A Dynamical Model of Solar Prominences with Current Sheet

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Abstract

Recent observations of solar prominences show that slow upward motions($\simeq$ 1 kms^{-1}) occur through quiescent prominences and a fast input of material with horizontal motions ($\simeq$ 5 kms^{-1}) occurs at both edges of prominences. A time-dependent dynamical model of solar prominences with current sheet is investigated. It is natural extension of the magnetostatic models proposed by Kippenhahn and Schlüter (KS) and Kuperus and Raadu (KR). It is shown that horizontal nonlinear oscillations can exist in the prominences. The global structure of current sheet including solar wind plasmas is also simulated by means of full MHD equations.

1. Introduction

Solar prominences are thin condensed sheets of cold material located in the low corona. They are suspended against gravity and above magnetic neutral lines between two opposite magnetic polarities (see Tandberg-Hanssen[1], 1974). Recent observations (Malherbe et al.[2], 1983; Schmieder et al.[3], 1984) showed that slow upward motions(0.5 km s^{-1} in Hα and 5.6 kms^{-1} in C$_{\text{IV}}$) occur in the prominences and a fast input of material with horizontal mo ($\simeq$5 km s^{-1}) occurs at both edges of prominences.

Previous solar prominence models such as Kippenhahn and Schlüter[4] (KS)(1957)and Kuperus and Raadu (KR)[5] (1974) are magnetostatic and do not take into account the plasma dynamics, except for eruptive prominence model (see Sakai and Nishikawa[6], 1983). Recently, Malherbe and Priest (1983)[7] proposed a qualitative dynamical model with magnetic configurations etiher of the KR or KS type to explain the observed upward motions.

In the present paper we investigate a time-dependent dynamical model of solar prominences with current sheet. It is natural extension of the magnetostatic models proposed by the KS or KR. We pay particular attention on the local solutions describing the dynamical accumulation of plasmas near the current sheet. Coupled basic equations with self-similar solutions are derived. It is recently shown that magnetic reconnection forced by external plasma flow induced by coalescence of two current loops can be well described by the self-similar solutions (Sakai et al., 1984)[8].

It is shown that for nearly one-dimensional accumulation of plasmas horizontal nonlinear oscillations can exist in the prominences. Finally the global structure of current sheet including solar wind plasmas is simulated by means of full MHD equations.

2. Basic equations for current sheet model

We consider solar prominences as vertical thin current sheets supported by magnetic field in the low corona shown in Fig. 1. We consider a dynamical condensation of plasmas in the current sheet only by j × B force, neglecting the effect of thermal instability. We assume the law of adiabatic compression and that the sheet is homogenous in the z-direction. The MHD equation including gravity gives

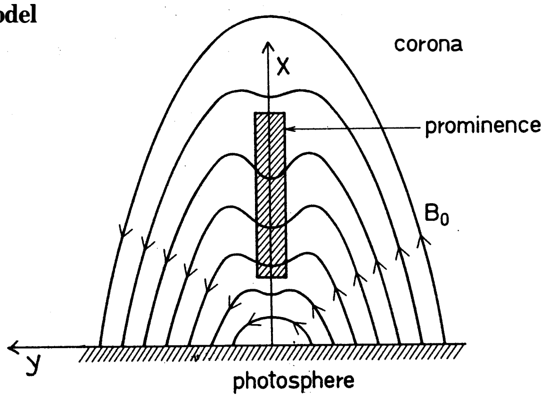

Fig. 1

$$\frac{\partial \rho}{\partial t} + div(\rho \boldsymbol{v}) = 0, \qquad (1)$$

$$\rho\left(\frac{\rho \boldsymbol{v}}{\rho t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v}\right) = -\nabla P + \frac{1}{4\pi}\, rot\, \boldsymbol{B} \times \boldsymbol{B} - \rho g \vec{e}_x, \qquad (2)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = rot(\boldsymbol{v} \times \boldsymbol{B}) + \frac{c^2}{4\pi\sigma} \Delta \boldsymbol{B}, \qquad (3)$$

$$\frac{\partial P}{\partial t} + \boldsymbol{v} \cdot \nabla P + \gamma P\, div\, \boldsymbol{v} = 0, \qquad (4)$$

where the gravity is given by $g(x) = GM_{\odot} R_{\odot}^{-2} (1 + x/R_{\odot})^{-2} = g_{\odot} (1 + x/R_{\odot}^{2})^{-2}$ and $g_{\odot} = GM_{\odot} R_{\odot}^{-2}$.

We assume that the input horizontal flows around the current sheet obey

$$v_y = \frac{\dot{a}}{a} y, \qquad (5)$$

where the dot means the time derivative and a(t) is a scale factor characterizing continuous change of thickness of current sheet. The upward flow is taken as

$$v_x = v_{x0}(t) + \frac{\dot{b}}{b} x, \qquad (6)$$

where v_{x0} and a scale factor b(t) are determined self-consistently. The magnetic fields are assumed that

$$B_x = B_{x0}(t)\, y/\lambda, \quad B_y = B_{n0}(t) + B_{y0}(t)\, x/\lambda, \quad B_z = B_{z0}(t) \quad . \qquad (7)$$

From the continuity equation (1) we find

$$\dot{\rho}/\rho + \dot{a}/a + \dot{b}/b = 0, \qquad (8)$$

where ρ is a function of time only. The equation (8) gives

$$\rho = \rho_0 / ab, \qquad (9)$$

where ρ_0 is a constant. From the induction equation (3) we obtain

$$B_{x0}(t)=B_0/a^2 \quad , \tag{10}$$
$$B_{y0}(t)=B_0/b^2 \quad , \tag{11}$$
$$B_{z0}(t)=B_{00}/ab \quad , \tag{12}$$
$$\frac{\partial B_{n0}}{\partial t}=-\left(\frac{v_{x0}}{\lambda}\frac{B_0}{b^2}+B_{n0}\frac{\dot{b}}{b}\right) \quad , \tag{13}$$

where B_0 and B_{00} are constants.
If we assume that the pressure P is given by

$$P(x,y,t)=P_{00}(t)-P_0(t)\frac{x}{\lambda}-P_{x0}(t)\frac{x^2}{\lambda^2}-P_{y0}(t)\frac{y^2}{\lambda^2} \quad , \tag{14}$$

we find fromthe equation (4)

$$P_{x0}(t)=P_0/a^{\gamma}b^{\gamma+2} \quad , \tag{15}$$
$$P_{y0}(t)=P_0/a^{\gamma+2}b^{\gamma} \quad , \tag{16}$$
$$\frac{\partial P_{00}}{\partial t}+\gamma P_{00}(\dot{a}/a+\dot{b}/b)-v_{x0}\frac{P_0(t)}{\lambda}=0 \quad , \tag{17}$$
$$\frac{1}{P_0(t)}\frac{\partial P_0}{\partial t}+2v_{x0}P_0/P_0(t)\lambda a^{\gamma}b^{\gamma+2}+\dot{b}/b+\gamma\left(\dot{b}/b+\dot{a}/a\right)=0, \tag{18}$$

where P_0 is a constant.

Finall we obtain basic equations for scale factors a(t) and b(t) from the equation of motions (2)

$$\ddot{a}=\frac{c_s^2}{\lambda^2a^{\gamma}b^{\gamma-1}}+\frac{V_A^2}{\lambda^2}\left(\frac{1}{b}-\frac{b}{a^2}\right) \quad , \tag{19}$$
$$\ddot{b}=\frac{c_s^2}{\lambda^2a^{\gamma-1}b^{\gamma}}-\frac{V_A^2}{\lambda^2}\left(\frac{a}{b^2}-\frac{1}{a}\right)+2\frac{g_\odot}{R_\odot}b, \tag{20}$$
$$\frac{\partial v_{x0}}{\partial t}+v_{x0}\frac{\dot{b}}{b}=\frac{P_0(t)}{\rho_0}\frac{ab}{\lambda}-\frac{B_{n0}(t)B_0}{4\pi\rho_0\lambda}\frac{(a^2-b^2)}{ab}-g_\odot \quad , \tag{21}$$

where we assumed that the gravitational acceleration g(x) is approximated by $g(x)=g_\odot(1-2x/R_\odot)$, when $x \ll R_\odot$. $c_s^2=2P_0/\rho_0$ and $V_A^2=B_0^2/4\pi\rho_0$.
If we neglect the pressure terms and the effect of gravity in the above equations we obtain

$$\ddot{a}=\frac{V_A^2}{\lambda^2}\left(\frac{1}{b}-\frac{b}{a^2}\right) \quad , \tag{22}$$
$$\ddot{b}=-\frac{V_A^2}{\lambda^2}\left(\frac{a}{b^2}-\frac{1}{a}\right) \quad , \tag{23}$$

which were first derived by Imshennik and Syrovatskii(1967)[9] for the investigation of plasma dynamics near X-type neutral point.

3. Self-similar solutions

We examine a few limiting cases for applications of solar prominence dynamics.

(case 1) $B_y=0$ and $g=0$.

In this case it is consistent to assume $V_{x0}=0$ and $P_0=0$. The above basic equations are simplified to

$$\ddot{a}=\frac{c_s^2}{\lambda^2 a^\gamma b^{\gamma-1}}-\frac{V_A^2 b}{\lambda^2 a^2}\quad, \tag{24}$$

$$\ddot{b}=\frac{c_s^2}{\lambda^2 a^{\gamma-1} b^\gamma}\quad. \tag{25}$$

(case 2) B_y=constant and g=constant.

In this case we can neglect the inhomogenous terms in v_x and By in Eq.(6) and (7). We find for a(t)

$$\ddot{a}=\frac{c_s^2}{\lambda^2 a^\gamma}-\frac{V_A^2}{\lambda^2 a^2}\quad. \tag{26}$$

and v_{x0} (t) is determined from

$$\frac{\partial V_{x0}}{\partial t}=\frac{V_A^2 B_n}{\lambda B_0 a(t)}-g_\odot\quad, \tag{27}$$

instead of Eq.(21).

When the ratio of adiabatic capacity γ is larger than 2 ($\gamma>2$), the equation (26) has the solutions of nonlinear oscillation. The period T of the nonlinear oscillation when $\gamma=3$ is given by

$$T=2\pi V_A^2/E^{3/2}\lambda^2\quad, \tag{28}$$

where E is a constant which is related with the initial conditions.

The minimum period T_{min} is given by

$$T_{min}=2\pi\beta^{3/2}\tau_A\quad, \tag{29}$$

where $\beta=8\pi P_0/B_0^2=c_s^2/V_A^2$, and $\tau_A=\lambda/V_A$.

when $\gamma=2$ and $v_A>c_s$, there occur only compressional motions. When E is small, we find

$$a(t)=\left(\frac{9}{2}\right)^{1/3}(v_A^2-c_s^2)^{1/3}\lambda^{-2/3}(t_0-t)^{2/3}\quad. \tag{30}$$

By means of Eq.(30) we obtain the upward flow v_{x0} from eq.(27)

$$v_{x0}=3\left(\frac{2}{9}\right)^{1/3}\frac{V_A^2}{\lambda^{1/3}}\frac{B_n}{B_0}\frac{(t_0-t)^{1/3}}{(V_A^2-c_s^2)^{1/3}}-g_\odot(t_0-t). \tag{31}$$

If we take t_0 as the value when $v_{x0}=0$, we find

$$t_0\simeq 0.84(V_A/c_s)^3\tau_A\quad. \tag{32}$$

The first termof Eq.(31) is always larger than the second term, so that we have upward flows. The time t_0 is the life time of the solar prominence which is order of 3.4 days if we take $V_A/c_2 \simeq 33.3$ an d $\lambda/V_A \simeq$ 10 seconds.

4. Simulation of global structure with solar wind plasmas

We show the results of computer simulation by means of full MHD equations, which includes the expanding supersonic solar wind plasmas. The dipole magnetic field is taken at x= 0.9 $R_\odot$. On the solar surface the flows are taken along the magnetic field. The other boundary conditions are free. Fig. 2 shows the global plasma flow and magnetic field-line pattern. Fig. 3 shows the inflow velocity profile across y axis. Fig. 4 is the density profile and Fig. 5 is the magnetic field B_x. As seen in Fig. 3 and 5 the inflow velocity and magnetic field produced by the sheet current are well described as self-similar solutions which space dependency is proportional to y. The more detail comparison between the theory and the results of simulations will be needed.

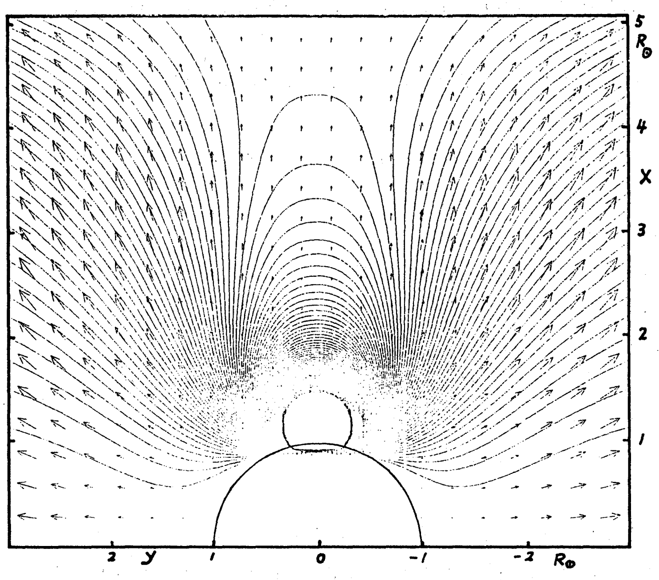

Fig. 2

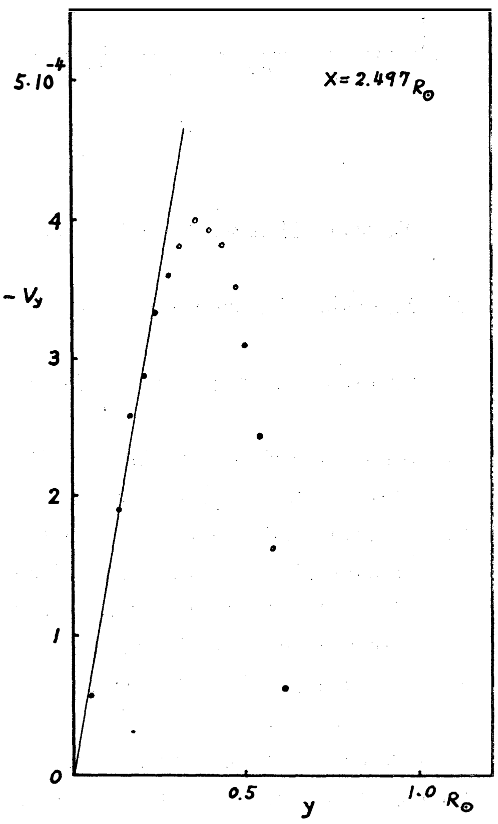

Fig. 3

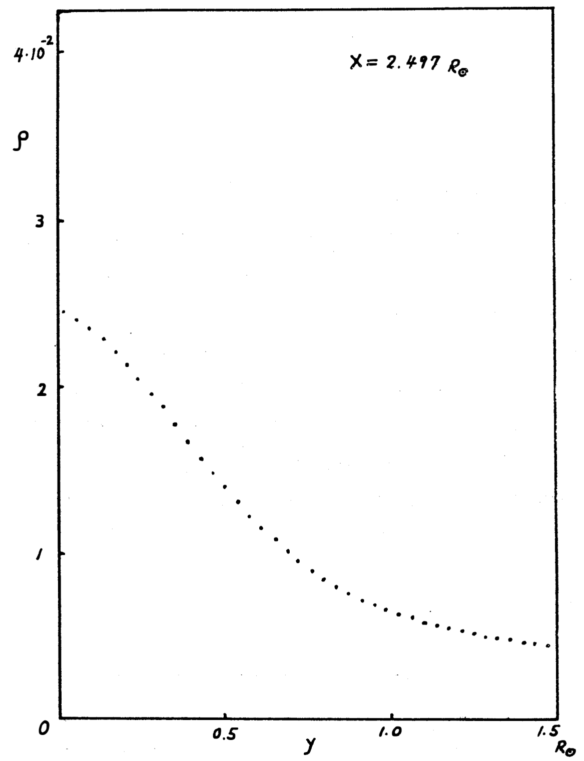

Fig. 4

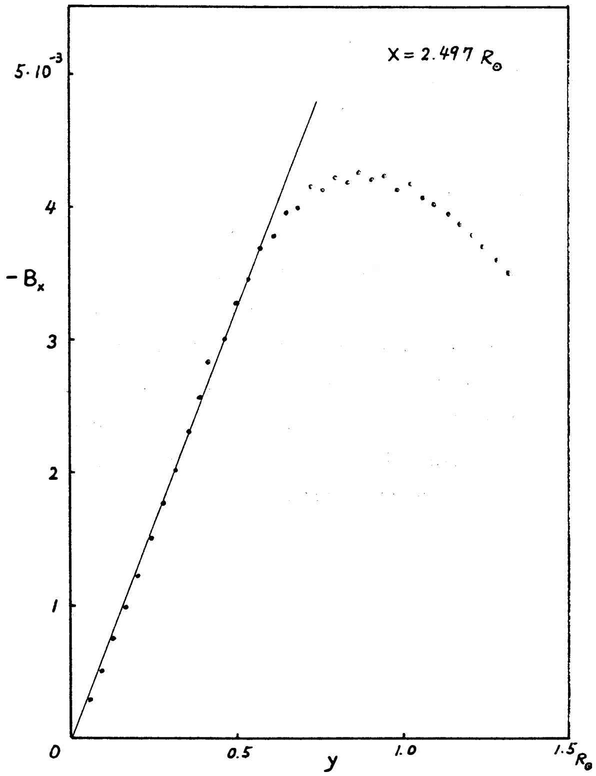

Fig. 5

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