A Game-theoretic Interpretation for the Free-rider Problem under the Lindahl Mechanism

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Abstract: An exposition of the free rider problem in a public good economy is presented by the use of a non–cooperative n–person game. It is shown that the personalized Lindahl prices are not the Nash equilibria of the game.

1. It was proved by Foley (1970) that in an economy with public goods there exist Lindahl equilibria and that they are contained in the core of the economy. Unlike in a market of private goods only, different agents must face different prices for each public good under the Lindahl equilibria. However, as is well-known in literature [e.g., Buchanan (1968)], each agent will have an incentive to 'free ride' on the benefit of the public good. As a result the Lindahl mechanism would not work, since no agent would behave as a price–taker for the Lindahl prices. The purpose of this note is to interpret the free rider problem under the Lindahl mechanism in terms of n–person game theory. This approach would be of some use in explaining the nature of the problem.

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2. We consider a very simple economy comprised of n consumers, one public good and freely transferable money. Let $N=\{1,\cdots,n\}$ be the set of all consumers. The public good is consumed collectively and equally by all individuals in N. The utility of individual i received from consuming q amount of the public good is given by $u_i(q)$, which is measured in terms of money. The cost of producing q amount of the public good is given by $C(q)$. We assume:

(a) $u_i(q)$ is differentiable on $(0,\infty)$ with $u'_i>0$ and $u''_i<0$,

(b) $C(q)$ is differentiable on $(0,\infty)$ with $C'>0$, $C''>0$, $C(0)=0$ and $\lim_{q\to+0} C'(q)=0$.

Lindahl equilibrium in this economy is a pair $(p^*;q^*)=(p_1^*,\cdots p_n^*;q^*)$ such that

$$u_i(q^*)-p_i^*q^*=\max_{q_i\geq 0}(u_i(q_i)-p_i^*q_i),\ \text{for all}\ i\in N \tag{1}$$

$$\sum_{i\in N} p_i^*q^*-C(q^*)=\max_{q\geq 0}(\sum_{i\in N} p_i^*q-C(q)) \tag{2}$$

At the Lindahl equilibrium $(p^*;q^*)$, each individual i receives a net benefit:

$$x_i=u_i(q^*)-p_i^*q^*+d_i(\sum_{j\in N} p_j^*q^*-C(q^*)),$$

where $d=(d_1,\cdots,d_n)$ represents a fixed proportion of profits satisfying $\sum_{i\in N} d_i=1$ and $d_i>0$ for all $i\in N$.

3. Now, suppose that each individual $i\in N$ decides 'his price' for the public good and announces it to the producer. The level of supply is, then, determined through profit maximization at the price of the sum announced voluntarily by all individuals. Let $P_i=[0,\bar{p}_i]$ be a closed interval with $\bar{p}_i$ being sufficiently large, from which individual i chooses his price $p_i(p_i\geq 0)$ for the public good. Define a set Q for each $p\in P=P_1\times\cdots\times P_n$ by

$$Q=\{q\geq 0:\sum_{i\in N} p_iq-C(q)=\max_{z\geq 0}(\sum_{i\in N} p_iz-C(z))\} \tag{3}$$

It is clear from assumption (b) that Q is nonempty and consists of a unique

point for each $p \in P$.

Then, we can define a payoff function $f(p)=(f_1(p),\cdots,f_n(p))$ as follows:

$$f_i(p)=u_i(q(p))-p_iq(p)+d_i\{\sum_{j\in N} p_jq(p)-C(q(p))\}, \tag{4}$$

where $q(p)$ is the unique point in Q for $p \in P$. Thus we have defined a game characterized by N, P and f. A Nash equilibrium point of the game is a point $p^0=(p_1^0,\cdots,p_n^0)\in P$ such that

$$f_i(p^0)=\max_{p_i\in P_i} f_i\ (p_1^0,\cdots,p_{i-1}^0,p_i,p_{i+1}^0,\cdots,p_n^0) \text{ for all } i\in N. \tag{5}$$

We call $p^* \in P$ a Lindahl price if the pair $(p^*;q(p^*))$ is a Lindahl equilibrium.

4. The behaviour of free riding under the Lindahl mechanism is expressed in this game as that of choosing a strategy $p_i=0$ or less than his Lindahl price p_i^*. The following proposition reveals the game theoretic nature of such behaviours.

Proposition. Let $p^* \in P$ be a Lindahl price. Then, p^* is not a Nash equilibrium point of the game if $q^*>0$, where $p^*=q(p^*)$.

Proof. We note initially that the function $q(p)$ is differentiable for all $p>0$, since $C'(q)=\sum_{i\in N} p_i$ for $q\in Q$ and $C'(q)$ is monotone increasing, differentiable with $C''(q)\neq 0$ for all $q>0$.

Suppose that p^* is a Nash equilibrium point. Then we have

$$f_{ip_i}(p^*)\geqq 0, \quad \text{if } 0<p_i^*\leqq \bar{p}_i \tag{6}$$

where $f_{ip_i}(p^*)$ denotes the value of the parial derivative of $f_i(p)$ at $p=p^*$ with respect to p_i. Since $(p^*;q^*)$ is a Lindahl equilibrium with $q^*>0$, we have

$$u_i'(q^*)\ =p_i^*>0,\ 0, \text{ for all } i\in N, \tag{7}$$

$$C'(q^*)=\sum_{i\in N} p_1^*.$$

Then,

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\begin{aligned}
f_{ip_i}(p^*) &= u_i'(q^*)q_{p_i}(p^*) - p_i^* q_{p_i}(p^*) - q(p^*) \\
&\quad + d_i(q(p^*) + \sum_{i \in N} p_i^* q_{p_i}(p^*) - C'(q^*)q_{p_i}(p^*)) \\
&= -(1-d_i)q(p^*) \\
&< 0, \quad \text{for all } i \in N. \qquad (8)
\end{aligned}
$$

From this and (6) we have $p_i^* = 0$, which contradicts (7). Q. E. D.

This proposition states that if the public good is to be produced, then there is an individual who can make himself better off by choosing a price other than the Lindahl one, provided that any other individual stays at the Lindahl price. In fact, every individual has an incentive not to pay up to p_i^*, as is seen in (8). It is this lack of stability that incites an individual to act as a free rider under the Lindahl mechanism.

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