by

$$\begin{aligned}
\pi_{i,t} &= \sum\nolimits_{j=1}^{N_A+N_L} q_{i,j,t}^{NCF} - C_i^V\left(\mathbf{p}_{i,t}^V, \mathbf{q}_{i,t}, \mathbf{x}_{i,t}^F, \mathbf{z}_{i,t}^Q, \tau_{i,t}\right) - \sum\nolimits_{j=1}^{M_F} p_{i,j,t}^F \cdot I_{i,j,t}, \\
&= \sum\nolimits_{j=1}^{N_A+N_L} b_j \cdot \left[\left\{1 + h_{i,j}\left(Q_{j,t-1}, \mathbf{z}_{i,j,t-1}^H\right)\right\} \cdot p_{G,t-1} \cdot q_{i,j,t-1} - p_{G,t} \cdot q_{i,j,t}\right] \\
&\quad - C_i^V\left(\mathbf{p}_{i,t}^V, \mathbf{q}_{i,t}, \mathbf{x}_{i,t}^F, \mathbf{z}_{i,t}^Q, \tau_{i,t}\right) \\
&\quad - \sum\nolimits_{j=1}^{M_F} p_{i,j,t}^F \cdot \left[x_{i,j,t}^F - (1 - \delta_{i,j,t}) \cdot x_{i,j,t-1}^F\right],
\end{aligned} \tag{7}$$

where $p_{i,j,t}^F$ $(j = 1, \cdots, M_F)$ are the prices of real resource fixed inputs. The first term in this equation, which is the sum of net cash flows, represents the total net revenue of financial goods. The second term is the variable cost of real resource variable inputs and the last term represents total expenditure on investments. We note that profit, as defined here, is neither exactly variable nor short-run profit in a conventional static model. It is essential for the definition of variable or short-run profit to specify the prices of financial goods and real resource fixed inputs. We define these prices below, as generalized user-cost prices.

To deal explicitly with the attitude to risk of a financial firm, we use a utility function describing the financial firm's preferences over profits defined by equation (7). The utility function of the ith financial firm during period t is given by $u_i(\pi_{i,t})$. We assume that the utility function u_i is strictly increasing, twice continuously differentiable, and strictly concave in $\pi_{i,t}$.

2.5 Dynamic Uncertainty Behavior

Since financial goods and real resource fixed inputs, both measured by stocks, are assumed to be held during each period, but change discretely at the boundaries of periods, their adjustment to optimal levels requires at least two periods. In an intertemporal decision, some risk, which may be defined as reductions in firm value due to changes in the business environment, is hardly avoidable. In this section, we model a financial firm's decision as a stochastic dynamic programming problem.

There are two specifications of the problem between which the important difference is in the relative timing of each decision-making period and the realization of uncertainty. In the first case, the decision is made after the uncertainty is realized, so that in each period the decision maker chooses directly the next period's state variable. In the second case, the decision is made before the uncertainty is realized, so that the decision maker chooses the current period's control variable, and the next period's state variable is a function of that variable and the current state variable. Since the adjustment cost of stock variables is assumed to be zero and more reliable information on the decision leads to rises in firm value, we assume the first case, in which the financial firm's decision is made at the end of the period after the uncertainty is realized, so that the financial firm chooses the next period's state variable directly.

The state variables are classified into the endogenous state variables and the exogenous ones. Let $\mathbf{y}_{i,t} = \left(\mathbf{q}'_{i,t}, \mathbf{x}^{F\prime}_{i,t}\right)' = \left(q_{i,1,t}, \cdots, q_{i,N_A+N_L,t}, x^F_{i,1,t}, \cdots, x^F_{i,M_F,t}\right)'$ $(t \geq 0)$ be the vectors of the endogenous state variables. Let $\mathbf{z}_{i,t} = \left(\mathbf{z}^{H\prime}_{i,t-1}, p_{G,t}, \mathbf{p}^{V\prime}_{i,t}, \tau_{i,t}, \mathbf{p}^{F\prime}_{i,t}, \boldsymbol{\delta}'_{i,t}\right)'(t \geq 0)$ denote the vectors of the exogenous state variables, where $\mathbf{z}^H_{i,j,t} = \left(\mathbf{z}^{R\prime}_{i,j,t}, \mathbf{z}^{Q\prime}_{i,j,t}, \mathbf{z}^{S\prime}_{i,j,t}, h^C_{i,j,t}, \mathbf{z}^{D\prime}_{i,j,t}\right)'(t \geq 0)$ for $j = 1, \cdots, N_A$, $\mathbf{z}^H_{i,j,t} = \left(\mathbf{z}^{R\prime}_{i,j,t}, \mathbf{z}^{Q\prime}_{i,j,t}, \mathbf{z}^{I\prime}_{i,j,t}, \mathbf{z}^{S\prime}_{i,j,t}, r^D_{i,t}, \kappa_{i,j,t}\right)'(t \geq 0)$ for $j = N_A + 1, \cdots, N_A + N_L$, $\mathbf{z}^H_{i,t-1} = \left(\mathbf{z}^{H\prime}_{i,1,t-1}, \cdots, \mathbf{z}^{H\prime}_{i,N_A+N_L,t-1}\right)'(t \geq 1)$, $\mathbf{z}^H_{i,-1} = \mathbf{z}^H_{i,0}$, and $\mathbf{z}_{i,0} = \left(\mathbf{z}^{H\prime}_{i,0}, p_{G,0}, \mathbf{p}^{V\prime}_{i,0}, \tau_{i,0}, \mathbf{p}^{F\prime}_{i,0}, \boldsymbol{\delta}'_{i,0}\right)'$. Let $\mathbf{z}^C_{i,t} = \left(\mathbf{p}^{V\prime}_{i,t}, \mathbf{z}^{Q\prime}_{i,t}, \tau_{i,t}\right)'(t \geq 0)$ be the vectors of the exogenous state variables with respect to the variable cost function, where $\mathbf{z}^Q_{i,t} = \left(\mathbf{z}^{Q\prime}_{i,1,t}, \cdots, \mathbf{z}^{Q\prime}_{i,N_A+N_L,t}\right)'(t \geq 0)$. Let $\mathbf{z}^\pi_{i,t} = \left(\mathbf{z}^{H\prime}_{i,t-1}, p_{G,t-1}, p_{G,t}, \mathbf{z}^{C\prime}_{i,t}, \mathbf{p}^{F\prime}_{i,t}, \boldsymbol{\delta}'_{i,t}\right)'(t \geq 1)$ be their vectors with respect to profit during period t (≥ 1), and $\mathbf{z}^\pi_{i,0} = \left(\mathbf{z}^{H\prime}_{i,0}, p_{G,0}, \mathbf{p}^{V\prime}_{i,0}, \tau_{i,0}, \mathbf{p}^{F\prime}_{i,0}, \boldsymbol{\delta}'_{i,0}\right)'$ their vector with respect to profit in period 0.

We assume that the stochastic process $\{\mathbf{z}_{i,t}\}_{t\geq 0}$ follows a stationary Markov process. Let $(Z, \boldsymbol{B}_Z)$ be a measurable space, where Z is a set of $\mathbf{z}_{i,t}$ and $\boldsymbol{B}_Z$ is a σ-algebra of its subsets. The stochastic properties of the exogenous state variables are represented by the stationary transition function, $Q : Z \times \boldsymbol{B}_Z \to [0, 1]$.[3] The interpretation is that $Q(\mathbf{z}_{i,t}, A_{i,t+1})$ is the probability that the next period's state lies in the set $A_{i,t+1}$, given that the current state is $\mathbf{z}_{i,t}$. Let $\left(Z^t, \boldsymbol{B}^t_Z\right) = (Z \times \cdots \times Z, \boldsymbol{B}_Z \times \cdots \times \boldsymbol{B}_Z)$, ($t$ times) denote the product space and let $\mathbf{z}_{i,0} \in Z$ be given. We define probability measures $\mu^t(\mathbf{z}_{i,0}, \cdot) : \boldsymbol{B}^t_Z \to [0, 1]$, $t = 1, 2, \cdots$, on these spaces as follows.[4] For any rectangle $A^t_i = A_{i,1} \times \cdots \times A_{i,t} \in \boldsymbol{B}^t_Z$, let

$$\mu^t\left(\mathbf{z}_{i,0}, A^t_i\right) = \int_{A_{i,0}} \cdots \int_{A_{i,t-1}} \int_{A_{i,t}} Q(\mathbf{z}_{i,t-1}, \mathbf{dz}_{i,t})\, Q(\mathbf{z}_{i,t-2}, \mathbf{dz}_{i,t-1}) \cdots Q(\mathbf{z}_{i,0}, \mathbf{dz}_{i,1}) \tag{8}$$

The probability measure $\mu^t(\mathbf{z}_{i,0}, \cdot)$ satisfies the properties of measures and $\mu^t(\mathbf{z}_{i,0}, Z^t) = 1$.

The decision to be carried out in period t can depend upon the informa-

[3] For further details of the stationary transition function, see Stokey and Lucas (1989: p.212).

[4] See Stokey and Lucas (1989: pp.220-225) for a full account of the probability measures.

tion that will be available at that time. This information is the sequence of the vectors of the exogenous state variables. Let $\mathbf{z}_i^t = (\mathbf{z}_{i,1}, \cdots, \mathbf{z}_{i,t}) \in Z^t$ denote their partial history in period 1 through t. Let $(Y, \boldsymbol{B}_Y)$ be a measurable space, where Y is a set of the vectors of the endogenous state variables $\mathbf{y}_{i,t}$ and $\boldsymbol{B}_Y$ is a σ-algebra of its subsets. We define a plan $\mathbf{y}_i^p$ as the set of a value $\mathbf{y}_{i,0}^p \in Y$ and a sequence of functions $\mathbf{y}_{i,t}^p : Z^t \to Y$, $t = 1, 2, \cdots$, where $\mathbf{y}_{i,t}^p(\mathbf{z}_i^t) = \left(\mathbf{q}_{i,t}^p(\mathbf{z}_i^t)', \mathbf{x}_{F,i,t}^p(\mathbf{z}_i^t)'\right)'$ is the value for $\mathbf{y}_{i,t+1} = \left(\mathbf{q}_{i,t+1}', \mathbf{x}_{i,t+1}^{F\prime}\right)'$ that will be chosen in period t if the partial history of the exogenous state variables in period 1 through t is $\mathbf{z}_i^t$.

The financial firm is assumed to choose this plan to maximize the expected value of the discounted inter-temporal utility of its profits stream. We also assume that the inter-temporal utility function is additively separable. The ith financial firm's optimization problem is then given by

$$\max_{\mathbf{y}_i^p} u_i\left[\pi_i\left(\mathbf{y}_{i,0}, \mathbf{y}_{i,0}^p(\mathbf{z}_{i,0}), \mathbf{z}_{i,0}^{\pi}\right)\right]$$

$$+ \lim_{T\to\infty} \sum_{t=1}^{T} \int_{Z^t} \beta_i^t \cdot u_i\left[\pi_i\left(\mathbf{y}_{i,t-1}^p\left(\mathbf{z}_i^{t-1}\right), \mathbf{y}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^{\pi}\right)\right] \mu^t\left(\mathbf{z}_{i,0}, \mathbf{dz}_i^t\right), \quad (9)$$

where $\beta_i^t = \prod_{s=0}^{t-1} \beta_{i,s} = \prod_{s=0}^{t-1} \dfrac{1}{1+r_{i,s}^D}$ is the cumulative discount factor and $r_{i,s}^D$ is the subjective rate of time preference.[5] Planned profits are represented by

$$\pi_i\left(\mathbf{y}_{i,t-1}^p\left(\mathbf{z}_i^{t-1}\right), \mathbf{y}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^{\pi}\right)$$

$$= \sum_{j=1}^{N_A+N_L} b_j \cdot \left[\left\{1 + h_{i,j}\left(Q_{j,t-1}^p, \mathbf{z}_{i,j,t-1}^H\right)\right\} \cdot p_{G,t-1} \cdot q_{i,j,t-1}^p\left(\mathbf{z}_i^{t-1}\right) - p_{G,t} \cdot q_{i,j,t}^p\left(\mathbf{z}_i^t\right)\right]$$

$$-C_i^V\left(\mathbf{y}_{i,t}^p\left(\mathbf{z}_i^t\right), \mathbf{z}_{i,t}^C\right) - \sum_{j=1}^{M_F} p_{i,j,t}^F \cdot \left[x_{F,i,j,t}^p\left(\mathbf{z}_i^t\right) - (1-\delta_{i,j,t}) \cdot x_{F,i,j,t-1}^p\left(\mathbf{z}_i^{t-1}\right)\right] (t \geq 1), \quad (10)$$

[5] On this optimization problem, see Stokey and Lucas (1989: pp.241-254).

$$\pi_i\left(\mathbf{y}_{i,0},\mathbf{y}^p_{i,0}\left(\mathbf{z}_{i,0}\right),\mathbf{z}^\pi_{i,0}\right)$$

$$=\sum\nolimits_{j=1}^{N_A+N_L} b_j\cdot\left[\left\{1+h_{i,j}\left(Q_{j,0},\mathbf{z}^H_{i,j,0}\right)\right\}\cdot p_{G,0}\cdot q_{i,j,0}-p_{G,0}\cdot q^p_{i,j,0}\left(\mathbf{z}_{i,0}\right)\right]$$

$$-C^V_i\left(\mathbf{y}^p_{i,0}\left(\mathbf{z}_{i,0}\right),\mathbf{z}^C_{i,0}\right)-\sum\nolimits_{j=1}^{M_F} p^F_{i,j,0}\cdot\left[x^p_{F,i,j,0}\left(\mathbf{z}_{i,0}\right)-\left(1-\delta_{i,j,0}\right)\cdot x^F_{i,j,0}\right]. \tag{11}$$

For stochastic optimization problems in sequence form, necessary conditions for an optimum can be derived by a variational approach. These conditions are called stochastic Euler equations. The stochastic Euler equations for the above optimization problem (9) are given by

$$-\frac{\partial u_{i,t}}{\partial \pi_{i,t}}\cdot\left(p_{G,t}+\frac{\partial C^V_{i,t}}{\partial q^{p*}_{i,j,t}}\right)$$

$$+\beta_{i,t}\cdot p_{G,t}\cdot\left\{1+b_C\cdot\left(h_{i,j,t}+\frac{\partial h_{i,j,t}}{\partial \ln q^{p*}_{i,j,t}}\right)\right\}\cdot\int_Z\frac{\partial u_{i,t+1}}{\partial \pi_{i,t+1}}\,Q\left(\mathbf{z}_{i,t},\mathbf{dz}_{i,t+1}\right)=0,$$

$$j=1,\cdots,N_A, \tag{12}$$

$$-\frac{\partial u_{i,t}}{\partial \pi_{i,t}}\cdot\left(p^F_{i,j,t}+\frac{\partial C^V_{i,t}}{\partial x^{p*}_{F,i,j,t}}\right)$$

$$+\beta_{i,t}\cdot\int_Z p^F_{i,j,t+1}\cdot\left(1-\delta_{i,j,t+1}\right)\cdot\frac{\partial u_{i,t+1}}{\partial \pi_{i,t+1}}\,Q\left(\mathbf{z}_{i,t},\mathbf{dz}_{i,t+1}\right)=0,$$

$$j=1,\cdots,M_F, \tag{13}$$

$$\frac{\partial u_{i,t}}{\partial \pi_{i,t}} \cdot \left(p_{G,t} - \frac{\partial C_{i,t}^{V}}{\partial q_{i,j,t}^{p*}} \right)$$

$$- \beta_{i,t} \cdot p_{G,t} \cdot \left(1 + h_{i,j,t} + \frac{\partial h_{i,j,t}}{\partial \ln q_{i,j,t}^{p*}} \right) \cdot \int_{Z} \frac{\partial u_{i,t+1}}{\partial \pi_{i,t+1}} Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}\right) = 0,$$

$$j = N_A + 1, \cdots, N_A + N_L, \quad (14)$$

where $\pi_{i,t} = \pi_i \left(\mathbf{y}_{i,t-1}^{p*}\left(\mathbf{z}_i^{t-1}\right), \mathbf{y}_{i,t}^{p*}\left(\mathbf{z}_i^{t}\right), \mathbf{z}_{i,t}^{\pi} \right)$, $u_{i,t} = u_i\left(\pi_{i,t}\right)$, $C_{i,t}^{V} = C_i^{V}\left(\mathbf{y}_{i,t}^{p*}\left(\mathbf{z}_i^{t}\right), \mathbf{z}_{i,t}^{C}\right)$, $b_C = \begin{cases} 0 & (j=1), \\ 1 & (j \neq 1), \end{cases}$ and $h_{i,j,t} = h_{i,j}\left(Q_{j,t}^{p*}, \mathbf{z}_{i,j,t}^{H}\right)$. $q_{i,1,t}^{p*} = q_{i,1,t}^{p*}\left(\mathbf{z}_i^{t}\right)$ denotes the optimal level of cash, $q_{i,j,t}^{p*} = q_{i,j,t}^{p*}\left(\mathbf{z}_i^{t}\right)(j = 2, \cdots, N_A)$ the optimal levels of assets with the exception of cash, $x_{F,i,j,t}^{p*} = x_{F,i,j,t}^{p*}\left(\mathbf{z}_i^{t}\right)(j = 1, \cdots, M_F)$ the optimal levels of real resource fixed inputs, and $q_{i,j,t}^{p*} = q_{i,j,t}^{p*}\left(\mathbf{z}_i^{t}\right)(j = N_A + 1, \cdots, N_A + N_L)$ the optimal levels of liabilities.

If the utility function $u_{i,t}$ is concave and continuously differentiable in $\mathbf{y}_{i,t-1}^{p*} = \left(\mathbf{q}_{i,t-1}^{p*\prime}, \mathbf{x}_{F,i,t-1}^{p*\prime}\right)^{\prime}$ and $\mathbf{y}_{i,t}^{p*}$, and integrable, and if each of the partial derivatives of $u_{i,t}$ with respect to $\mathbf{y}_{i,t-1}^{p*}$ is absolutely integrable, then the stochastic Euler equations (12)-(14) with the following transversality conditions,

$$\lim_{t \to \infty} \beta_i^t \cdot \int_{Z} \frac{\partial u_{i,t+1}}{\partial \pi_{i,t+1}} \cdot \frac{\partial \pi_{i,t+1}}{\partial y_{i,j,t}^{p*}} \cdot y_{i,j,t}^{p*}\, Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}\right) = 0,\ j = 1, \cdots, N_A + N_L + M_F, \quad (15)$$

are sufficient conditions for an optimal plan $\mathbf{y}_i^{p*} = \left\{ \mathbf{q}_{i,0}^{p*}, \mathbf{x}_{F,i,0}^{p*}, \left\{ \mathbf{q}_{i,t}^{p*}, \mathbf{x}_{F,i,t}^{p*} \right\}_{t=1}^{\infty} \right\}$.

2.6 Conjectural User-Revenue Price

The user-cost prices of financial goods presented by Hancock (1985, 1991) have the advantage of the unambiguous classification of financial goods into inputs and outputs based on the sign of each of the user-cost prices. However, this classification is not always consistent with the classification based on the sign of each of the partial derivatives of the variable cost function with respect to financial goods. In addition, for Hancock's user-cost prices to provide meaningful estimates of prices of financial goods, strong assumptions must be maintained. In particular, it is necessary to assume that the financial firm's attitude to risk is neutral, that financial goods markets are competitive, and that there is informational symmetry between buyers and sellers. The last two assumptions imply that the rates of holding revenues (or costs) are given. As a consequence, if any of these assumptions is violated, the price measures based on Hancock's user-cost prices will yield biased estimates of prices of financial goods, and the classification of financial goods into inputs and outputs will be incorrect.

In the light of these problems, we derive generalizations of Hancock's user-cost prices. Rearranging the stochastic Euler equations (12) and (14) with respect to the partial derivatives of the variable cost function, we get the following equations:

$$\frac{\partial C_{i,t}^{V}}{\partial q_{i,j,t}^{p*}} = b_j \cdot p_{G,t} \cdot \left[\beta_{i,t} \cdot \left\{1 + b_C \cdot \left(h_{i,j,t} - s_{i,j,t} \cdot \eta_{i,j,t} \cdot (1 + CV_{i,j,t})\right)\right\}\right.$$

$$\left. \cdot \int_Z \frac{\partial u_{i,t+1} / \partial \pi_{i,t+1}}{\partial u_{i,t} / \partial \pi_{i,t}} Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}\right) - 1\right]$$

$$= b_j \cdot p_{G,t} \cdot \left[\beta_{i,t} \cdot (1 + b_C \cdot h_{i,j,t}) \cdot \int_Z \frac{\partial u_{i,t+1} / \partial \pi_{i,t+1}}{\partial u_{i,t} / \partial \pi_{i,t}} Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}\right) - 1\right]$$

$$- b_j \cdot b_C \cdot p_{G,t} \cdot \beta_{i,t} \cdot s_{i,j,t} \cdot \eta_{i,j,t} \cdot (1 + CV_{i,j,t}) \cdot \int_Z \frac{\partial u_{i,t+1} / \partial \pi_{i,t+1}}{\partial u_{i,t} / \partial \pi_{i,t}} Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}\right), \text{ and}$$

$$j = 1, \cdots, N_A + N_L, \quad (16)$$

where

$$b_j = \begin{cases} 1 & (j = 1, \cdots, N_A), \\ -1 & (j = N_A + 1, \cdots, N_A + N_L), \end{cases}$$

$$b_C = \begin{cases} 0 & (j = 1), \\ 1 & (j \neq 1), \end{cases}$$

$$s_{i,j,t} = \frac{q_{i,j,t}^{p*}}{Q_{j,t}^{p*}},$$

$$\eta_{i,j,t} = \begin{cases} -\left(\dfrac{\partial r_{i,j,t}}{\partial \ln Q_{j,t}^{p*}} + \dfrac{\partial r_{i,j,t}^{Q}}{\partial \ln Q_{j,t}^{p*}} + \dfrac{\partial h_{i,j,t}^{S}}{\partial \ln Q_{j,t}^{p*}} - \dfrac{\partial h_{i,j,t}^{D}}{\partial \ln Q_{j,t}^{p*}} \right) \\ \qquad (j = 2, \cdots, N_A), \\ -\left(\dfrac{\partial r_{i,j,t}}{\partial \ln Q_{j,t}^{p*}} + \dfrac{\partial r_{i,j,t}^{Q}}{\partial \ln Q_{j,t}^{p*}} + \dfrac{\partial h_{i,j,t}^{I}}{\partial \ln Q_{j,t}^{p*}} - \dfrac{\partial h_{i,j,t}^{S}}{\partial \ln Q_{j,t}^{p*}} \right) \\ \qquad (j = N_A + 1, \cdots, N_A + N_L), \end{cases}$$

and

$$CV_{i,j,t} = \sum\nolimits_{k \neq i}^{N_F} \frac{\partial q_{k,j,t}^{p*}}{\partial q_{i,j,t}^{p*}}.$$

The parameter b_j distinguishes assets from liabilities, and b_C identifies cash. $s_{i,j,t}$ is the ith financial firm's market share of the jth financial good in period t. $\eta_{i,j,t}$ is the negative elasticity of the rate of the ith financial firm's holding revenue (or cost) with respect to the logarithm of the quantity of the jth financial good in the market in period t. $CV_{i,j,t}$ is the conjectural derivative, which is the way the ith financial firm thinks all other firms' jth financial good changes as the ith financial firm's jth financial good changes in period t.

We, then, define generalizations of Hancock's user-cost prices as follows:

$$\begin{aligned} p_{i,j,t}^{SUR} &= b_j \cdot p_{G,t} \cdot \left[\beta_{i,t} \cdot (1 + b_C \cdot h_{i,j,t}) \cdot \int_Z \frac{\partial u_{i,t+1} / \partial \pi_{i,t+1}}{\partial u_{i,t} / \partial \pi_{i,t}} Q\left(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}\right) - 1 \right] \\ &= b_j \cdot p_{G,t} \cdot \left\{ \beta_{i,t}^{S} \cdot (1 + b_C \cdot h_{i,j,t}) - 1 \right\}, \quad j = 1, \cdots, N_A + N_L, \qquad (17) \end{aligned}$$

$$
\begin{aligned}
p_{i,j,t}^{CUR} &= b_j \cdot p_{G,t} \cdot \Big[\beta_{i,t} \cdot \Big\{1 + b_C \cdot \big(h_{i,j,t} - s_{i,j,t} \cdot \eta_{i,j,t} \cdot (1 + CV_{i,j,t})\big)\Big\} \\
&\qquad \cdot \int_Z \frac{\partial u_{i,t+1} / \partial \pi_{i,t+1}}{\partial u_{i,t} / \partial \pi_{i,t}} Q(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}) - 1\Big] \\
&= b_j \cdot p_{G,t} \cdot \Big[\beta_{i,t}^S \cdot \Big\{1 + b_C \cdot \big(h_{i,j,t} - s_{i,j,t} \cdot \eta_{i,j,t} \cdot (1 + CV_{i,j,t})\big)\Big\} - 1\Big] \\
&= p_{i,j,t}^{SUR} - \gamma_{i,j,t}, \quad j = 1, \cdots, N_A + N_L, \qquad (18)
\end{aligned}
$$

where

$$
\beta_{i,t}^S = \beta_{i,t} \cdot \int_Z \frac{\partial u_{i,t+1} / \partial \pi_{i,t+1}}{\partial u_{i,t} / \partial \pi_{i,t}} Q(\mathbf{z}_{i,t}, \mathbf{dz}_{i,t+1}), \qquad j = 1, \cdots, N_A + N_L, \quad (19a)
$$

$$
\gamma_{i,j,t} = b_j \cdot b_C \cdot p_{G,t} \cdot \beta_{i,t}^S \cdot s_{i,j,t} \cdot \eta_{i,j,t} \cdot (1 + CV_{i,j,t}), \qquad j = 1, \cdots, N_A + N_L. \quad (19b)
$$

The first price $p_{i,j,t}^{SUR}$ is the stochastic user-revenue price of the jth financial good of the ith financial firm in period t, which contains Hancock's user-cost price as a special case because the financial firm's attitude toward risk is not assumed to be neutral. If it is assumed, then $\frac{\partial u_{i,t+1}}{\partial \pi_{i,t+1}} \Big/ \frac{\partial u_{i,t}}{\partial \pi_{i,t}} = 1$, so that $p_{i,j,t}^{SUR} = b_j \cdot p_{G,t} \cdot \big[\beta_{i,t} \cdot (1 + b_C \cdot h_{i,j,t}) - 1\big] = -b_j \cdot p_{G,t} \cdot \frac{r_{i,t}^D - b_C \cdot h_{i,j,t}}{1 + r_{i,t}^D}$; that is, the stochastic user-revenue price equals Hancock's negative user-cost price. The stochastic user-revenue price allows us to think about the case of the risk-averse financial firm.

The second price $p_{i,j,t}^{CUR}$ is the conjectural user-revenue price of the jth financial good of the ith financial firm in period t, which is more general than the stochastic user-revenue price $p_{i,j,t}^{SUR}$ because it is not assumed that the financial goods markets are competitive and that there is informational symmetry between buyers and sellers. If the jth financial good market is competitive, then $CV_{i,j,t} = -1$, so that $p_{i,j,t}^{CUR} = p_{i,j,t}^{SUR}$; that is, the conjectural user-revenue price equals the stochastic user-revenue price. In addition to the case of the risk-averse financial firm, the conjectural user-revenue price allows us to consider the case that the financial firms are strategically interdependent and that there are informational asymmetries between buyers and sellers that lead to adverse selection. From equations (16) and (18), furthermore, we obtain the following equations:

$$\frac{\partial C_{i,t}^{V}}{\partial q_{i,j,t}^{p*}} = p_{i,j,t}^{CUR} = p_{i,j,t}^{SUR} - \gamma_{i,j,t}, \quad j = 1, \cdots, N_A + N_L. \tag{20}$$

Equations (20) mean that the classification of financial goods into inputs and outputs based on the sign of each of the conjectural user-revenue prices is consistent with the classification based on the sign of each of the partial derivatives of the variable cost function with respect to financial goods. If the conjectural user-revenue price has a plus (or minus) sign, the partial derivative of the variable cost function also has a plus (or minus) sign, so that the financial good is defined as an output (or fixed input).

Using equations (20) with respect to the financial goods defined as outputs, we obtain the following equations:

$$\left(p_{i,j,t}^{SUR} - \frac{\partial C_{i,t}^{V}}{\partial q_{i,j,t}^{p*}} \right) / p_{i,j,t}^{SUR} = \left\{ \gamma_{i,j,t} \cdot \left(b_j \cdot p_{G,t} \cdot \beta_{i,t}^{S} \right)^{-1} \right\} \Bigg/ \left(1 + b_C \cdot h_{i,j,t} - \frac{1}{\beta_{i,t}^{S}} \right)$$

$$= \left\{ b_C \cdot s_{i,j,t} \cdot \eta_{i,j,t} \cdot (1 + CV_{i,j,t}) \right\} \Bigg/ \left(1 + b_C \cdot h_{i,j,t} - \frac{1}{\beta_{i,t}^{S}} \right), \quad j = 1, \cdots, N_A + N_L. \tag{21}$$

Equations (21) are generalizations of the Lerner index of monopoly power of the financial firms' oligopoly under dynamic uncertainty. If $s_{i,j,t} = 1$ and $CV_{i,j,t} = 0$, the ith financial firm is a monopoly in the jth financial good market in period t. If $CV_{i,j,t} = 0$, the ith financial firm is a Cournot firm, which expects that all other financial firms' outputs will not change as its own output changes. If $CV_{i,j,t} = -1$, the ith financial firm is a competitive firm, whose price-marginal cost margin is zero. Higher values of $CV_{i,j,t}$ correspond to larger gaps between price and marginal cost and thus to less intense rivalry. The correspondence between the generalized Lerner index (GLI) and $h_{i,j,t}$ or $\beta^S_{i,t}$, however, is not clear from the right-hand side of equations (21) because its numerator also generally varies if $h_{i,j,t}$ or $\beta^S_{i,t}$ changes.[6] Therefore, we write the left-hand side of equations (21) as follows:

$$\left(p^{SUR}_{i,j,t} - \frac{\partial C^V_{i,t}}{\partial q^{p*}_{i,j,t}}\right) / p^{SUR}_{i,j,t} = 1 - \frac{\partial C^V_{i,t}}{\partial q^{p*}_{i,j,t}} / b_j \cdot p_{G,t} \cdot \left\{\beta^S_{i,t} \cdot (1 + b_C \cdot h_{i,j,t}) - 1\right\}, \quad j = 1, \cdots, N_A + N_L.$$

From these equations, under the assumption that the partial derivatives of the variable cost function are constant, higher values of $h_{i,j,t}$ correspond to higher values of the GLI if the jth financial good is an asset ($b_j = 1$), whereas higher values of $h_{i,j,t}$ correspond to lower values of the GLI if the jth financial good is a liability ($b_j = -1$). These findings are consistent with our intuition about the relationship between $h_{i,j,t}$ and the degree of competition. Furthermore, under the same assumption, higher values of $\beta^S_{i,t}$ correspond to higher values of the GLI if the jth financial good is an asset, whereas higher values of $\beta^S_{i,t}$ correspond to lower values of the GLI if the jth financial good is a liability. Since the inverse of $\beta^S_{i,t}$ can be interpreted as the risk-free rate (RFR), according to asset pricing theory, lower values of the RFR correspond to higher values of the GLI if the jth financial good is an asset, whereas lower values of the RFR correspond to lower values of the GLI if the jth financial good is a liability.

The concept of conjectural variation is popular in both applied theoretic and empirical industrial organization. Theorists of industrial organization, however, take a dim view of its ad hoc assumptions about the conduct of

[6] For example, if $h_{i,j,t}$ in equation (3) or $\beta^S_{i,t}$ in equation (19a) changes with $r^D_{i,t}$, then $q^{p*}_{i,j,t}$ varies. Therefore, $s_{i,j,t}$, $\eta_{i,j,t}$, and $CV_{i,j,t}$ in the numerator of the right-hand side of equations (21) will also change.

firms, its lack of a game-theoretic foundation, and the forcing of dynamics into an essentially static model with the strategy space and the time horizon of the underlying game only loosely defined (cf. Fellner, 1949; Friedman, 1983, p.110; Daughety, 1985; Makowski, 1987; Tirole, 1989, pp. 244–245). These shortcomings are often perceived as the cost that the modeler must pay for realism without compromising simplicity and tractability. However, it is fortunate that Dockner (1992), Cabral (1995), and Pfaffermayr (1999) show that the concept of conjectural variation can be supported by a consistent theoretical foundation, if it is considered to be a reduced form of a dynamic game.[7] Their findings can be used to justify a static conjectural variations analysis for both modeling dynamic interactions and estimating the degree of oligopoly power. From the same viewpoint as theirs, we believe that the use of the conjectural user-revenue model is rationalized by considering it as a reduced form of an (unmodeled) dynamic game.

3 Empirical Sketch

A theoretical extension is not the only contribution of the conjectural user-revenue model (CURM) described in the last section. The CURM could also give a basis for future empirical research. To realize this contribution, we sketch an empirical research procedure based on the CURM.

3.1 Empirical Specification

In order to estimate the CURM, we need to specify the variable cost function defined by (5) and the utility function described in section 2.4. The subjective

[7] Using an infinite horizon adjustment cost model, Dockner (1992) demonstrates that any steady state closed-loop (subgame-perfect) equilibrium coincides with a static conjectural variation equilibrium with nonzero conjectures. Cabral (1995) proves that, in linear oligopolies and for an open set of values of the discount factor, there exists an exact correspondence between the conjectural variation solution and the solution of quantity-setting repeated game with minimax punishments during T periods. Pfaffermayr (1999) follows an idea put forward by Cabral (1995) and demonstrates that the conjectural variation model can be interpreted as the joint-profit-maximizing steady-state reduced form of a price-setting supergame in a differentiated product market under optimal punishment strategies.

rate of time preference found in (9) is also specified if it is estimated. A point duly to be considered in specifying the variable cost function is the argument between the calibration and the flexible functional approach put forward by Lau (1978), Diewert and Wales (1988), and Barnett et al. (1991). They point out that the former approach uses very restrictive specifications for tastes and technology, whereas the latter approach cannot be implemented using time-series data if the model has a large number of goods.

Taking into account this argument, Diewert and Wales (1988) proposed the concept of a semiflexible form, which is a special case of a flexible form but which requires fewer free parameters. While this concept is only one possible parametric specification, Barnett et al. (1991) asserted that a general modeling approach that meets the following requirements is needed. (1) Such an approach must be able to incorporate as much theoretical flexibility as can be supported by the available sample size, so that the depth of the parameterization must be dependent upon sample size. (2) The approach must be able to impose the economic theory that it supports globally. (3) Iterative estimation algorithms capable of producing inferences having known properties must exist.

Is the first requirement compatible with the second one? The answer is: yes, under difficult and limited conditions. Although some sophisticated models proposed by Diewert and Wales (1987, 1988), Barnett et al. (1991), and Barnett and Hahm (1994) do make it possible, the estimation of these models is not easy because the second requirement imposes highly nonlinear restrictions on estimation parameters. In specifying the variable cost function, these arguments should be borne in mind.

In specifying the utility function, we need to take into consideration a restriction on the specification stemming from the dynamic model. In the static model assuming the single period optimization, the indirect utility function or the expenditure function would be estimated by applying duality theory.[8] In the CURM, however, it is difficult to derive the same function because the CURM is a dynamic-uncertainty model. For this reason, we must estimate parameters of the utility function indirectly through the stochastic

[8] See, for example, Hughes, Lang, Mester, and Moon (1995).

Euler equations described by (12), (13), and (14). Unlike the specification of the variable cost function assuming direct estimation, the specification of the utility function, which does not have a directly estimated equation, is forced to keep the parameterization to a requisite minimum. Although a very general specification of the utility function to represent risk is the hyperbolic absolute risk aversion (HARA) class, feasible specifications may include Box-Cox, quadratic, power, and logarithmic function.[9]

3.2 Estimation Procedure

The CURM can be estimated using the stochastic Euler equations (12) to (14). For more efficient estimates, however, we hope to estimate the variable cost function, (5), and/or the short-run derived demand function simultaneously in addition to these equations. In estimating these equations and functions simultaneously, the first consideration is that all these include endogenous state variables as explanatory variables. These explanatory variables, therefore, correlate with error terms of each equation and function. This causes nonlinear least squares estimates to be inconsistent.[10]

Classifying roughly, there are two procedures to resolve this problem. One procedure is the nonlinear maximum likelihood (NLML) procedure including the Jacobian factor, and another is the nonlinear instrumental variable (NLIV) procedure. The former procedure is familiar as the nonlinear full-information maximum likelihood (NLFIML) procedure.[11] The latter is well known as the nonlinear three-stage least squares (NL3SLS) procedure and the generalized method of moment (GMM) procedure.[12] In theory, the NL3SLS procedure is a special case of the GMM procedure because the estimators of both procedures are coincident under serial independence and conditional homoscedasticity in the errors.[13]

Which of these procedures is chosen depends on whether the estimated

[9] For further details of HARA, see Ingersoll (1987, pp.37–40).
[10] On this problem, see Davidson and MacKinnon (1993, pp.211–215).
[11] See, for example, Amemiya (1985, Chapter 8) and Gallant (1987, Chapter 6).
[12] See, for example, Davidson and MacKinnon (1993, Chapters 7, 17, 18).
[13] See, for example, Davidson and MacKinnon (1993, pp.651–667).

equations or functions are linear in the endogenous state variables. If linear, the NLFIML procedure is consistent, even if the error terms are not, in fact, distributed as multivariate normal, and asymptotically equivalent to the NL3SLS procedure.[14] If nonlinear, although the NLFIML procedure will be more efficient, asymptotically, than the NLIV procedure, the NLFIML procedure may be inconsistent if the assumption of normality is not met.[15] The NLIV procedure, on the other hand, does not need this assumption for consistency.[16] For this reason, if the estimated equations or functions are nonlinear in the endogenous state variables, in the light of robustness to the assumption of the distribution of the errors, the NLIV procedure would be more desirable than the NLFIML procedure. Furthermore, since the GMM procedure is superior to the NL3SLS procedure under serial correlation and conditional heteroscedasticity in the errors, the GMM procedure would be the most desirable procedure. Since the stochastic Euler equations, (12), (13), and (14), would be in most cases nonlinear in the endogenous state variables, the GMM procedure should be used to estimate the CURM.

4 Conclusion

We have derived an index of the degree of competition as a generalization of the Lerner index of monopoly power of financial firms' oligopoly under dynamic uncertainty by extending the user-cost approach of Hancock (1985, 1991) in two ways. First, our model allows financial firms to behave strategically as well as competitively. Second, we do not assume that financial firms are risk-neutral. Although the user-cost approach has been used to determine whether a financial product is an input or an output on the basis of its net contribution to the revenues of the financial institution, this classification is not always consistent with the classification based on the sign of each of the partial derivatives of the variable cost function with respect to financial

[14] See Amemiya (1977).

[15] See Amemiya (1977), White (1982), Phillips (1982), Hausman (1983, p.444), and Davidson and MacKinnon (1993, p.667).

[16] See Amemiya (1977), Hausman (1983, p.444), and Davidson and MacKinnon (1993, p.667).

goods. However, our index is consistent with the classification based on the variable cost function.

Some issues will need to be investigated in future work. First, although the user-cost approach has been used mainly for the banking industry (we also derive our model for the banking industry in this paper), we will try to derive this index of the degree of competition using the user-cost approach for various financial institutions like the life insurance industry, the property-liability insurance industry and the securities industry.[17] Second, we need to estimate our index of the degree of competition. But, as Berger and Humphrey (1992) pointed out, some applications of the user cost approach in banking showed that classifications of inputs and outputs were not robust to the choice of opportunity-cost estimates nor were they robust over time. For this reason, we need to treat the classifications of inputs and outputs with special care when we estimate our model.

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